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# النتائج الجديدة الحاصلة من معادلة Dawy – Stewartson

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**الملخص:**

تهدف هذه الدراسة إلى الحصول على حلول الموجات المنتقلة بالإستفادة من أسلوب (G'/G)، وذلك من معادلة «Dawy–Stewartson »، باستخدام برنامج Maple. وبالإستفادة من هذه المعادلة قد تم التوصّل إلى فئة جديدة من حلول الموجات المنتقلة.

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ORIGINAL ARTICLE

# New exact solutions for Davey–Stewartson system

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**Abstract** In this work, we construct the travelling wave solutions by a new application of $(G'/G)$-expansion method to Davey–Stewartson system by using the Maple package. Then new types of exact travelling wave solutions are obtained for these equations.


## 1. Introduction

The theory of nonlinear dispersive wave motion has recently undergone much study. Phenomena in physics and other fields are often described by nonlinear evolution equations and play a crucial role in applied mathematics and physics. Recently, searching for explicit solutions of nonlinear evolution equations by using various methods has become the main goal for many authors, and many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the tanh-function expansion and its various extensions (Parkes and Duffy, 1996; Fan, 2000), the Jacobi elliptic function expansion (Liu et al., 2001; Fu et al., 2001). Very recently, Wang et al. (2008) introduced a new method called the $(G'/G)$-expansion method to look for travelling wave solutions of nonlinear evolution equations (Neirameh et al., 2011). The $(G'/G)$-expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in $(G'/G)$, and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (ODE).

## 2. Description of $(G'/G)$-expansion method

Considering the nonlinear partial differential equation in the form

$$P(u, u_x, u_t, u_y, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

where $u = u(x,y,t)$ is an unknown function, $P$ is a polynomial in $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $(G'/G)$-expansion method.

**Step1**: Combining the independent variables $x$,$y$ and $t$ of Eq. (1) into one variable $\xi = k(x + y - vt)$, we suppose that

$$u(x, y, t) = u(\xi), \quad \xi = k(x + y - vt) \tag{2}$$

The travelling wave variable (2) permits us to reduce Eq. (1) to an ODE for $G = G(\xi)$, namely

$$P(u, ku', -kvu', ku', k^2v^2u'', -vu'', u'', \ldots) = 0 \tag{3}$$

**Step2**: Suppose that the solution of ODE (3) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right) + \ldots, \tag{4}$$

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where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{5}$$

$\alpha_m, \ldots, \lambda$ and $\mu$ are constants to be determined later $\alpha_m \neq 0$, the unwritten part in (4) is also a polynomial in $(G'/G)$, but the degree of which is generally equal to or less than $m-1$, the positive integer $m$ can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

**Step3**: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order $(G'/G)$ together, the left-hand side of Eq. (3) is converted into another polynomial in $(G'/G)$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_m, \ldots, \lambda$ and $\mu$.

**Step4**: Assuming that the constants $\alpha_m, \ldots, \lambda$ and $\mu$ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known to us, then substituting $\alpha_m, \ldots, v$ and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution Eq. (1).

## 3. Application for Davey–Stewartson equations

In this section we consider the Davey–Stewartson equations in the following form

$$\begin{cases} iq_t + \frac{1}{2}\delta^2(q_{xx} + \delta^2 q_{yy}) + \lambda|q|^2 q - \phi_x q = 0 \\ \phi_{xx} - \delta^2\phi_{yy} - 2\lambda(|q|^2)_x = 0. \end{cases} \tag{6}$$

We may choose the following travelling wave transformation:

$$\begin{aligned} q &= u(\xi)e^{i(\alpha x+\beta y+\gamma t)}, \quad \xi = k(x+y-\alpha t) \\ \phi &= v(\xi)e^{i(\alpha x+\beta y+\gamma t)}, \quad \xi = k(x+y-\alpha t) \end{aligned} \tag{7}$$

where $\alpha$, $\beta$, $\gamma$ are arbitrary constants. Equations above become

$$\begin{cases} \frac{1}{2}\delta^2 k^2(1+\delta^2)u'' + k(1 + i\delta^2\alpha + i\delta^4\beta)u' \\ \quad +\left(i\alpha - \frac{1}{2}\delta^2\alpha^2 - \frac{1}{2}i\delta^4\beta^2\right)u + \lambda u^3 - v'u = 0 \\ k^2(1-\delta^2)v'' - 2\lambda(u^2)' = 0. \end{cases}$$

By integrating from the second equation of the system above we have

$$\begin{aligned} v' &= \tfrac{2\lambda}{k^2(1-\delta^2)}u^2 \\ v &= \tfrac{2\lambda}{k^2(1-\delta^2)}\int u^2 d\xi. \end{aligned} \tag{8}$$

And substituting (8) into first Eq. (6) we obtain

$$\begin{aligned} &\tfrac{1}{2}\delta^2 k^4(1-\delta^4)u'' + k^3(1-\delta^2)(1 + i\delta^2\alpha + i\delta^4\beta)u' \\ &+k^2(1-\delta^2)\left(i\alpha - \tfrac{1}{2}\delta^2\alpha^2 - \tfrac{1}{2}i\delta^4\beta^2\right)u - \lambda u^3 = 0 \end{aligned} \tag{9}$$

Suppose that the solution of ODE (9) can be expressed by a polynomial in $(G'/G)$ as follows:

$$u(\xi) = \alpha_m\left(\frac{G'}{G}\right) + \ldots, \tag{10}$$

Considering the homogeneous balance between $u^3$ and $u''$ in (9), we required that $3m = m + 2 \Rightarrow m = 1$. We may still choose the solution of Eq. (10) in the form:

$$u(\xi) = \alpha_1\left(\frac{G'}{G}\right) + \alpha_0 \tag{11}$$

Therefore

$$u^3 = \alpha_1^3\left(\frac{G'}{G}\right)^3 + 3\alpha_1^2\alpha_0\left(\frac{G'}{G}\right)^2 + 3\alpha_1\alpha_0^2\left(\frac{G'}{G}\right) + \alpha_0^3$$

By using (5) it is derived that

$$u' = -\alpha_1\left(\frac{G'}{G}\right)^2 - \alpha_1\lambda\left(\frac{G'}{G}\right) - \alpha_1\mu$$

$$u'' = 2\alpha_1\left(\frac{G'}{G}\right)^3 + 3\alpha_1\lambda\left(\frac{G'}{G}\right)^2 + (\alpha_1\lambda^2 + 2\alpha_1\mu)\left(\frac{G'}{G}\right) + \alpha_1\lambda\mu$$

By substituting relations above into Eq. (9) and collecting all terms with the same power of $(G'/G)$ together, the left-hand side of Eq. (9) is converted into another polynomial in $(G'/G)$. Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for $\alpha_1$, $\alpha_0$, $v$, $\lambda$, $\mu$ and $c$ as follows:

$$\delta^2 k^4(1-\delta^4)\alpha_1 - D\alpha_1^3 = 0$$

$$\frac{3}{2}\delta^2 k^4(1-\delta^4)\alpha_1\lambda - k(1 + i\delta^2\alpha^2 + i\delta^4\beta^2)\alpha_1 - 3\lambda\alpha_1^2\alpha_0 = 0$$

$$\begin{aligned} &\frac{1}{2}\delta^2 k^4(1-\delta^4)(\alpha_1\lambda^2 + 2\alpha_1\mu) - k(1 + i\delta^2\alpha^2 + i\delta^4\beta^2)\alpha_1\lambda \\ &\quad + k^2(1-\delta^2)\left(i\alpha - \frac{1}{2}\delta^2\alpha^2 - \frac{1}{2}\delta^4\beta^2\right)\alpha_1 - 3\lambda\alpha_1\alpha_0^2 = 0 \end{aligned}$$

$$\begin{aligned} &\frac{1}{2}\delta^2 k^4(1-\delta^4)\alpha_1\lambda\mu - k(1 + i\delta^2\alpha^2 + i\delta^4\beta^2)\alpha_1\mu \\ &\quad + k^2(1-\delta^2)\left(i\alpha - \frac{1}{2}\delta^2\alpha^2 - \frac{1}{2}\delta^4\beta^2\right)\alpha_0 - \lambda\alpha_0^3 = 0 \end{aligned}$$

Solving algebraic equations above by the Maple package we have

$$\alpha_1 = \pm\frac{\sqrt{-\lambda(-1+\delta^4)}k^2\delta}{\lambda}$$

$$\alpha_0 = \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}}$$

$$\begin{aligned} \mu = &\frac{1}{12k^6\delta^4(-1+\delta^4)^2}(4 + 8i\delta^2\alpha^2 + 8i\delta^4\beta^2 + 24\delta^4k^3\lambda i\alpha^2 \\ &- 24\delta^8k^3\lambda i\alpha^2 + 24\delta^6k^3\lambda i\beta^2 - 24\delta^{10}k^3\lambda i\beta^2 + 3\delta^4k^6\lambda^2 - 12\delta^2k^4 i\alpha \\ &+ 12\delta^6k^4 i\alpha + 12\delta^4k^4 i\alpha - 12\delta^8k^4 i\alpha + 8i^2\delta^6\alpha^2\beta^2 + 24\delta^2k^3\lambda \\ &- 24\delta^6k^3\lambda - 6\delta^8k^6\lambda^2 + 3\delta^{12}k^6\lambda^2 + 6\delta^4k^4\alpha^2 - 6\delta^8k^4\alpha^2 + 6\delta^6k^4\beta^2 \\ &- 6\delta^{10}k^4\beta^2 - 6\delta^6k^4\alpha^2 + 6\delta^{10}k^4\alpha^2 - 6\delta^8k^4\beta^2 + 6\delta^{12}k^4\beta^2 + 4i^2\delta^4\alpha^4 + 4i^2\delta^8\beta^4). \end{aligned}$$

$\lambda$ is an arbitrary constant. By substituting $\alpha_1$, $\alpha_0$ into Eq. (10) we obtain

$$\begin{aligned} u(\xi) = &\pm\frac{\sqrt{-\lambda(-1+\delta^4)}k^2\delta}{\lambda}\left(\frac{G'}{G}\right) \pm \frac{1}{6} \\ &\times\frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}}, \end{aligned} \tag{12}$$

Substituting the general solutions of Eq. (5) as follows

$$\frac{G'}{G} = \frac{1}{2}\sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right) - \frac{\lambda}{2}.$$

Into (12) we have three types of travelling wave solutions of the $(3+1)$-dimensional Burgers system (6) as follows:

When $\lambda^2 - 4\mu\ 0$

$$u(\xi) = \pm\frac{1}{2}\frac{\sqrt{-\lambda(\lambda^2 - 4\mu)(-1+\delta^4)k^2\delta}}{\lambda} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right) \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}.$$

So

$$q = \left(\pm\frac{1}{2}\frac{\sqrt{-\lambda(\lambda^2-4\mu)(-1+\delta^4)}k^2\delta}{\lambda} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}\right)e^{i(\alpha x+\beta y+\gamma t)}$$

where $C_1$, and $C_2$, are arbitrary constants. So from (8) we obtain $v$ as

$$v = \frac{2\lambda}{k^2(1-\delta^2)}\int\left(\pm\frac{1}{2}\frac{\sqrt{-\lambda(\lambda^2-4\mu)(-1+\delta^4)k^2\delta}}{\lambda} \times \left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}\right)^2$$

$$d\xi = \frac{1}{2}(\lambda^2-4\mu)(1+\delta^2)k^2\delta^2\left(\frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)+1\right)}{\sqrt{\lambda^2-4\mu}} - \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)-1\right)}{\sqrt{\lambda^2-4\mu}} + \frac{2\sqrt{C_1^2-C_2^2}arctg\left(\frac{tgh(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi)C_1+C_2}{\sqrt{C_1^2-C_2^2}}\right)}{\sqrt{\lambda^2-4\mu}C_1}\right.$$

$$+\frac{32C_1 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)C_2^2}{4\sqrt{\lambda^2-4\mu}(C_1^2-C_2^2)\left(C_1 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)\right)}$$

$$-\frac{16C_1^3 tgh(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi)}{4\sqrt{\lambda^2-4\mu}(C_1^2-C_2^2)\left(C_1 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)\right)}$$

$$-\frac{16 tgh(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi)C_2^4}{4\sqrt{\lambda^2-4\mu}(C_1^2-C_2^2)\left(C_1 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)\right)}$$

$$+\frac{32C_1C_2^2 arctg\left(\frac{tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)C_1+C_2}{\sqrt{C_1^2-C_2^2}}\right)}{4(C_1^2-C_2^2)\sqrt{(\lambda^2-4\mu)(C_1^2-C_2^2)}} - \frac{16C_1^3 arctg\left(\frac{tgh\left(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi\right)C_1+C_2}{\sqrt{C_1^2-C_2^2}}\right)}{4(C_1^2-C_2^2)\sqrt{(\lambda^2-4\mu)(C_1^2-C_2^2)}}$$

$$\left.-\frac{16C_2^4 arctg\left(\frac{tgh(\frac{1}{4}\sqrt{\lambda^2-4\mu}\xi)C_1+C_2}{\sqrt{C_1^2-C_2^2}}\right)}{4C_1(C_1^2-C_2^2)\sqrt{(\lambda^2-4\mu)(C_1^2-C_2^2)}}\right)$$

$$+\frac{1}{18k^4\delta^2(1-\delta^2)(1-\delta^4)}(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)\xi + \frac{\lambda^3}{2k^2(1-\delta^2)}\xi$$

$$\pm\frac{2k}{3(1-\delta^2)}(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)\times\ln(C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi) \mp \frac{\lambda}{(1-\delta^2)}\sqrt{-\lambda(\lambda^2-4\mu)(-1+\delta^4)}\delta$$

$$\times\frac{\ln(C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi)}{\sqrt{\lambda^2-4\mu}} \mp\frac{\lambda^2(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{3k^3\delta(1-\delta^2)\sqrt{(1-\delta^4)\lambda}}\xi.$$

$$\phi = v(\xi)e^{i(\alpha x+\beta y+\gamma t)}.$$

In particular, if $C_1 \neq 0$, $C_2 = 0$, $\lambda \prec 0$, $\mu = 0$, $u$, become

$$u(\xi) = \pm\frac{1}{2}\sqrt{-\lambda(-1+\delta^4)}k^2\delta tgh\frac{1}{2}\lambda\xi \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}.$$

Hence

$$q = \left(\pm\frac{1}{2}\sqrt{-\lambda(-1+\delta^4)}k^2\delta tgh\frac{1}{2}\lambda\xi \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}\right)e^{i(\alpha x+\beta y+\gamma t)}.$$

and

$$v = \frac{2\lambda}{k^2(1-\delta^2)}\int\left(\pm\frac{1}{2}\sqrt{-\lambda(-1+\delta^4)}k^2\delta tgh\frac{1}{2}\lambda\xi \pm\frac{1}{6}\frac{k(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}\right)^2 d\xi$$

$$= \lambda k^2\delta^2(1+\delta^2)\left(\frac{1}{2}\lambda\xi - tgh\left(\frac{1}{2}\lambda\xi\right)\right)$$

$$+\frac{1}{18}\frac{(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)^2}{\delta^2k^4(1-\delta^2)^2(1+\delta^2)}\xi + \frac{\lambda^3}{2k^2(1-\delta^2)}\xi$$

$$\pm\frac{(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{6k(1-\delta^2)}\ln\left(\cosh\left(\frac{1}{2}\lambda\xi\right)\right)$$

$$\mp\frac{2\lambda\delta\sqrt{\lambda(1-\delta^4)}}{(1-\delta^2)}\ln\left(\cosh\left(\frac{1}{2}\lambda\xi\right)\right)$$

$$\mp\frac{\lambda^2(3\delta^2k^3\lambda-3\delta^6k^3\lambda+2+2i\delta^2\alpha^2+2i\delta^4\beta^2)}{3\delta k^3(1-\delta^2)\sqrt{(1-\delta^4)\lambda}}\xi.$$

In this case and the following cases $v$ is calculated as above.

When $\lambda^2 - 4\mu \prec 0$

$$
\begin{aligned}
u(\xi) = &\pm \frac{1}{2} \frac{\sqrt{-\lambda(\lambda^2-4\mu)(-1+\delta^4)}k^2\delta}{\lambda} \\
&\times \left( \frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} \right) \\
&\pm \frac{1}{6} \frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2}
\end{aligned}
$$

So we have $v$ as

$$
\begin{aligned}
v = &\frac{2\lambda}{k^2(1-\delta^2)} \int \Bigg( \pm \frac{1}{2} \frac{\sqrt{-\lambda(\lambda^2-4\mu)(-1+\delta^4)}k^2\delta}{\lambda} \\
&\times \left( \frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} \right) \\
&\pm \frac{1}{6} \frac{k(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{\sqrt{-\delta^2k^4(-1+\delta^4)\lambda}} - \frac{\lambda}{2} \Bigg)^2 d\xi \\
= &\frac{k^2\delta^2(\lambda^2-4\mu)(1+\delta^2)}{\sqrt{4\mu-\lambda^2}C_1(C_1^2-C_2^2)^2\left(C_1 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)\right)} \\
&\times \left(4C_1^5 \ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) - 1\right) C_2\right. \\
&+ \frac{(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)^2}{18\delta^2k^4(1-\delta^2)^2(1+\delta^2)}\xi \\
&+ \frac{\lambda^3}{2k^2(1-\delta^2)}\xi \\
&\pm \frac{k}{3(1-\delta^2)}(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2) \\
&\times \left( \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) + 1\right)C_1}{(C_1-C_2)\sqrt{4\mu-\lambda^2}} \right. \\
&+ \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) + 1\right)C_2}{(C_1-C_2)\sqrt{4\mu-\lambda^2}} \\
&- \frac{2\ln\left(C_1 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)\right)C_1^2}{(C_1^2-C_2^2)\sqrt{4\mu-\lambda^2}} \\
&- \frac{2\ln\left(C_1 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)\right)C_2^2}{(C_1^2-C_2^2)\sqrt{4\mu-\lambda^2}} \\
&\left. + \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) - 1\right)C_1}{(C_1+C_2)\sqrt{4\mu-\lambda^2}} + \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) - 1\right)C_2}{(C_1+C_2)\sqrt{4\mu-\lambda^2}} \right) \\
&\mp \frac{\lambda\delta\sqrt{\lambda(\lambda^2-4\mu)(1-\delta^4)}}{(1-\delta^2)} \left( \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) + 1\right)C_1}{(C_1-C_2)\sqrt{4\mu-\lambda^2}} \right. \\
&+ \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) + 1\right)C_2}{(C_1-C_2)\sqrt{4\mu-\lambda^2}} \\
&- \frac{2\ln\left(C_1 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)\right)C_1^2}{(C_1^2-C_2^2)\sqrt{4\mu-\lambda^2}} \\
&- \frac{2\ln\left(C_1 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)^2 + C_1 + 2C_2 tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right)\right)C_2^2}{(C_1^2-C_2^2)\sqrt{4\mu-\lambda^2}} \\
&\left. + \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) - 1\right)C_1}{(C_1+C_2)\sqrt{4\mu-\lambda^2}} + \frac{2\ln\left(tgh\left(\frac{1}{4}\sqrt{4\mu-\lambda^2}\xi\right) - 1\right)C_2}{(C_1+C_2)\sqrt{4\mu-\lambda^2}} \right) \\
&\mp \frac{\lambda^2(3\delta^2k^3\lambda - 3\delta^6k^3\lambda + 2 + 2i\delta^2\alpha^2 + 2i\delta^4\beta^2)}{3k^3\delta(1-\delta^2)\sqrt{(1-\delta^4)\lambda}}\xi.
\end{aligned}
$$

When $\lambda^2 - 4\mu = 0$

$$
u(\xi) = \pm \frac{\sqrt{-\lambda(-1+\delta^4)}k^2\delta C_2}{\lambda(C_1+C_2\xi)},
$$

$$
v = \frac{-2}{(C_1+C_2\xi)}(1+\delta^2)k^2\delta^2.
$$

So

$$
q = \pm \frac{\sqrt{-\lambda(-1+\delta^4)}k^2\delta C_2}{\lambda(C_1+C_2\xi)} e^{i(\alpha x+\beta y+\gamma t)},
$$

$$
\phi = \frac{-2(1+\delta^2)k^2\delta^2}{(C_1+C_2\xi)} e^{i(\alpha x+\beta y+\gamma t)}.
$$

Where $C_1$ and $C_2$ are arbitrary constants.

## 4. Conclusion

In this paper, we explore a new application of the $(G'/G)$ expansion method and obtain new types of exact travelling wave solutions to the Davey–Stewartson equations. This paper presents a wider applicability for handling nonlinear evolution equations using the $(G'/G)$ expansion method. The new type of exact travelling wave solution obtained in this paper might have a significant impact on future researches.

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