Analyzing the Impact of Discretization Techniques on Real Time Simulation of DC Servomotor Using FPGA

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Abstract: This paper explains the strategies to create a hardware-based real time model of DC servomotor by utilizing the FPGA technology that can accurately simulate the behavior of the servomotor in real-time. Such FPGA based hardware model is useful for testing control algorithms, validating designs, and optimizing performance for various applications because of its reconfiguration capabilities. Continuous-time model is discretized using both Backward Euler (BE) and Trapezoidal (TRZ) methods for the real-time implementation on FPGA. The discretized models are coded using 'C', converted to hardware descriptive language using Vivado high level synthesis tools, and the performance is analyzed with change in step-size by comparing with the transfer function (TF) model. With 100 µsec step-size, TRZ response is found to be matching with the TF model, however, a step-size of 0.6 µsec was required for the BE. Also analyzed the closed loop speed control performance of the hardware-based real time DC servomotor models with discrete PID controller, again by varying the step-size. Both the BE and TRZ models could track the reference speed within 2 msec, because of the PID controller, however faster dynamics was observed in case of TRZ as compared to BE, especially with larger step-size. These analysis shows the effect of step-size and the discretization technique for the real-time modeling, however, with a suitable selected values, the developed FPGA model can be utilized efficiently for the development of suitable control algorithm.

Keywords: Backward Euler, DC Servomotor, FPGA, High Level Synthesis, Discrete PID Controller, Real-Time Simulation, Trapezoidal, Vivado

INTRODUCTION

DC servo motors are versatile devices widely used in various applications across a wide range of industries and technologies [1]. Its crucial role in robotic systems, computer numerical control machines, medical devices, aerospace applications, solar tracking systems, automotive systems etc. highlight its versatility and precision control capabilities. Its various applications in electric vehicles such as power steering, power brake, power windows, power seating, cooling fans, heating, ventilation, and air conditioning systems, etc. contributes to improved energy efficiency, enhanced vehicle control, and increased overall performance. As technology continues to advance, the role of these motors in various applications may expand even further.

Research on DC servomotors spans various aspects, including control algorithms, modelling and simulation, real-time implementations, hardware implementations, optimization techniques, with applications in different fields. Real-time simulation of DC servomotors is a crucial step in the development and deployment of control systems. It allows rapid control prototyping and testing, hardware in the loop and software in the loop testing etc. without the need for complete physical hardware setups [2]-[4]. It enables thorough testing, analysis, and optimization of control algorithms, ultimately leading to more reliable and efficient real-world implementations.

Simulating a DC servomotor in real-time involves using software tools and models to replicate the behaviour of the motor and its control system. Various papers are available in the literature, with different software tools, to investigate the impact of the control algorithms to enhance the precision and responsiveness of DC servomotors. A LabVIEW based PID controller for the position control of DC servomotor is explained in detail in paper [5]. Similar approach with LabVIEW, but a fuzzy-PI controller is developed in the paper [6] to control the speed of the DC servomotor.

The paper in [7] presents a MATLAB based simulation model for the DC servomotor and PID controller, to enhance the performance of the motor. Application of such MATLAB based motor with PID controller to humanoid robotic arm is explained in [8]. The tuning of such PID controller using genetic algorithm for the position and speed control of the MATLAB based DC servomotor is presented in [9] and [10]. Whereas, the papers [11] and [12] explains the advantages of other control techniques such as fuzzy logic [11], ANN [12] and a hybrid fuzzy and position-velocity controller [13] for the speed control of DC servomotor, using MATLAB /Simulink models.

While LabVIEW [5], [6], and MATLAB/Simulink [7]-[13] models are powerful tools for designing and simulating control algorithms, transitioning to 'C' code is often necessary for real-world deployment on embedded systems or microcontrollers [14],[15]. Using high level synthesis (HLS) tools, ‘C’ code can be converted to hardware descriptive language (HDL) which is used for the implementation on Field Programmable Gate Array (FPGA).

FPGAs provide hardware-level parallelism and can execute real time simulations of systems and control algorithms with minimal latency [16],[17]. This is crucial in real-time applications where rapid response times are essential [18-21], such as robotics and automation. Prototyping systems using FPGA can be a cost-effective solution compared to designing custom ASICs (Application-Specific Integrated Circuits). It offers a balance between performance and cost, as compared to ASICs, making them suitable for iterative development [22].

DC servomotors often require integration with sensors for feedback on position, speed, or torque. FPGA-based prototypes facilitate the integration of these sensors in real-time, helping to validate and refine the feedback control loop. Thus well-suited for hardware-in-the-loop (HIL) testing [23], [24] where the DC servomotor in FPGA can be connected to a physical or a simulated control system. It is valuable for educational purposes, providing hands-on experience in control system design.

Such prototyping also allows engineers and researchers to develop and optimize control algorithms for the DC servomotor, with different specifications, in a real-time environment. This is feasible because FPGAs are versatile and can be easily reprogrammable, hence the same FPGA platform can be repurposed to adapt to different DC servomotor applications such as control of robotic arm, conveyor system, or any other automated process. While FPGA-based implementations offer these advantages, in general, it requires expertise in hardware design and FPGA programming. However, HLS tools will facilitate the coding with high level languages such as ‘C’.

DC servomotor can be modelled in real time on FPGA using ‘C' code by replacing the continuous system into its discretized form [25]-[27]. There are various discretization methods available in literature [28]- [30] such as Backward Euler (BE) method, Trapezoidal (TRZ) method etc. Ultimately, it's crucial to evaluate the performance of available models in the specific application context to determine which discretization method better meets the accuracy and stability requirements of the system. The paper [31] explains in detail the discretization issue with the BE method for the real time simulation of induction motor, especially with the steady state error.

TRZ method is a second order approximation thus can be computationally more efficient and accurate than BE, especially for systems where the solution exhibits rapid changes [32]. In the case of DC servo motors, because of its stiff dynamics, the solution changes rapidly [1], thus TRZ method may provide more accurate solutions, which need to be verified.

This paper explains the real time modelling of DC Servomotor using both BE and TRZ method on FPGA, and compare their performance with the transfer function (TF) model. Motor is modeled using ‘C’ code and converted to HDL using Vivado HLS tool for its implementation on FPGA [33]. It is observed that even with a step-size of 100 µsec, the response from TRZ is found to be at par with the TF model. However, the response from BE is equivalent to that only by reducing the step-size to 6µsec. Also, analysed the performance of these discrete time models on FPGA by connecting it in loop with the discrete PID controller [34] for the speed control. In this case also, observed a faster dynamic with the TRZ method as compared to the BE method thus verified the superiority of TRZ.

Novelty of our paper is the modeling of DC servomotor using ‘C’ with both BE and TRZ discretization techniques and further the implementation on FPGA using Vivado HLS. These prototypes on FPGA can be connected in loop with the controllers for the design and development of advanced control algorithms for various applications. Feasibility of such HIL technique is also analysed by connecting in loop with the PID implementation on FPGA.

MODELLING OF DC SERVO MOTOR

Equivalent circuit of a DC servomotor can be represented as shown in Fig. 1. Using Kirchhoff’s Voltage Law on this circuit, the mathematical equation can be written as

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + E_b(t) = E_a(t) \quad (1)$$

Where R_a and L_a are the resistance and inductance of the armature circuit, E_a is the applied DC voltage, E_b is the generated back emf and i_a is the current flowing through the armature circuit.

The back emf E_b can be written in terms of angular velocity w_m and back emf constant, K_b as,

$$E_b(t) = K_b w_m(t) \quad (2)$$

Equations (1) and (2) can be combined as,

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b w_m(t) = E_a(t) \quad (3)$$

Torque developed by the motor, T_m, can be written in terms of Moment of Inertia, J_m, and Frictional constant, D_m, of motor as,

$$T_m(t) = J_m \frac{dw_m(t)}{dt} + D_m w_m(t) \quad (4)$$

Also, the torque can be expressed in terms of armature current i_a and Torque constant. K_t as,

$$T_m(t) = K_t i_a(t) \quad (5)$$

Equation (6) can be derived by combining (4) and (5),

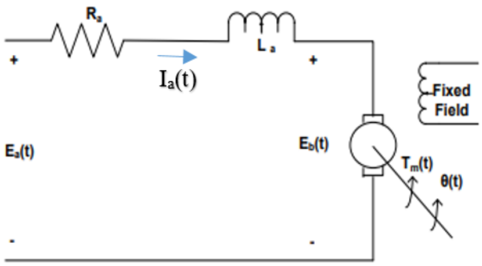

Figure 1. Equivalent circuit of a DC servomotor

$$K_t i_a(t) = J_m \frac{dw_m(t)}{dt} + D_m w_m(t) \tag{6}$$

and then rearranging (3) and (6), the following (7) and (8) can be derived.

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) - \frac{K_b}{L_a} w_m(t) + \frac{1}{L_a} E_a(t) \tag{7}$$

$$\frac{dw_m(t)}{dt} = \frac{K_t}{J_m} i_a(t) - I_a(s) = w_m(t) \tag{8}$$

Equations (7) and (8) can be discretized using a numerical method to simulate the dynamic behavior of the DC servo motor. It's important to note that the accuracy of the simulation depends on the chosen step-size and the appropriateness of the numerical method for the specific dynamics of the system.

For the TF model, assuming zero initial conditions, (3) and (6) can be written using Laplace transform as follows:

$$R_a I_a(s) + L_a s I_a(s) + K_b W_m(s) = E_a(s) \tag{9}$$

$$K_t I_a(s) = J_m s W_m(s) + D_m W_m(s) \tag{10}$$

Substituting $I_a(s)$ from (10) to (9), and after the simplification, the TF can be expressed as,

$$\mathrm{G}(s) = \frac{W_m(s)}{E_a(s)} = \frac{K_t}{J_m L_a s^2 + (R_a J_m + L_a D_m)s + (R_a D_m + K_b K_t)} \tag{11}$$

A. Discretization Methods

During transient analysis, the total time interval of interest is discretized into small step-size, 'h', and the circuit is solved at each time-step or sample. An ordinary differential equation, $\frac{dx}{dt}$, which exists due to the presents of energy storage elements, can be solved numerically by representing it as, slope$=\frac{x_k - x_{k-1}}{t_k - t_{k-1}}$. Here, k is the current sample and k-1 is the previous sample, $t_k - t_{k-1}$ is the step-size, 'h'. However, this expression may be representing the slope at the samples k, k-1 etc. Depends upon that, various discretization techniques like BE and TRZ methods are defined.

In the BE method, $\frac{x_k - x_{k-1}}{h}$ is the slope at 'k'. Thus $\frac{dx}{dt} = f(k)$ can be written in discrete form as, $\frac{x_k - x_{k-1}}{h} = f(x_k)$. However, in the TRZ method, $\frac{dx}{dt} = \frac{1}{2}(f(x_k) + f(x_{k-1}))$ and thus, $\frac{x_k - x_{k-1}}{h} = \frac{1}{2}(f(x_k) + f(x_{k-1}))$ is representing the differential equation.

The choice between TRZ and BE method depends on factors such as system dynamics, stability requirements, and computational efficiency. In the context of DC servo motors, the choice between TRZ and BE modelling may also depend on other factors such as the control algorithm used, the requirements for speed and precision, and the specifics of the motor's behavior.

B. Discretized Model of DC Servomotor

Using the BE discretization technique, (7) and (8) can be written as,

$$i_a(k) = i_a(k-1) - \frac{hR_a}{L_a} i_a(k) - \frac{hK_b}{L_a} w_m(k) + \frac{h}{L_a} E_a(k) \tag{12}$$

$$w_m(k) = w_m(k-1) + \frac{hK_t}{J_m} i_a(k) - \frac{hD_m}{J_m} w_m(k) \tag{13}$$

Which can be written in matrix form as,

$$\begin{bmatrix} 1 + \frac{hR_a}{L_a} & \frac{hK_b}{L_a} \\ -\frac{hK_t}{J_m} & 1 + \frac{hD_m}{J_m} \end{bmatrix} \begin{bmatrix} i_a(k) \\ w_m(k) \end{bmatrix} = \begin{bmatrix} i_a(k-1) + \frac{h}{L_a} E_a(k) \\ w_m(k-1) \end{bmatrix} \tag{14}$$

Equation (14) is of 'Ax=b' form where 'A' is the system matrix, 'x' is the variables, $i_a(k)$ and $w_m(k)$, which need to be determined, and 'b' is the vector which depends upon the input voltage and the previous sample stored values. Here, 'A' depends only upon the system parameters, and thus remains same in all samples, whereas 'b', need to be determined for each sample. Similarly, using the TRZ discretization technique, (7) and (8) can be written as,

$$i_a(k) = i_a(k-1) + \frac{h}{2}[f_1(k) + f_1(k-1)] \tag{15}$$

where $f_1(k) = -\frac{R_a}{L_a} i_a(k) - \frac{K_b}{L_a} w_m(k) + \frac{1}{L_a} E_a(k)$

$$w_m(k) = w_m(k-1) + \frac{h}{2}[f_2(k) + f_2(k-1)] \tag{16}$$

where $f_2(k) = \frac{K_t}{J_m} i_a(k) - \frac{D_m}{J_m} w_m(k)$

Equations (11) and (12) in matrix form is

$$\begin{bmatrix} 1 + \frac{hR_a}{2L_a} & \frac{hK_b}{2L_a} \\ -\frac{hK_t}{2J_m} & 1 + \frac{hD_m}{2J_m} \end{bmatrix} \begin{bmatrix} i_a(k) \\ w_m(k) \end{bmatrix} = \begin{bmatrix} i_a(k-1) + \frac{h}{2L_a} E_a(k) + \frac{h}{2} f_1(k-1) \\ w_m(k-1) + \frac{h}{2} f_2(k-1) \end{bmatrix} \tag{17}$$

By solving (17), which is again of the form '$Ax = b$', $i_a(k)$ and $w_m(k)$ can be determined.

Thus, at every sample, k, armature current and angular velocity can be determined from their previous sample value with the present sample voltage E_a.

DC motor modelling program is written in 'C', using both BE and TRZ methods, and it is observed that the TRZ method gives good accuracy as compared to the BE method. That is because the TRZ method is a second-order method thus the local error per sample is proportional to the square of the step size. And can have better stability and convergence properties for stiff differential equations, which is the case with DC servomotor. The stiff mechanical system in DC servomotors ensures that the motor responds quickly and accurately to changes in the input command, and thus most suitable for tracking and positioning applications.

The control of DC servomotors often involves feedback mechanisms, such as position or speed feedback, to achieve the desired performance characteristics. In this paper, speed of the DC servomotor is observed and controlled by connecting a properly tuned Proportional-Integral-Derivative (PID) controller in the loop. The motor's actual speed is compared with the reference speed, and the error is fed to the PID controller model. The output of the PID controller becomes the DC input to the DC servomotor.

C. Discretized Model of PID Controller

PID controller output can be expressed as,

$$c(t) = K_p e(t) + K_i \int e(t)d(t) + K_d \frac{de(t)}{dt} \qquad (18)$$

Where $e(t)$ is the error in speed and K_p, K_i and K_d are the proportional, integral and derivative constants respectively.

It can be discretized as,

$$c(k) = K_p e(k) + K_i \text{int\_e(k)} + K_d \text{Deriv\_e(k)} \qquad (19)$$

Where at each sample, k, int_e(k) represents the integral of the error and Deriv_e(k) is the derivative of the error. Using BE method, int_e(k) can be written as,

$$\text{int\_e(k)} = \text{int\_e(k} - 1) + \text{h} * e(k) \qquad (20)$$

similarly, derivative of the error can be discretised as,

$$\text{Deriv\_e(k)} = \frac{1}{h}\left(e(k) - e(k-1)\right) \qquad (21)$$

where $(\text{k} - 1)$ is the previous sample and 'h' is the step-size.

PID controller implementation can be represented as shown in Fig. 2

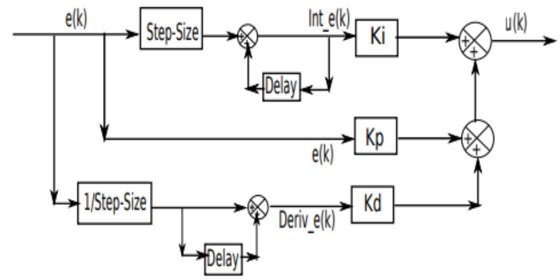

Figure 2. Block diagram of Discretized PID controller

The output of the PID controller becomes the input to the DC servomotor for the speed control. The overall block diagram for the speed control of DC servomotor is shown in Fig. 3.

3. REAL TIME COMPUTATION ALGORITHM

The DC servomotor modelling using both BE and TRZ is coded using 'C'. The total required simulation time is divided into multiple samples by considering a suitable step-size, 'h'.

The computations are performed at every sample, to get the armature current and speed as output by taking the armature voltage as input and the algorithm is given below:

- Formulate the system matrix, 'A', using (14) or (17), where (14) is for BE and (17) for TRZ.
- Read the armature voltage, E_a, at current sample, and the armature current i_a, and angular velocity w_m from the previous sample.
- Formulate the vector 'b' using the above mentioned values.
- Use LU decomposition technique to solve the armature current and angular velocity.
- Decompose 'A' to 'LU' where 'L' is the unit lower triangular matrix and 'U' is the upper triangular matrix.
- Apply forward substitution technique and then the backward substitution technique to determine the armature current and angular velocity.
  - Store the armature current and angular velocity at current sample, for the next time-step.
  - Determine the speed. Make available the armature current and speed at the output.

The algorithm can be represented as flowchart as shown in Fig. 4.

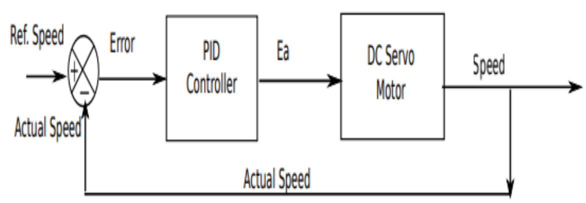

Figure 3. Block diagram for the closed loop speed control of DC servomotor with PID controller

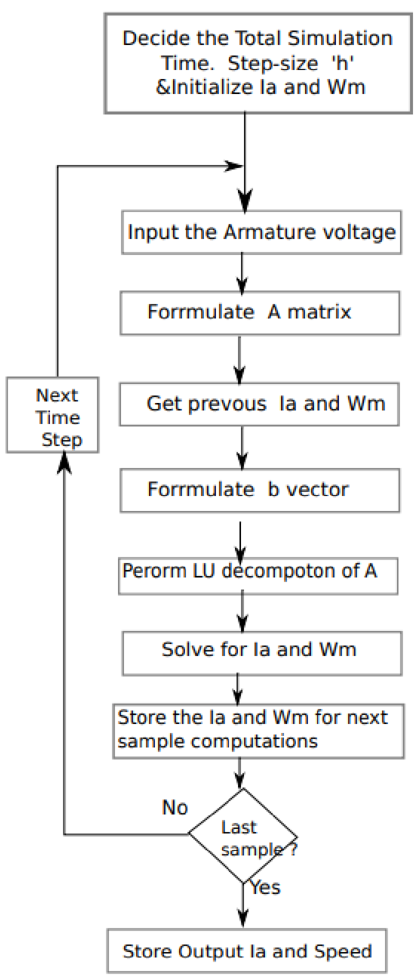

Figure 4. Flow chart for the computational algorithm

4. IMPLEMENTATION ON FPGA

The DC Servomotor is implemented on Xilinx's PYNQ FPGA board using Vivado tools. PYNQ board features Programmable Logic (PL) with an ARM Cortex-A9, 666 MHz Processing System (PS) [35].

The computational program is written in 'C' code, as explained in section-3, converted to HDL, and corresponding intellectual property (IP) is generated using Vivado HLS tool [25]. Similarly, 'C' code is written for the discrete PID controller implementation, using (19) to (21), which is converted to HDL and then IP using the HLS tool. Here, both DC servomotor and PID controller IPs are instantiated in the PL for the closed loop speed control.

In this design, error signals are generated in the PS by comparing the input reference speed with the actual speed from the DC servomotor IP. The PID controller IP in PL takes error as input and generate control signal for the DC servomotor IP. The DC servomotor IP take this control signal as input directly from the PID controller in PL through AXI4-Stream interface (axis). It transfers data in a sequential streaming manner, and thus AXI4-Stream Data FIFO is also included, which provides a buffer between AXI4-Stream data master and slave in the design. Then the speed output of the DC servomotor is given back to the PS for calculating the error.

Since the input port of PID control IP and output port of DC servomotor IP need to be communicated with the PS, AXI4-Lite slave interface (s_axilite) is used during the IP design, which will allow the PS to access the respective port in the IP through the memory-mapped instructions. However as mentioned before, AXI4-Stream interface (axis) is used for the outport port of PID and input port of DC servomotor IPs. The main advantage of this streaming implementation is that it reduces memory-mapped communications and thus reduces the communication latency. The top-level function code for the PID controller IP, with the interface mentioned, are shown below.

```
void PID_IP(float error, float CS)
{
        #pragma HLS INTERFACE s_axilite port=error
        #pragma HLS INTERFACE axi port=CS
         #pragma HLS INTERFACE s_axilite port=return
```

Similarly in the case of DC servomotor IP, the top-level function code, with the interface mentioned, are

```
void DC_SM_IP(float CS, float Speed, float Ia)
{
        #pragma HLS INTERFACE axi port=CS
        #pragma HLS INTERFACE s_axilite port=Speed
         #pragma HLS INTERFACE s_axilite port=Ia
         #pragma HLS INTERFACE s_axilite port=return
```

Both of these customized IPs are integrated with other hardware libraries such as ZYNQ-PS, AXI-fabric etc. available in the Vivado HLx environment to implement the overall hardware system in FPGA. The corresponding block design using Vivado HLx is as shown in Fig. 5.

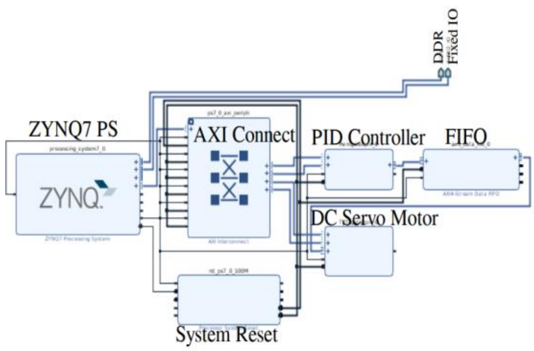

Figure 5. Block Design using Vivado HLx for the closed loop speed control of DC servomotor with PID controller

Then the HDL wrapper is created for the designed system and generated the bit-stream for the FPGA, after the synthesis and implementation on the specified FPGA hardware. Then exports the design to the Xilinx's Software Development Kit (SDK) to develop the application program in the on-chip processor. Application program in 'C' includes the memory read and write operations and the input-output operation for the communication between the PS and the PL [27].

In our implementation, at every sample, the generated error signals are passed into the PID_IP in PL using the memory-mapped instructions. The instruction Xil_Out32 (u32Addr, u32Value) perform an output operation by writing the 32-bit error value to the address specified. Similarly, 32-bit servomotor outputs, speed and Ia, are read from the address specified using the instruction Xil_In32 (u32Addr). The snippets of our application program code for writing into and read from the implemented hardware with the signal '1' for starting the communication is shown below:

```
Xil_Out32(PID_IP_BASEADDR+0x10, error);
Xil_Out32(PID_IP_BASEADDR+0x00,1);
while(0==(2 & Xil_In32(DC_SM_IP_BASEADDR+0x00)));
Speed=Xil_In32(DC_SM_IP_BASEADDR+0x20);
Ia=Xil_In32(DC_SM_IP_BASEADDR+0x24);
```

Here, *0x10*, *0x20* etc. are the offset addresses created during the IP generation in Vivado HLS, which is available at SDK with the exported design. *0x10* offset address is for the handshaking signal between PL and PS. Signal '1' indicates the 'start' operation for the PL and '2' indicates output is ready by the PL. Once it is ready, the output can be read from the respective address. Then it is stored in the DDR available at the PS or to make it available at the output pins. Now the next iteration starts and it continue till reaching the final desired time.

The above mentioned application program need to build to generate the 'elf' file. Configure the FPGA board with the generated bit-stream and then load the 'elf' file for running the overall system. Flow chart for the implementation on FPGA using Vivado tools, for the closed loop speed control with PID controller is shown in Fig. 6.

5. Results and Discussions

The DC Servomotor is modelled using 'C' code with both BE and TRZ methods and the performance is compared with the TF model in MATLAB. Following parameters [9][10] are considered for the modelling of DC Servomotor. R_a = 2.45Ω, L_a = 0.035H, K_b = K_t =1.2volt/(rad/sec), J_m =0.022Kg-m^2/rad, D_m=0.5*10^{-3} N-m/(rad/sec).

Total time of 0.25 sec is considered as the simulation time for both BE and TRZ methods using 'C' and for TF in MATLAB. Step-size, 'h', is varied from 20msec to 0.6μsec, compared and analysed the performance of BE and TRZ methods with the TF model. The speed at 0.12 sec for all the three models with change in step-size is shown in Table I.

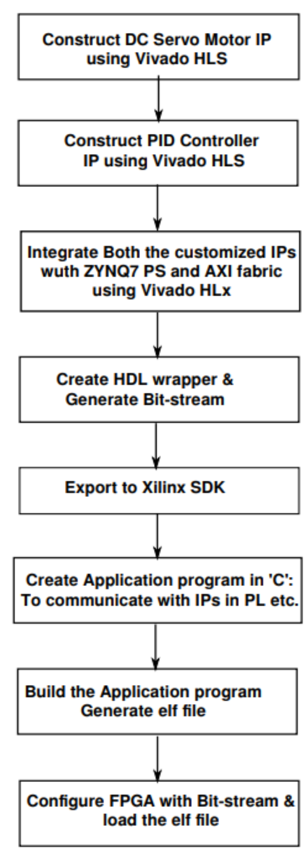

Figure 6. Flow chart for the implementation on FPGA, using Vivado tools, for the closed loop speed control with PID controller.

It is clear from the table that, when the step-size is 20msec, speed at 0.12 sec corresponds to BE is much deviated from the transfer function model. However, TRZ method gives more accurate result as compared to the BE method. When the step-size decreases, the accuracy of both the discretized models improves. At a step-size of 100 µsec, TRZ result becomes same as that of TF and at 0.6 µsec step-size, the result corresponds to BE method also matches with the TF model.

It indicates that being a second order approximation, TRZ model is more accurate as compared to the BE method and the accuracy increases with the reduction in step-size. However, for a real-time implementation, all communications between the input-output devices with the model, and all the computations in the model need to be completed in the selected step-size. So need to be compromised for the selection of the step-size, and thus second order TRZ method is more advantageous as compared to the BE method.

Dynamic response for all the three models are plotted in Fig. 7 to 9 with different step-size. It is clear from Fig. 7 that, with a step-size of 10mse, initial dynamics with both BE and TRZ experiences considerable deviations from that of the TF model, but, TRZ follows more closely to the TF model, as compared to BE.

Fig. 8 shows that, when the step-size is decreased to 5msec, the error reduced considerably, however, still the TRZ result is more nearer to the TF model than the BE.

Now by reducing the step-size further to 6µsec, both the BE and TRZ becomes accurate, which is clear from Fig. 9. Only a small deviation is present in the dynamics, which can be seen only by zooming the initial portion, and is shown as a separate rectangle along with the normal view in Fig. 9.

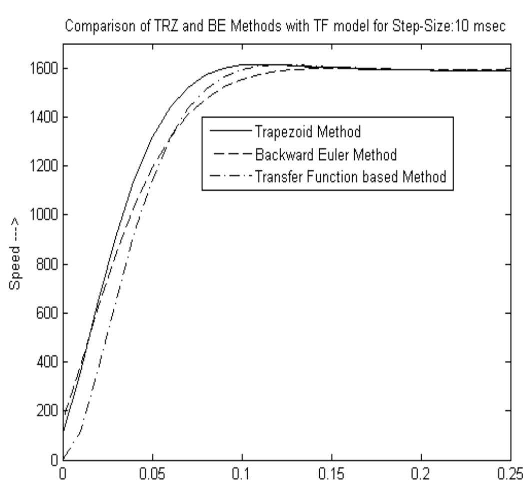

Figure 7. Comparison of TRZ and BE methods with TF for a step-size of 10msec

TABLE I: MEASURED SPEED AT 0.12 SEC FOR ALL THE THREE MODELS FOR VARIOUS STEP-SIZE.

Step-size	Speed Measured at 0.12 sec		
	BE method	TRZ method	Transfer Function
20 msec	1568.1	1613.9	1611.4
10 msec	1586.5	1613.2	1611.4
5 msec	1598.1	1612.3	1611.4
2 msec	1605.8	1611.7	1611.4
100 µsec	1611.3	1611.4	1611.4
20 µsec	1611.3	1611.4	1611.4
6 µsec	1611.3	1611.4	1611.4
0.6 µsec	1611.4	1611.4	1611.4

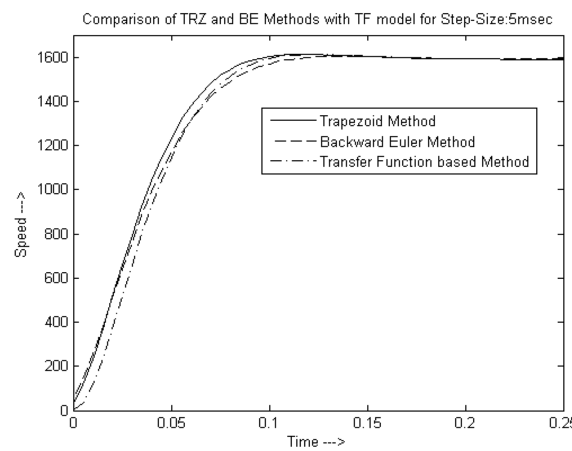

Figure 8. Comparison of TRZ and BE methods with TF for a step-size of 5msec

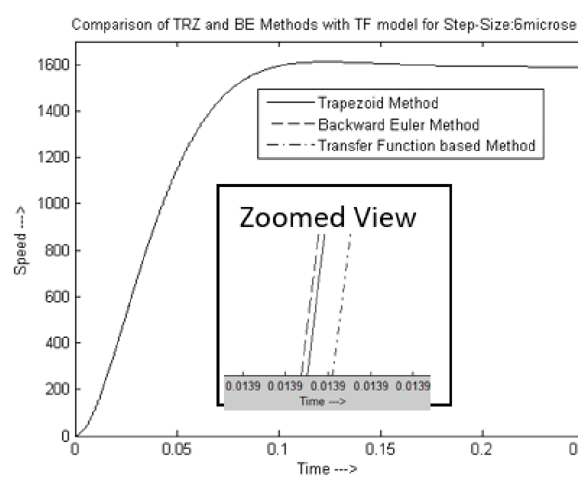

Figure 9. Comparison of TRZ and BE methods with TF for a step-size of 6µsec

Discrete PID controller with Kp=20.875, Ki=0.2138, and Kd=0.2195 [9] is also modelled using ‘C’ and connected in closed loop with the DC servomotor model for its speed control. Reference speed considered for the analysis is 1500 rad/sec. In this case also, compared the performance of both BE and TRZ modelled DC servomotor and the observations are shown in Fig. 10 and 11, with the step-size varying from 50µsec to 6 µsec.

Fig. 10 and 11 clearly shows that, similar to the previous cases, for higher step-size, TRZ gives better performance. However, when the step-size is reduced, BE is also in par with the TRZ. This shows that, TRZ is a better method of discretization especially with a larger step-size.

Also, analysed the performance by changing the reference speed from 1500 rad/sec to 1600 rad/sec at 2.5msec and the corresponding plot is shown in Fig. 12.

Fig. 10 and 11 clearly shows that with less than 2 msec both BE and TRZ models is able to achieve the reference speed. Similar conclusion can be inferred from Fig. 12, also with change in speed.

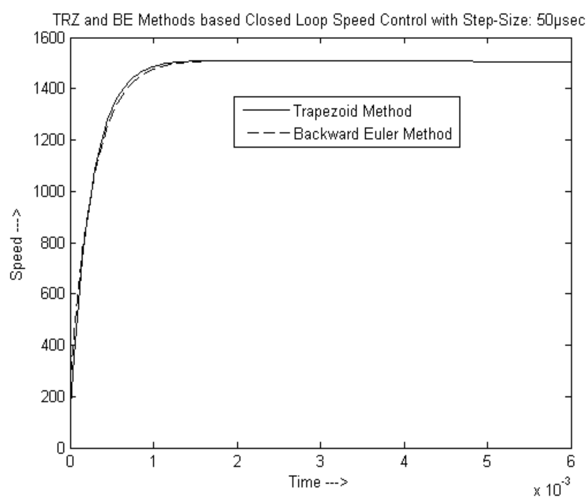

Figure 10. Comparison of TRZ and BE methods for the closed loop speed control with a step-size of 50µsec

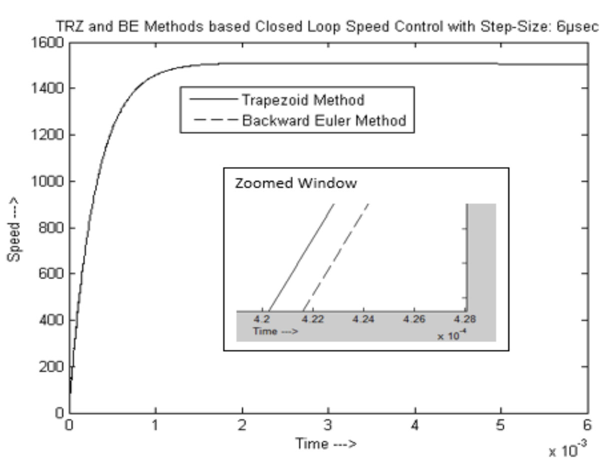

Figure 11. Comparison of TRZ and BE methods for the closed loop speed control with a step-size of 6µsec

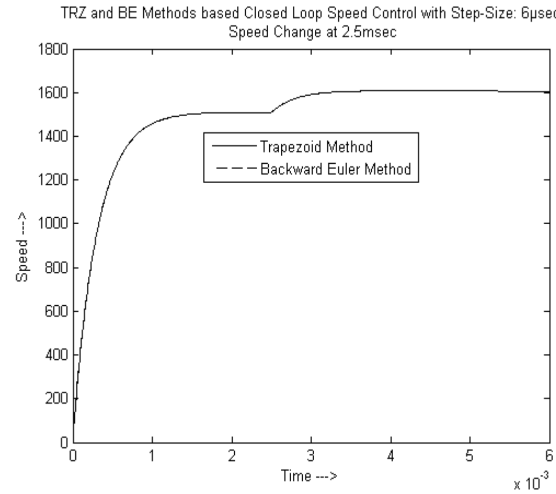

Figure 12. Tracking performance of both TRZ and BE method with speed changed from 1500 rad/sec to 1600 rad/sec at 2.5msec

These analysis shows that by including the discrete PID controller with proper tuning, in loop with the FPGA based DC servomotor, as expected, a faster dynamic with zero steady state error could be achieved. This strategy can be used for testing and development of various control algorithms for DC servomotor, for diverse applications, without involving the physical motors of numerous possible specifications.

6. CONCLUSION

The Trapezoidal (TRZ) and the Backward Euler (BE) methods are two implicit methods commonly used for time discretization in the numerical solution of ordinary differential equations (ODE). Relative accuracy of these methods can vary depending on the specific ODE being solved, the step-size chosen, and other numerical considerations. This paper compares these two discretization methods for the modelling of DC servomotor implemented on FPGA with that of the widely used transfer function method using MATLAB. Step-size is changed from 20msec to 0.6 µsec and observed that, TRZ method gives more accurate results as compared to BE method especially with larger step-size. However, shows that the designed FPGA based models of DC servomotor is as accurate as the TF model, thus can be used for further design purposes.

Speed control performance of DC servomotor is also analyzed by connecting the models of DC servomotor in closed loop with a properly tuned discrete PID controller for a step-size variation from 50 µsec to 6µsec. In this case also, with 50µsec, TRZ was superior, however at 6µsec, the performance of BE is well matched with the TRZ. These analysis explains in detail the effect of step-size on discretization techniques which will help us to choose the method and the step-size.

With a properly decided step-size and the discretization method, FPGA based DC servomotor model can be developed which can be effectively utilized for the design and testing of robust and optimized control systems without involving the physical systems. Same hardware model can be used for numerous applications of DC servomotor even with different specifications because of the reconfigurable property of the FPGA.

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