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Four-level atom via intensity dependent atom-field coupling

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ABSTRACT:

We study the interaction of a four level-atom with a single mode in a cavity, involving intensity dependent coupling with additional a kirr-like medium. The wave function is obtained when the atom is initially in superposition of its upper and ground states and the field in coherent state. We evaluate the Mandel Q parameter, phase probability distribution and the field entropy. The influence of the Kerr medium, the detuning and the presence of the intensity dependent coupling on their evolution was discussed.

INTRODUCTION

The JCM model via intensity-dependent coupling is a theoretical model proposed by Buck and Sukumar to describe the dependence of the interaction between light and atoms on light intensity. This mode not only solves but also displays various quantum effects such as the phenomena of collapses and revivals of atomic inversion, the squeezing effect and phase characteristics of light field [Buzek 1989, 1989b; Oliveira et al 2001; Katriel et al 1991, Obada et al 2007; Abdel-Aty 2007; Obada et al 1998]. Various generalizations have been proposed to modify the JCM. Multi-photon transition, intensity-dependent coupling parameters, multimode interaction, three-level atom and one(two) modes [Obada et al 1998], and N-level atom and (N-1) modes, with single photon transition and multiphoton transitions and intensity dependent coupling parameters are some of the modifications presented [Buzek 1989b].

Many authors have also investigated the JCM in different contexts and predicted new interesting results. For instance, the phase and coherent properties of the field in the non-resonant JCM, and Obada et al 1989 have verified the classical beat phenomena in the JCM with a Kerr nonlinearity for the field initially prepared in any super-position of Fock states. They have studied the time evolution and squeezing properties of the deformed JCM which corresponds to the usual model with intensity dependent coupling controlled by two additional parameters to be determined by experiment. Adopting the framework of non-resonant JCM, Oliveira et al [Oliveira et al 2001] has investigated the non-classical statistical properties of a coherent states defined in a finite-dimensional Hilbert space.

In this paper, we study a four-level atom interaction with a single mode in presence of the intensity dependent coupling. The organization of this paper is as follows: In section 2, we introduce our Hamiltonian model and give exact expression of wave function $|\Psi(t)\rangle$. In section 3, we employ the analytical results obtained in section 2 to investigate the properties of Mandel

Q parameter. We devote section 4 to discuss the phase probability distribution of cavity field due to the Pegg-Barnett phase formalism. Finally, we give our conclusion in the end of this paper.

THE MODEL

Consider the four -level atom system with states |1>, |2>, |3> and |4> whose energy levels are ω_1,ω_2,ω_3 and $\omega 4$, respectively, coupled to a single mode cavity field with Rabi frequency Ω. This configuration consists of a three-level in in a cascade scheme with the transitions |1>⟶ |2> and |2>⟶ |3> with the detuning Δ_1 and Δ_2. The ground level |1> is coupled to an upper fourth level with detuning $\Delta 3$. In the rotating wave approximation, the Hamiltonian for a single mode electromagnetic field interaction with a four-level atom via intensity dependent coupling and in the presence of a Kerr-like medium is written as

$$\begin{aligned} \hat{H} &= \sum_{j=1}^{4} \omega_j \sigma_{jj} + \Omega \hat{a}^\dagger \hat{a} + \chi \hat{a}^{\dagger 2} \hat{a}^2 + \lambda_1 \left(\hat{R} \sigma_{12} + \hat{R}^\dagger \sigma_{21} \right) \\ &+ \lambda_2 \left(\hat{R} \sigma_{23} + \hat{R}^\dagger \sigma_{32} \right) + \lambda_3 \left(\hat{R} \sigma_{41} + \hat{R}^\dagger \sigma_{14} \right), \end{aligned} \quad (1)$$

where, σ_{ij} are the lowering and raising operators between levels i and j defined by σ_{ij} =|i><j|, i,j=1,2,3,4, a and $a^\dagger$ are the annihilation and creation operators of the field mode, χ denotes the dispersive part of the third-order nonlinearity of the Kerr-like medium, λ_J(j=1,2,3) are the usual coupling constants between the field and the atom. The operators R and $R^\dagger$ are constructed from the single-mode field operators a and $a^\dagger$ are defined as

$$\hat{R} = \hat{a} f(\hat{n}), \qquad \hat{R}^\dagger = f(\hat{n}) \hat{a}^\dagger, \quad (2)$$

where $n=a^\dagger a$ is the photon number operator and f(n) represents an arbitrary intensity dependent atom-field coupling. for this Hamiltonian system we obtain the conservations of atomic probability and excitation number of the single mode. Using these constants of motion, we see that the interaction Hamiltonian of the considered atomic system can be written as

$$\begin{aligned} \hat{H}_I &= \Delta_1 \sigma_{22} + (\Delta_1 + \Delta_2) \sigma_{33} + \Delta_3 \sigma_{44} + \chi \hat{a}^{\dagger 2} \hat{a}^2 \\ &+ \lambda_1 \left(\hat{R} \sigma_{12} + \hat{R}^\dagger \sigma_{21} \right) + \lambda_2 \left(\hat{R} \sigma_{23} + \hat{R}^\dagger \sigma_{32} \right) + \lambda_3 \left(\hat{R} \sigma_{41} + \hat{R}^\dagger \sigma_{14} \right), \end{aligned} \quad (3)$$

where, the detuning parameters are given by

$$\begin{aligned} \Delta_1 &= \omega_2 - \omega_1 - \Omega, \\ \Delta_2 &= \omega_3 - \omega_2 - \Omega, \\ \Delta_3 &= \omega_4 - \omega_1 - \Omega \end{aligned} \quad (4)$$

The coupling atom-field wave function of the system $|\Psi(t)\rangle$ at an arbitrary time t, can be written in the form

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} q_n \left\{ A(t)\,|1,n\rangle + B(t)\,|2,n-1\rangle + C(t)\,|3,n-2\rangle + D(t)\,|4,n-1\rangle \right\}, \qquad (5)$$

where, q_n describes the amplitude of the state $|n\rangle$ of the field mode and the quantities A, B, C and D are the probability amplitudes.

Using the Schrödinger equation $i\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}_1 |\Psi(t)\rangle$ leads to the following coupling system of differential equations

$$\begin{aligned} i\dot{A} &= \upsilon_1 A + f_1 B + f_3 D, \qquad (6)\\ i\dot{B} &= \upsilon_2 A + (\upsilon_2 + \Delta_1)B + f_2 C,\\ i\dot{C} &= f_2 B + (\upsilon_3 + \Delta_1 + \Delta_2)C,\\ i\dot{D} &= f_3 A + (\upsilon_2 + \Delta_3)D, \end{aligned}$$

where,
$\upsilon_1 = \chi n(n-1), \upsilon_2 = \chi(n-1)(n-2)$ and $\upsilon_3 = \chi(n-2)(n-3)$,

with
$f_1 = \lambda_1\sqrt{n}f(n), f_2 = \lambda_2\sqrt{n-1}f(n-1)$ and $f_3 = \lambda_3\sqrt{n}f(n)$

The solution of the coupled system of differential equation (Eq. 6), can be obtained as;(we set $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$)

$$\begin{aligned} A &= -\sum_{j=1}^{4} W_j \left(\mu_j + \upsilon_2 + \Delta\right) exp[i\mu_j t], \qquad (7)\\ B &= \sum_{j=1}^{4} \frac{W_j}{f_1} \left(\mu_j^2 + y_1\mu_j + y_2\right) exp[i\mu_j t],\\ C &= -\sum_{j=1}^{4} \frac{W_j}{f_1 f_2} \left(\mu_j^3 + z_1\mu_j^2 + z_2\mu_j + z_3\right) exp[i\mu_j t],\\ D &= \sum_{j=1}^{4} f_3 W_j exp[i\mu_j t], \end{aligned}$$

where,

$$\begin{aligned} y_1 &= \upsilon_1+\upsilon_2+\Delta, \quad y_2=\upsilon_1(\upsilon_2+\Delta)-f_3^2, \quad z_1=y_1+\upsilon_2+\Delta, \\ z_2 &= y_2+y_1(\upsilon_2+\Delta)-f_1^2, \quad z_3=y_2(\upsilon_2+\Delta)-f_1^2(\upsilon_2+\Delta), \end{aligned} \tag{8}$$

and μ_j satisfies the fourth-order equation

$$\mu_j^4+\Gamma_1\mu_j^3+\Gamma_2\mu_j^2+\Gamma_3\mu_j+\Gamma_4=0, \tag{9}$$

with,

$$\begin{aligned} \Gamma_1 &= z_1+\upsilon_3+2\Delta, \quad \Gamma_2=z_2+z_1(\upsilon_3+2\Delta)-f_2^2, \\ \Gamma_3 &= z_3+z_2(\upsilon_3+2\Delta)-y_1f_2^2, \quad \Gamma_4=z_3(\upsilon_3+2\Delta)-y_2f_2^2. \end{aligned} \tag{10}$$

The coefficients W_j in Eq. (7), depend on the initial conditions of the atomic system, we assume the initial state of the system as

$$|\Psi(0)\rangle=\sum_{n=0}^{\infty}q_n\left\{\cos\left(\frac{\eta}{2}\right)|1\rangle\otimes|n\rangle+exp[i\phi]\sin\left(\frac{\eta}{2}\right)|4\rangle\otimes|n\rangle\right\}, \tag{11}$$

where, the atom enters the cavity in a coherent super position and finds there a coherent field state $|\alpha\rangle$. Also, the initial state of the field is given by

$$|\alpha\rangle=\sum_{n=0}^{\infty}q_n|n\rangle, \qquad q_n=exp[-\overline{n}/2]\sqrt{\overline{n}^n/n!}, \tag{12}$$

where, $\overline{n}$ is the initial mean number of photons for the field mode.
Using the above initial conditions, the coefficients W_j are given by

$$W_j=\frac{1}{\mu_{jm}\mu_{jl}\mu_{jn}}\left\{-r_1\mu_l\mu_m\mu_n+r_2(\mu_l\mu_m+\mu_l\mu_n+\mu_m\mu_n)-r_3(\mu_l+\mu_m+\mu_n)+r_4\right\}, \tag{13}$$

where, $\mu_{ij}=\mu_i-\mu_j$ and $j\neq l\neq m\neq n=1,2,3,4$ with,

$$\begin{aligned} r_1 &= \frac{1}{f_3}\sin\left(\frac{\eta}{2}\right), \quad r_2=-\cos\left(\frac{\eta}{2}\right)-r_1(\upsilon_3+\Delta), \\ r_3 &= -y_1r_2-y_2r_1, \quad r_4=-z_3r_1-z_2r_2-z_1r_3. \end{aligned} \tag{14}$$

MANDEL Q PARAMETER

One of the most remarkable neoclassical effect is the sub-Poissonian photon statistics of the state. To determine such effects we consider the Mandel Q parameter defined by [Mandel (1979)]

$$Q = \frac{\langle \hat{n}^2 \rangle - (\langle \hat{n} \rangle)^2}{, \langle \hat{n} \rangle} - 1, \qquad (15)$$

For -1 <=Q< 0 (Q>0), the statistics is sub-Poissonian(super-Poissonian); Q=0 stands for Poissnian statistics. For our system, we see that

$$\begin{aligned} \langle \hat{n} \rangle &= \overline{n} - 1 + \sum p_n \left(|A|^2 - |C|^2 \right), \\ \langle \hat{n}^2 \rangle &= \overline{n}^2 - \overline{n} + 1 + \sum p_n \left((2n-1)|A|^2 - (2n-3)|C|^2 \right), \end{aligned} \qquad (16)$$

where, $p_n = |q_n|^2$, $q_n = e^{-|\alpha|^2} \frac{\alpha^n}{\sqrt{n}}$ and $|\alpha|^2$ is the initial photon distribution, A and B are given by Eq. (7) .

To discuss the influence of the intensity dependent atom-field coupling f(n), the Kerr medium and the detuning on the evolution of Mandel Q parameter against the scaled time λt, we consider the atom to be initially prepared in state |1>, i.e. we take η=0, and the field to be initially in a coherent state. Also, we consider fixed value of the initial mean number of photons for the field mode n=10, and the relative phase φ=0. As shown in Fig. 1, corresponding to the absence of the intensity dependent coupling (f(n)=1) and Fig. 2, corresponding to the presence of the intensity dependent coupling (f(n)=√n). We see that for exact resonant and the absence of the Kerr medium (Figs.1a and 2a) the Mandel Q parameter is super-Poissonian, As soon as the Kerr medium takes place (Figs.1b and 2b) the statistics settle to the positive values, i.e. the statistics settle to the super-Poissonion. On the other hand, for the effect of the detuning (Figs.1c and 2c) the fluctuations of Q parameter settle a bit to the positive values, i.e. the statistics of the cavity field settle to the super-Poissnion.

PHASE PROBABILITY DISTRIBUTION

Now we give briefly the relations of the Pegg-Barnett formalism [Barnett, Pegg (1986); Pegg, Barnett (1988)], which will be used throughout the paper. This formalism is based on introducing a finite (s+1)-dimensional space Ψspanned by the number states |0>, |1>, ...,|s>. The expectation values of the different hermitian phase operators can be evaluated in the finite dimensional space Ψ and at the final stage the limit s→∞ is taken. The Hermitian phase operator of the single-mode case is defined as

$$\Phi_\theta = \sum_{m=0}^{s} \theta_m \left| \theta_m \right\rangle \left\langle \theta_m \right| , \qquad (17)$$

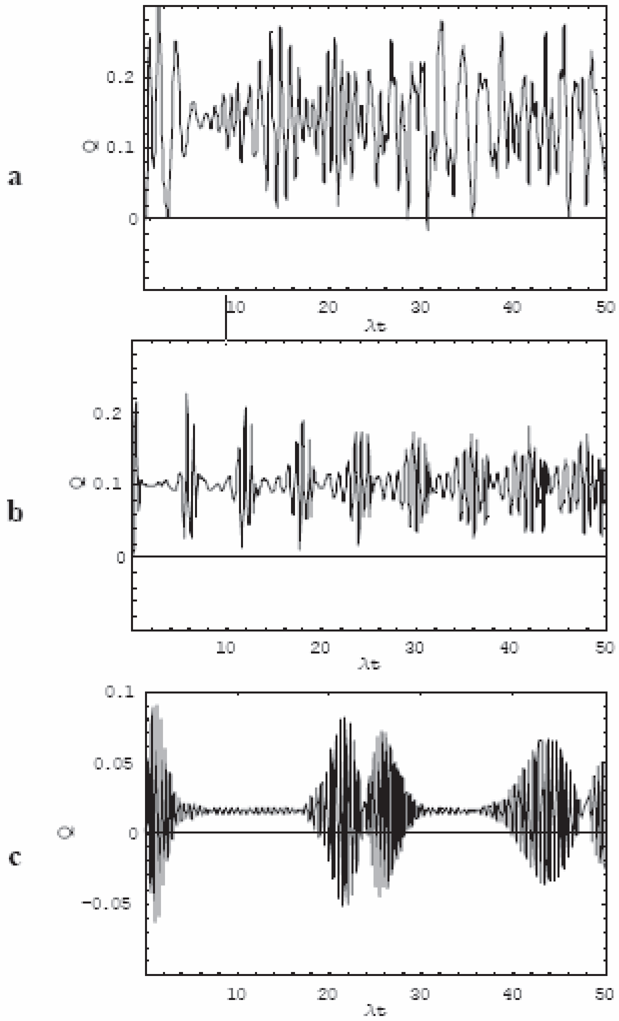

Fig.1: The evolution of Mandel Q parameter as a function of scaled time λt in the absence of the intensity dependent coupling $f(n) = 1$. Calculations assume that the atom prepared initially in state $|1\rangle$ and the field in the coherent state with the initial average photon numbers $\bar{n} = 10$. In (a: $\chi/\lambda = \Delta/\lambda = 0.0$), (b: $\chi/\lambda = 0.5$ and $\Delta/\lambda = 0$) and (c: $\chi/\lambda = 0.0$ and $\Delta/\lambda = 10$).

where the states $|\theta_m\rangle$ are the eigenstates of the phase operator of (17) and they from a complete orthonormal basis of s+1 states in Ψ. The Pegg and Barnett formalism allows to define phase distribution as,

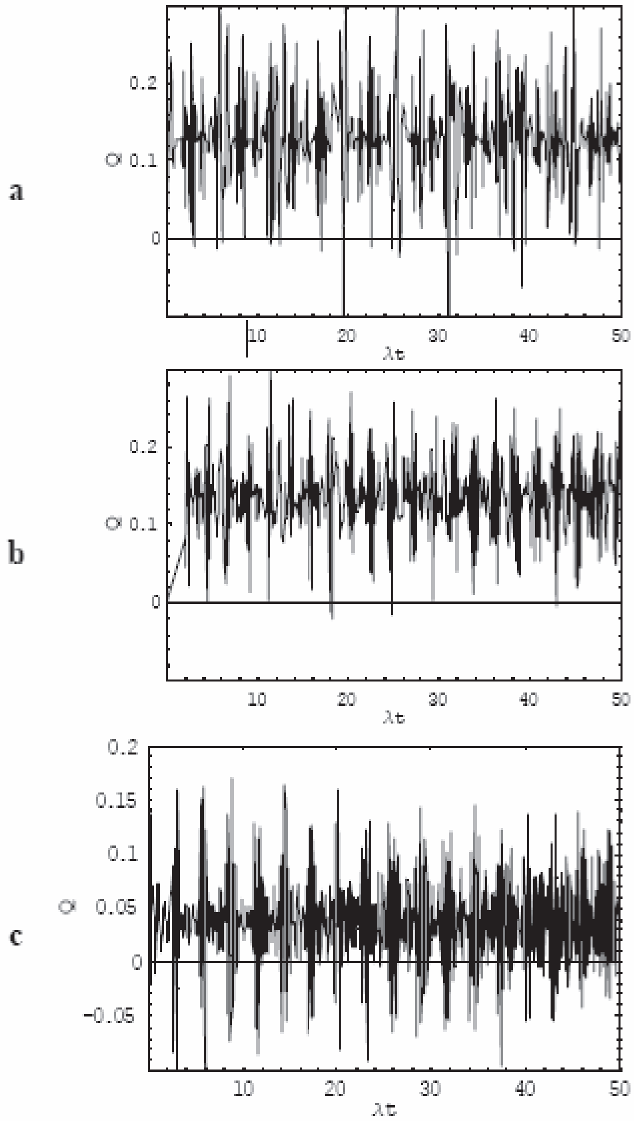

Fig.2: The evolution of Mandel Q parameter as a function of scaled time λt in the presence of the intensity dependent coupling $f(n)=\sqrt{n}$. Calculations assume that the atom prepared initially in state $|1\rangle$ and the field in the coherent state with the initial average photon numbers $\overline{n}=10$. Where in (a: $\chi/\lambda=\Delta/\lambda=0.0$), (b: $\chi/\lambda=0.5$ and $\Delta/\lambda=0$) and (c: $\chi/\lambda=0.0$ and $\Delta/\lambda=10$).

$$P(\theta_m) = |\langle \theta_m | \psi(t) \rangle|^2 , \tag{18}$$

By using Eqs. (5) and (18) the phase probability distribution is given by

$$p(\theta, t) = \frac{1}{2\pi} \left(1 + 2 \sum_{n>l} [R_1 \cos((n-l)\theta) + R_2 \sin((n-l)\theta)] \right) \tag{19}$$

where

$$\begin{aligned} R_1 &= \sum_{z=1}^{4} \sum_{j=1}^{4} \cos \left[t \left(\mu_j(n) - \mu_z(l) \right) \right] \sum_{\nu=1}^{4} Y_{\nu j}(n) Y_{\nu z}(l), \\ R_2 &= \sum_{z=1}^{4} \sum_{j=1}^{4} \sin \left[t \left(\mu_j(n) - \mu_z(l) \right) \right] \sum_{\nu=1}^{4} Y_{\nu j}(n) Y_{\nu z}(l), \end{aligned} \tag{20}$$

In Fig. 3, we have plotted the phase probability distribution as a function of θ and scaled time λt for the same corresponding data used in Fig. 1. In the absence of the intensity dependent atom-field coupling f(n)=1 (left plot), and for exact resonant and the absence of the Kerr medium it is remarked that P(θ,t) exhibits symmetric splitting as λt varies as shown in Fig. 2a. When λt=0, P(θ,t) has a single peak structure corresponding to the initial coherent state. The peaks are symmetric about θ=0 so that the mean phase always remains equal to zero. The time behavior of the phase probability distribution carries some information about the collapse and revival of Rabi oscillations. When the phase peaks are well separated the Rabi oscillations collapse and each time as the peaks meet they produce a revival, see Fig. 3a. It is observed that the symmetry shown in the absence of the nonlinear medium for the phase distribution is no longer present once the nonlinear medium is added (see Fig. 3b). As soon as the detuning is considered non zero (see Fig. 3c) we have the same behavior of the phase probability distribution as in Fig. 3a, but with damping of one peak of the phase distribution. When we consider the intensity coupling effect (f(n)=√n), the number of oscillations becomes more and more at the same time, also P(θ,t) exhibits symmetric splitting in the case (χ/λ=Δ/λ=0 see Fig. 4a). When the Kerr medium takes place, as well as, the detuning, it is almost keep a small amplitude compared with the other peaks.

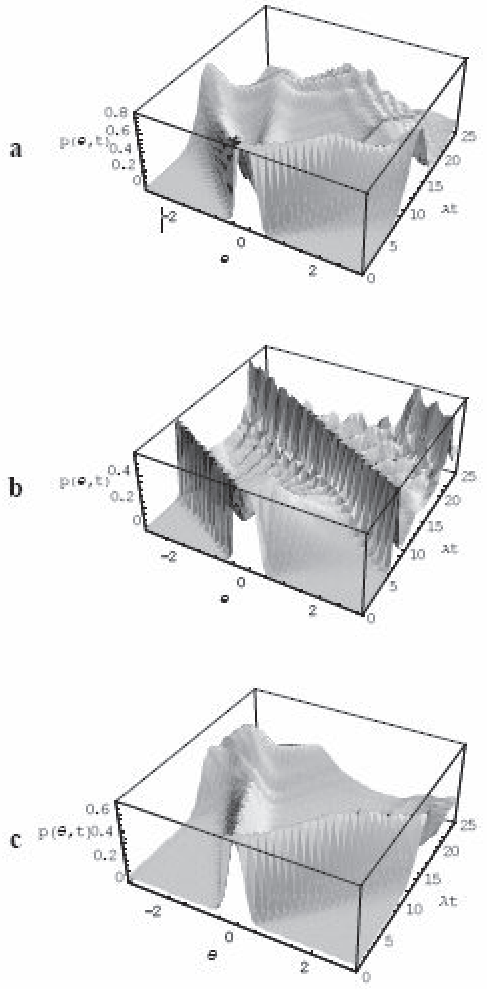

Fig.3: Plot of the phase probability distribution of the cavity field as a function of θ and scaled time λt in the absence of the intensity dependent coupling $f(n)=1$. Calculations assume that the atom prepared initially in state $|1\rangle$ and the field in the coherent state with the initial average photon numbers $\overline{n}=10$. In (a:. $\chi/\lambda=\Delta/\lambda=0.0$), (b: $\chi/\lambda=0.01$ and $\Delta/\lambda=0$) and (c: $\chi/\lambda=0.0$ and $\Delta/\lambda=5$).

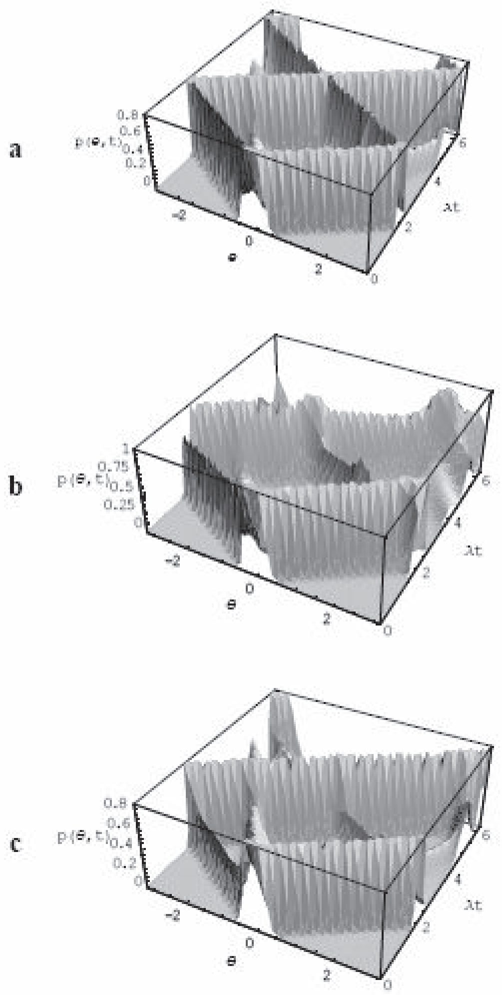

Fig.4: Plot of the phase probability distribution of the cavity field as a function of θ and scaled time λt in the presence of the intensity dependent coupling $f(n)=\sqrt{n}$. Calculations assume that the atom prepared initially in state $|1\rangle$ and the field in the coherent state with the initial average photon numbers $\bar{n}=10$. In (a:. $\chi/\lambda=\Delta/\lambda=0.0$), (b: $\chi/\lambda=0.01$ and $\Delta/\lambda=0$) and (c: $\chi/\lambda=0.0$ and $\Delta/\lambda=5$).

CONCLUSION

In this paper we have considered a Hamiltonian model which represents the interaction between the cavity field and a four-level atom with intensity dependent coupling, taking into account the effect of Kerr-like medium and the detuning. Exact solution for the wave function in Schrődinger picture is given from which we have managed to discuss the Mandel Q parameter behavior and phase probability distribution.

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ذرةالمستوى الرباعي من خلال أقتراب ذرة-المجال معتمدة الشدة

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قمنا بدراسة نمط واحد حول تفاعلات ذرة المستوى الرباعي في فجوة متضمنة أقتران معتمد الشدة بوسط أحنا في شبية بالكيير Kerr ، وتم الحصول على دالة الموجة عندما تكون الذرة متراكبة مبدئياً لحالاتها الدنيا والسفلى ومجال في حالة متماسكة. قومنا مؤشر مندل والتوزيع الاحتمالي لمرحلة ومجال الانتروبي. كما تمت مناقشة تأثير وسط كيير.