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Performance Comparison of Channel Estimation Techniques for Mobile Communication Systems

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Abstract: In this paper, the performance comparison of channel estimation techniques for Direct Sequence CDMA systems over a frequency selective fading channel disturbed by additive white Gaussian noise (AWGN) is presented. Three types of channel estimation techniques, including *moving averaging* (MA), *minimum mean square error* (MMSE) and *maximum signal-to-noise ratio* (MAXSNR), are presented, and a special attention to the MA approach is paid with optimal averaging interval because of its implementation simplicity and performance is very close to the other two techniques. Since the optimal averaging interval of the MA approach varies with different channel fading rate characterized by the Doppler-shift, a Doppler-shift estimation and its modification are therefore proposed and analyzed, resulting in an asymptotic Doppler-shift estimate which is shown to be working properly at lower SNR and in a large Doppler-shift dynamic range. Furthermore, the optimal averaging interval of the MA approach, obtained by minimizing the mean-squared error of the channel estimate is introduced by means of the frequency domain analysis. Simulation results show that the proposed adaptive channel technique can provide much better channel estimation accuracy with a slight increase in computational complexity, which enables RAKE receiver to achieve better performance when the mobile user suffers from various Doppler-shift rates.

Keywords: DS-CDMA, Channel estimation, Coherent detection, RAKE reception

1. Introduction

The Adaptive estimation of time-variant mobile radio channel is very important in coherent DS-CDMA mobile systems [1]. For coherent reception, transmission of a common pilot channel together with the data channels is often required [2][3]. Since the symbol patterns of the common pilot channel are known a priori, the mobile users can extract the information of the time-variant mobile radio channel by observing the received common pilot channel instantaneously.

For the initial channel estimates resulting from several successive pilot symbols, a moving averaging (MA) procedure is often necessary to remove the interference resulting from non-ideal PN sequence correlation and AWGN, in which the initial channel estimates are equally averaged. In the case of time-invariant channel, more accurate channel estimate will be achieved when longer MA time interval is taken [4]. The mobile communication systems often suffer from time-variant channels [5]. If the MA time interval used for the channel estimation is too long, the resultant estimate may fail to track the variation of the time-variant channel [6]. Therefore, tradeoff should be made between the interference suppression and the channel tracking capability.

Since the time-variant radio channel is often characterized by Doppler-shift or equivalently corresponding vehicular speed, this paper intends to explore the optimal MA time interval in the presence of a specific Doppler-shift parameter and compares this approach with other two approaches called the *minimum mean square error* (MMSE) and the *maximum signal-to-noise-ratio* (MAXSNR). If the Doppler-shift can be estimated in real-time, an adaptive channel estimation scheme can be figured out based on the optimal MA channel estimation approach, to adaptively achieve the best reception performance for mobile users with different mobile speeds.

Because modern mobile systems must support the vehicular speed from 0 up to 500 km/h, Chen and You [4] presented the techniques of adaptive channel estimation and power adjustment based on the fading-rate measurement for DS-CDMA systems. The fading-rate measurement was made based on the numerical result of a proposed cost function and the optimal MA time interval

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with respect to different fading-rate was obtained by simulation and systematic testing. Benthin and Kammeyer [6] gave the evaluation on the effects of the MA time interval into the receiver performance in terms of symbol error rate.

For the estimation of Doppler-shift, the literature has proposed various methods [7]-[13] using the statistical characteristics of the received signals, including level crossing rate of the averaged signal level [7], and squared deviation of the logarithmically compressed signal envelope [8]. Other methods extract the information of the maximum value of Doppler shift through eigen-value subspace [9], multi-dimensional scaling [10], wavelets [11], carrier frequency offset (CFO) [12], and measure the Doppler bandwidth adaptively [13]. These methods would only find limited applications due to their computational complexities and huge amount of storage. As a comparison, this paper proposes a Doppler-shift estimate based on the instantaneous channel estimates from the pilot symbols. Theoretical analysis and simulation results show that the proposed estimation method has some attractive features, including accuracy and considerably reduced computational complexity, which simplify the hardware implementation.

A channel estimation of MIMO communication systems using techniques STBC and SFBC for pilot and data subcarriers under various fading channels are analyzed and compared in [28]. The performance of pilot-based, semi-blind, blind, and adaptive-blind channel estimation methods using a cluster-based COST 2100 channel model is introduced in [29]. A simple and practical channel estimator for multipath multi-cell massive MIMO TDD systems with pilot contamination, which poses significant challenges to channel estimation is presented [30]. A computationally efficient hybrid steepest descent Gauss–Seidel (SDGS) joint detection, which directly estimates the user's transmitted symbol vector, and can quickly converge to obtain an ideal estimation value with a few simple iterations is proposed in [31]. The performance of practical channel estimators is analyzed for multi-user massive MIMO systems. The pilots are generated using Zadoff-Chu sequences. For the assessment of channel estimator Normalized Mean Square Error (NMSE) is used as one of the performance measurement metrics [32].

This paper is arranged as follows. Section 2 introduces the channel estimation model and discusses the algorithms employed in the channel estimation, including the MA, the MMSE and MAXSNR. Section 3 investigates the estimation of Doppler-Shift and the theoretical derivation of the optimal MA time interval. Theoretical analysis results for the Doppler-shift estimate and for the performance of the optimal MA approach are also provided in this Section. In Section 4, simulation results are introduced to justify the theoretical analysis, while we conclude in Section 5.

2. CDMA Channel Estimation Model

We consider the DS-CDMA system downlink with the common pilot channel transmitted simultaneously with a single or multiple data channels, since this is a common case in the mobile communication systems [2][3]. The transmission signal is mapped onto in-phase and quadrature branches which are independently spread by their respective orthogonal channelization codes. The pilot signal and the data signals are then scrambled in a complex manner, followed by base-band chip wave-shaping filtering and up-conversion for the radio transmission. The equivalent low-pass transmitted signal can be thus expressed as

$$s(t) = \sum_{i=-\infty}^{+\infty}\left[\sum_{k=1}^{K}\sum_{m=-\infty}^{+\infty}\sqrt{\varepsilon_k}\,d_k(m)c_k(i-mL_k) + \sum_{n=-\infty}^{+\infty}\sqrt{\varepsilon_p}\,d_p(n)c_p\left(i-nL_p\right)\right]g(t-iT_c) \qquad (1)$$

where

$\{d_k(m)\}$ the complex data symbol sequence for the k-th user,

$\{d_p(n)\}$ the complex common pilot symbol sequence for all the mobile users in the cell,

$c_k(i)$, $c_p(i)$ the equivalent complex spreading sequences of the data channel for the kth user and the common pilot channel, respectively, consisting of orthogonal variable spreading factor (OVSF) codes and complex scrambling codes,

L_k, L_p the number of chips per data symbol and pilot symbol, respectively,

ε_k, ε_p the power transmitted for the k-th user and the pilot channel, respectively,

$g(t)$ the chip signal shaping filter impulse response,

T_c the chip duration of the spread signal.

The transmitted signals will propagate through a multi-path frequency selective channel, which can be modeled as a tapped delay line [1]. The RAKE-type architecture is usually employed at the receiver to collect the signal energy from the resolvable multi-path components. The equivalent lowpass received signal can be given by

$$r(t) = \sum_{l=0}^{L-1} h_l(t)s(t-\tau_l) + z(t) \qquad (2)$$

where L is the maximum number of resolvable multi-path signals, $h_l(t)$ denotes the independent complex-Gaussian fading coefficient of the l-th multi-path [14], with a Rayleigh distributed magnitude and a uniformly distributed phase; τ_l is the relative delay of the l-th multi-path; and $z(t)$ is a complex-valued stationary *additive white Gaussian noise* (AWGN) process with zero mean and two-sided power spectral density $N_0/2$.

Since the pilot symbol patterns are known a priori by the receiver, the channel estimation can be carried out by de-spreading the common pilot channel in each finger of the RAKE receiver

$$r_{p,l} = \frac{1}{T_b}\int_{nT_s+\tau_i}^{(n+1)T_s+\tau_i} r(t)\sum_{j=1}^{L_p} g^*(t-\tau_l - jT_c)\, c_p^*(j)dt$$

$$= \frac{1}{T_b}\int_{nT_s+\tau_i}^{(n+1)T_s+\tau_i} \sqrt{\varepsilon_p}\ d_p(n)\, h_l(t)\, dt + z'_{p,l}(n) \quad l = 1,\cdots,L \qquad (3)$$

where superscript "$*$" denotes the complex conjugation, T_s is the pilot symbol duration, $z'_{p,l}(n)$ is the sample composed of multiple access interference (MAI) of other users, multi-path interference (MPI) of the pilot channel and additive Gaussian noise during the process of de-spreading in the l-th finger. It is logical to assume that the channel parameters keep unchanged during a symbol period of T_s. Thus equation (3) can be rewritten as

$$r_{p,l}(n) = \sqrt{\varepsilon_p} d_p(n) h_l(n) + z'_{p,l}(n) \qquad l = 1,\ \ldots, L \quad (4)$$

An instantaneous estimate $\tilde{h}_{l,T}(n)$ for the channel coefficient $h_l(n)$ is obtained by properly normalizing $r_{p,l}(n)$ as follows:

$$\tilde{h}_{l,T}(n) = \frac{r_{p,l}(n)}{\sqrt{\varepsilon_p} d_p(n)} = h_l(n) + \frac{z_{p,l}(n)}{\sqrt{\varepsilon_p}} \quad l = 1,\ \ldots, L \quad (5)$$

where $z_{p,l}(n) = z'_{p,l}(n)/d_p(n)$.

For a RAKE receiver to perform coherent detection and to form the decision variable, the despread data symbols are multiplied by the complex conjugate of the channel fading coefficients estimates in each finger and then are added to the outputs from other fingers to carry out the *maximum ratio combing* (MRC) [15].

Equation (5) indicates that the instantaneous channel estimate, $\tilde{h}_{l,T}(n)$, is corrupted by the noise component $z_{p,l}(n)$. In practice, an improved channel estimate in some statistical sense can be attained by further process of the instantaneous estimates. In the following parts of the paper, we assume that the code sequences in the systems are perfectly orthogonal and present a perfect autocorrelation and that the noise component $z_{p,l}(n)$ is statistically independent Gaussian distributed with zero mean and variance σ_z^2. For simplicity, we consider the channel estimation for the l-th multi-path in the l-th finger of the RAKE receiver.

A. Moving Averaging (MA) Technique

If the channel is kept unchanged over $2N + 1$consecutive pilot symbols, an improved channel estimate can be achieved by a *moving averaging* (MA) procedure in which the instantaneous estimates $\tilde{h}_{l,T}(i)$ are arithmetically averaged in the range $n - N \le i \le n + N$ as follows:

$$\hat{h}_{l,\mathrm{MA}}(n) = \frac{1}{2N+1}\sum_{i=n-N}^{n+N} \tilde{h}_{l,T}(i) = h_l(n) + \frac{\sum_{i=n-N}^{n+N} z_{p,l}(i)}{(2N+1)\sqrt{\varepsilon_p}} \quad (6)$$

Obviously, $\hat{h}_{l,\mathrm{MA}}(n)$ is an unbiased estimate of the channel coefficient $h_l(n)$ with its variance of $\sigma_z^2/(2N+1)\varepsilon_p$. As the averaging interval goes to infinity, the variance of the estimate approaches zero. Consequently, for a time-invariant channel, a large averaging interval would remove the noise efficiently.

B. MMSE Technique

To obtain a channel estimate in the sense that the mean-square value of the estimation error is minimized, the MMSE criterion is to be employed in the channel technique. [16].

For an estimate $\hat{h}_l(n)$ of the channel coefficient at time n, an observed vector $\boldsymbol{x}_l$ of the instantaneous channel estimates $\tilde{h}_{l,T}(i)$ in the range $n - N \le i \le n + N$ is formed at input of the channel technique

$$\boldsymbol{x}_l = [\tilde{h}_{l,T}(n-N) \ \cdots \ \tilde{h}_{l,T}(n) \ \cdots \ \tilde{h}_{l,T}(n+N)] = \boldsymbol{h}_l + \boldsymbol{z}_{p,l} \quad (7)$$

where $\boldsymbol{h}_l = [h_l(n-N) \ \cdots \ h_l(n) \ \cdots \ h_l(n+N)]$ and

$$\boldsymbol{z}_{p,l} = \frac{1}{\sqrt{\varepsilon_p}}[z_{p,l}(n-N) \ \cdots \ z_{p,l}(n) \ \cdots \ z_{p,l}(n+N)].$$

The channel technique is assumed to be linear and can be characterized by a weight vector

$$\boldsymbol{w} = [w_{n-N} \ \cdots \ w_n \ \cdots \ w_{n+N}]^{\boldsymbol{T}} \quad (8)$$

The channel estimate $\hat{h}_l(n)$, obtained at the channel estimation technique output, can be expressed as

$$\hat{h}_l(n) = \boldsymbol{x}_l \boldsymbol{w} \quad (9)$$

The weight vector of the MMSE technique is given as:

$$\boldsymbol{w}_{\mathrm{MMSE}} = (\boldsymbol{R} + \frac{\sigma_z^2}{\varepsilon_p}\boldsymbol{I}_{2N+1})^{-1}\boldsymbol{r} \quad (10)$$

where the superscript "$^{-1}$" is the inversion of the matrix, matrix $\boldsymbol{I}_{2N+1}$ stands for the $(2N+1)\times(2N+1)$ identity matrix, $\boldsymbol{r} = [r_N \ \cdots \ r_0 \ \cdots \ r_N^*]^{\boldsymbol{T}}$, $r_i = E\{\ h_l(n)h_l^*(n-i)\}$ denotes the autocorrelation function of the l-th multi-path channel coefficients for a lag of i and has the property $r_{-i} = r_i^*$, and $\boldsymbol{R}$ is the correlation matrix of the l-th multi-path channel coefficients $\boldsymbol{h}_l$.

$$\boldsymbol{R} = \begin{bmatrix} r_0 & r_1 & \cdots & r_{2N} \\ r_1 & r_0 & \cdots & r_{2N-1} \\ \vdots & \vdots & \ddots & \vdots \\ r_{2N} & r_{2N-1} & \cdots & r_0 \end{bmatrix} \quad (11)$$

C. MAXSNR Technique

The linear channel technique can also be designed to attain an estimate for channel coefficient $h_l(n)$ with *maximum signal-to-noise ratio* (MAXSNR) [17]. Based on the instantaneous channel estimate vector $\boldsymbol{x}_l$ defined above, the output signal-to-noise ratio of the channel technique, can be written as

$$SNR_o = \frac{P_o}{N_o} = \frac{\varepsilon_p}{\sigma_z^2}\frac{\boldsymbol{w^H R w}}{\boldsymbol{w^H w}} \quad (12)$$

where P_o and N_o are the average power of the signal component and the noise component of the channel technique output, respectively, $\boldsymbol{R}$ is the correlation matrix of $\boldsymbol{h}_l$.

If the channel technique vector $\boldsymbol{w}$ satisfies the condition $\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{w} = 1$, the weight vector of the MAXSNR technique is given as

$$\boldsymbol{w}_{\mathrm{MAXSNR}} = \boldsymbol{q}_{\max} \quad (13)$$

where $\boldsymbol{q}_{\max}$ is the eigenvector associated with the largest eigenvalue of the correlation matrix $\boldsymbol{R}$.

3. Optimal MA Channel Estimation

As shown in the above derivation, the MA technique has the least implementation complexity among the channel techniques, in that it obviates the need of matrix inversion required by the MMSE technique and the need of eigenvector computation required by the MAXSNR estimation. However, the MA technique can provide channel estimates with noise variance decreasing inversely with the averaging interval. Accordingly, the MA channel technique is preferred for practical applications because of its simple implementation and high efficiency.

The high efficiency of the MA technique suppressing noise assuming that the channel is time-invariant over the large averaging interval. However, in practice, a typical mobile wireless fading channel is time-variant with its fading rate characterized by maximum Doppler-shift f_d. This implies a tracking with respect to the instantaneous value of channel coefficient in the MA technique. As will be illustrated later in the simulation results, a large averaging interval will lead to considerable tracking errors in the channel estimates which would severely degrade the receiver performance, even though the large averaging interval also results in less noise variance. Consequently, there is an optimal averaging interval of the MA technique to provide the estimate with a minimum cost function consisting of the mean-square value of the noise plus the squared tracking error under a given fading rate of the time-variant channel. Moreover, the maximum Doppler-shift, depends on the mobile speed and the carrier frequency, varies with the change of the mobile speed. In such cases, it is necessary for practical applications to estimate maximum Doppler-shift of the time-variant channel and then select the optimal moving averaging interval of the MA technique adaptively.

In the following parts of this section, we will propose an estimate $\hat{f}_d$ for maximum Doppler-shift of a wireless channel. Furthermore, theoretical performance analysis of the proposed estimate is provided and a method of modification for improved estimation accuracy is suggested. An important result is that we present the optimal MA interval in a closed-form equation that minimizes the cost function consisting of the mean-squared noise plus the squared tracking error. It should be noted that the following development is established on the assumption of Jake's channel model, while the derivation is applicable to any other wireless channel models.

A. Estimation of the Maximum Doppler-shift $\hat{\boldsymbol{f}}_{\boldsymbol{d}}$

To estimate the maximum Doppler-shift f_d of the channel based on the instantaneous estimates $\tilde{h}_{l,T}(i)$ in the range $-N \le i \le N$, a variable G defined as [22] is used:

$$G = \frac{(2N+1)\sum_{i=-N}^{N}(y_{i+1}-y_i)^2}{2N\sum_{i=-N}^{N} y_i^2} \quad (14)$$

where y_i denotes the magnitude of $\tilde{h}_{l,T}(i)$. As will be shown, G can be used to construct the estimate of maximum Doppler-shift $\hat{f}_d$.

For Rayleigh fading with Jake's Doppler spectrum, the autocorrelation function of the channel coefficient $\{h_l(i)\}$ is given by [20]

$$r(jT_s) = E\{\, h_l(i)h_l^*(i-j)\} = E_0 J_0(2\pi f_d jT_s) \quad (15)$$

where E_0 is the average power of $h_l(i)$. The autocorrelation function of $\{x_i\}$, formed by the magnitude of the channel coefficient $\{h_l(i)\}$ can be represented in terms of a hypergeometric function [20]

$$R_y(jT_s) = \frac{\pi E_0}{4} F[-\tfrac{1}{2},\ -\tfrac{1}{2};\ \ 1,\ J_0^2(2\pi f_d jT_s)] \quad (16)$$

Let $\{h_l(i)\}$ be a channel coefficient sequence with the sampling duration of T_s, having Rayleigh distributed magnitude $\{x_i\}$ and average power E_0. A proposed variable is defined as

$$G_A = E\{(x_{i+1}-x_i)^2\}/E\{x_i^2\} \quad (17)$$

Then, the following equation holds

$$G_A = (\pi f_d T_s)^2 + O((\pi f_d T_s)^4) \quad (18)$$

where $O(\cdot)$denotes higher-order infinitesimal.

The variable G approximates the variable G_A when the channel estimates $\tilde{h}_{l,T}(i)$ are noise-free, and an estimate $\hat{f}_d$ for the maximum Doppler-shift when $2\pi f_d T_s \ll 1$,

$$\hat{f}_d = \frac{\sqrt{G}}{\pi T_s}. \quad (19)$$

The corresponding estimate for vehicular speed is $\hat{v} = \hat{f}_d\lambda$, where λ is the carrier wavelength.

B. Performance Analysis of the Estimate $\hat{\boldsymbol{f}}_{\boldsymbol{d}}$

For the maximum Doppler-shift estimation shown in equation (19), if the terms $O((\pi f_d T_s)^4)$ are neglected, which may introduce some estimation errors. However, the multiplicative gains of these terms are very small (the largest multiplicative gain is $1/4$). Besides, the term $(\pi f_d T_s)^4$ would be a small number for practical situations. For example, if the pilot symbol rate is 16ks/s and the mobile speed is 400 km/h, the number $\frac{(\pi f_d T_s)^4}{4}$ is as small as 1.1×10^{-4}. Therefore, the estimate $\hat{f}_d$ given in (19) is a good approximation of maximum Doppler-shift f_d.

To gain an insight on the performance of the estimate $\hat{f}_d$, we examine the fractional error of the estimate. The variable G_A can be expressed as a function of maximum Doppler frequency f_d

$$G_A = f(f_d) = 2 - \frac{\pi}{2} F[\,-\tfrac{1}{2},\ -\tfrac{1}{2};\ 1;\ J_0^2(2\pi f_d T_s)] \quad (20)$$

The variance of the estimates $\hat{f}_d$ can be obtained as [19]:

$$\sigma_{\hat{f}_d}^2 \approx \left|\frac{1}{f'(f_d)}\right|^2 \sigma_G^2 = \frac{4\sigma_G^2}{\left|\pi^2 T_s\, J_0(2\pi f_d T_s)\, J_1(2\pi f_d T_s) F[1/2,1/2;2;J_0^2(2\pi f_d T_s)\right|^2} \quad (21)$$

The fractional error of the estimates $\hat{f}_d$ is defined as a ratio of its standard deviation $\sigma_{\hat{f}_d}$ to f_d

$$\frac{\sigma_{\hat{f}_d}}{f_d} = \frac{2\sigma_G}{\left|\pi^2 f_d T_s\, J_0(2\pi f_d T_s)\, J_1(2\pi f_d T_s) F[1/2,1/2;2;J_0^2(2\pi f_d T_s)\right|} \quad (22)$$

where σ_G denotes the standard deviation of G [19].

Let $\{h_l(i)\}$ be the channel coefficient sequence, having Rayleigh distributed magnitude $\{x_i\}$, average power E_0 and uniformly distributed phase. Let $\{z_l(i)\}$ be a complex additive white Gaussian noise sequence with zero mean and variance of σ_z^2. $\{\tilde{h}_l(i)\}$ is a sequence defined as $\tilde{h}_l(i) = h_l(i) + z_l(i)$ and has its magnitude sequence $\{y_i\}$. T_s is the sampling duration of the sequences. We introduce a variable G_y, which is defined as

$$G_y = E\{(y_{i+1} - y_i)^2\}/E\{y_i^2\} \quad (23)$$

Then G_y and G_A satisfy the inequality

$$G_y \geq G_A \quad (24)$$

where the equation holds true if $\sigma_z^2 = 0$.

If the instantaneous estimate $\tilde{h}_{l,T}(i)$ involves noise components, the estimation error of $\hat{f}_d$ given in equation (19) is nonnegative, resulting in a biased estimate.

The above derivation suggests a method of modifying G_y to approach G_A to improve the estimate $\hat{f}_d$ with the knowledge of the received SNR of the multi-path. However, it is difficult to get an analytic relation between the two hypergeometric functions $F[-1/2, -1/2;\ 1;\ x]$ and $F[-1/2, -1/2;\ 1;\ \alpha x]$ for modifying G_y exactly to obtain G_A. Therefore, we consider the modification based on the linear approximation of the function $F[-1/2, -1/2;\ 1;\ x]$ in the range $x = [0,\ 1]$ to get a quasi-unbiased estimate $\hat{f}_d$. The modification equation of G_y to approach G_A is given as

$$G_A \approx \frac{G_y - 0.5(1-\alpha)}{\alpha} \quad (25)$$

where α can be expressed as

$$\alpha = \left(1 + \frac{1}{SNR}\right)^{-2} \quad (26)$$

where $SNR = E_0/\sigma_z^2$ is the multipath signal-to-noise ratio.

C. Optimal Averaging Interval of the MA Technique

We investigate the optimal averaging interval of the MA technique based on the instantaneous estimate $\tilde{h}_{l,T}(n)$ with the knowledge of the estimate $\hat{f}_d$ developed above. In a time-variant Rayleigh fading channel, the channel estimates $\hat{h}_{l,MA}(n)$ at the output of the MA technique can be expressed as:

$$\hat{h}_{l,MA}(n) = \frac{1}{2N+1}\sum_{i=n-N}^{n+N} \tilde{h}_{l,T}(i) = \frac{\sum_{i=n-N}^{n+N} h_l(i)}{2N+1} + \frac{\sum_{i=n-N}^{n+N} z_{p,l}(i)}{(2N+1)\sqrt{\varepsilon_p}} = h_i(n) + \delta(n) + z_i(n) \quad (27)$$

where $\delta(n) \triangleq \frac{\sum_{i=n-N}^{n+N} h_l(i)}{2N+1} - h_l(n)$, is the tracking error caused by the averaging procedure, $z_l(n) \triangleq \frac{\sum_{i=n-N}^{n+N} z_{p,l}(i)}{(2N+1)\sqrt{\varepsilon_p}}$, denotes the error induced by the AWGN. It is obvious that the tracking error $\delta(n)$ is independent from the noise component $z_l(n)$, assuming that the channel coefficients and the noise components are mutually independent.

The linear process producing $\delta(n)$ has the following transform function [18]

$$H_\delta(z) = \frac{z^N - z^{-N-1}}{(2N+1)(1-z^{-1})} - 1 \quad (28)$$

The *magnitude-squared frequency response* (MSFR) can be expressed as

$$|H_\delta(j\omega)|^2 = \frac{(M^2+1) - M^2\cos(\omega) - 2M\cos(N\omega)}{M^2(1-\cos(\omega))} + \frac{2M\cos((N+1)\omega) - \cos((2N+1)\omega)}{M^2(1-\cos(\omega))} \quad (29)$$

where $M = 2N+1$. Equation (29) can further be expanded in a Taylor series when ω is a small number. Dropping the terms beyond the fourth degree, we have

$$|H_\delta(j\omega)|^2 \approx \frac{1}{36}N^2(N^2+2N+1)\omega^4 = \frac{1}{36}N^2(N+1)^2\omega^4 \quad (30)$$

To obtain the mean-square value of the components $\delta(n)$, Doppler power spectrum of the fading channel is required to be specified. Assuming that Jakes' channel model is employed, the Doppler power spectrum, denoted by $f_d(\omega)$, is given by [20]

$$f_d(\omega) = \begin{cases} \frac{2E_0}{\omega_d}\left[1 - \left(\frac{\omega}{\omega_d}\right)^2\right]^{-\frac{1}{2}} & |\omega| < \omega_d \\ 0 & |\omega| > \omega_d \end{cases} \quad (31)$$

where E_0 is the average power of the *l*-th multi-path, ω_d is the maximum Doppler-shift. Therefore, the mean-square value of the tracking error induced by the averaging procedure is

$$\varepsilon_\delta = \frac{1}{2\pi}\int_{-\omega_d}^{\omega_d} |H_\delta(j\omega)|^2 f_d(\omega)\, d\omega = \frac{1}{2\pi}\frac{N^2(N+1)^2T_s^4}{36} \cdot \frac{2E_0}{\omega_d}\int_{-\omega_d}^{\omega_d}\frac{\omega^4}{\sqrt{1-(\omega/\omega_d)^2}}\, d\omega = \frac{N^2(N+1)^2}{96}(\omega_d T_s)^4\sigma_l^2 \quad (32)$$

Because the noise variance produced by the MA channel technique is $\frac{\varepsilon_z = \sigma_z^2}{[(2N+1)\varepsilon_p]}$, the mean-square value of the total estimation error is expressed as:

$$\varepsilon_{Total} = \varepsilon_\delta + \varepsilon_z = \frac{N^2(N+1)^2}{96}(\omega_d T_s)^4 E_0 + \frac{\sigma_z^2}{(2N+1)\varepsilon_p} \quad (33)$$

The mean-square value of the tracking error is proportional to N^4, while the noise variance is inversely proportional to $2N+1$. The optimal MA interval M and the minimum mean-square value of the total estimation error are given by:

$$M = \sqrt[5]{b/4a}. \quad (34)$$

$$min\ (\varepsilon_{Total}) \approx 1.65\sqrt[5]{ab^4} \quad (35)$$

where $M = 2N+1$, $a = 6.51 \times 10^{-4}(\omega_d T_b)^4 E_0$, $b = \frac{\sigma_z^2}{\varepsilon_p}$.

TABLE I depicts the optimal averaging interval of the MA technique for the mobile channel with different normalized maximum fading rate $\omega_d T_s$ and different SNR of the multi-path with $\varepsilon_p = 1$.

TABLE I: Optimal Averaging Interval of the MA Channel Technique

E_0/σ_z^2 \ $\omega_d T_s$	-6db	-3db	0db	3db	6db
0.036	61	54	46	40	35
0.11	25	22	19	16	15
0.25	13	11	10	9	7
0.36	10	8	7	6	5

4. Simulation Results

Several simulations are carried out for the performance evaluation of the channel estimation algorithms and verify the theoretical analyses. The single user transmission is considered in the simulations for the downlink of a DS-CDMA system like WCDMA system with the carrier of 2GHz and the chip rate of 4.096Mcps [3]. QPSK modulation is employed and the multi-path Rayleigh fading channel model recommended by the ITU-R M.1225 is adopted. In the simulations, synchronization of the chip, symbol and frame timing is assumed to be perfect at the receiver.

To evaluate the performance of the MA technique, the mean-squared estimation error of the MA technique is compared with the MMSE technique in Figure 1 for typical vehicular speeds and a fixed pilot symbol rate equal to 16ks/s. As can be seen, the mean-squared error of the MA technique decreases as the averaging interval increases from $3T_s$ to $7T_s$ and to $9T_s$ for the mobile speed taking 500km/h and 350km/h, respectively. This is because the mobile channel is almost constant in the time range considered and a larger averaging interval results in less noise variance. In contrast, as the averaging interval increases further, the mean-squared error of the MA technique increases dramatically due to the significant variation of the mobile channel during the averaging interval considered and considerable tracking error yielded. From Figure 1, the optimal averaging interval of the MA technique is $7T_s$ for 500 km/h and $9T_s$ for 350 km/h, respectively. It is worth to note that the mean-squared error of the MA technique with the optimal averaging interval is close to that of the MMSE estimation, which is designed for the minimum mean-squared error. Thus, it can be concluded that by using the optimal averaging interval, the MA technique can provide estimation performance almost as good as that done by the MMSE technique.

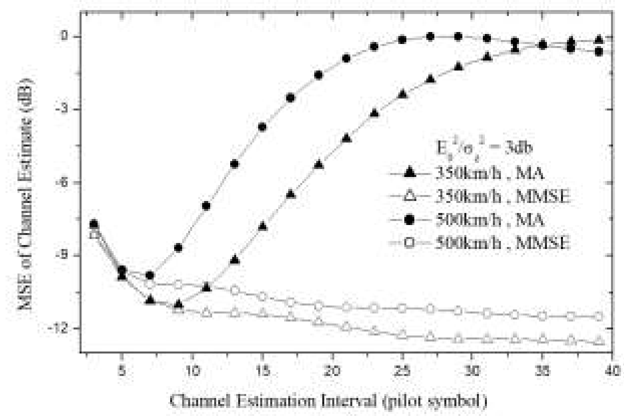

Figure 1. The total mean-squared estimation error versus different MA interval

Figure 2 shows the coded BER performance of the RAKE receivers employing different channel techniques proposed in Section II. The convolutional code of rate 1/3 and of constraint length 9 is used for the evaluation. The transmitted user's data takes the rate of 240kb/s with the interleaver depth equal to the frame length of 10ms. The spreading factor of the common pilot channel is 256. For the RAKE receiver employing the MA technique, using larger averaging intervals degrades the receiver performance severely, because a significant tracking error in the channel estimate cannot be avoided. As expected, the optimal averaging interval induces the best performance of the RAKE receiver, with almost the same performance as can be done by the RAKE receiver employing the MMSE or MAXSNR technique.

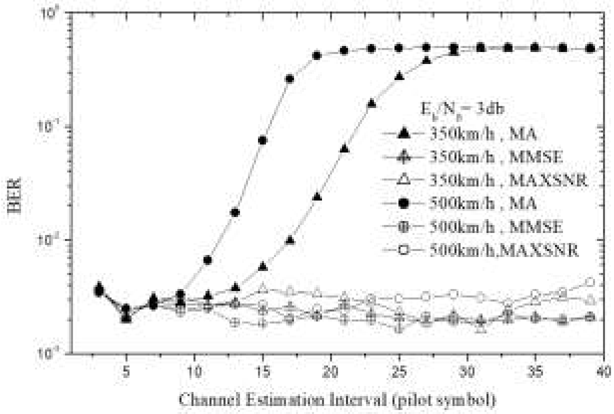

Figure 2. The coded BER of the RAKE receiver versus different MA interval

To evaluate the performance of the maximum Doppler-shift estimate $\hat{f}_d$, the fractional error of the estimate for various vehicular speeds is shown in Figure 3. E_b is the transmitted bit energy. It follows from the Figure that the fractional error is large when E_b/N_0 is lower than 2dB and approaches zero as E_b/N_0 increases to upwards of 5dB. It is also worth noting that the estimation error of $\hat{f}_d$ at lower speeds is relatively large due to the adverse effect of noise on the estimates given in (19).

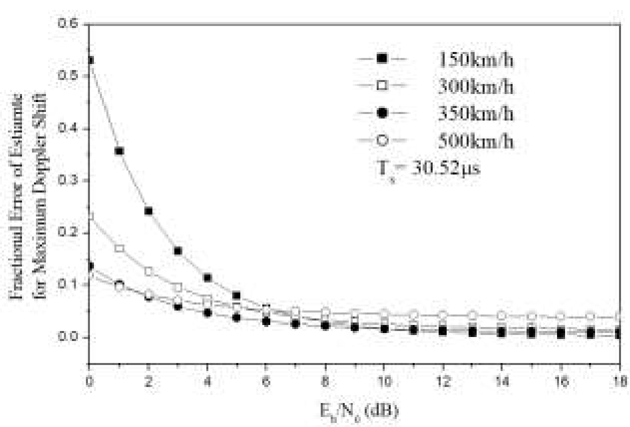

Fig. 3. The fractional error of the maximum Doppler-shift estimate ($T_s = 30.52\mu s$).

Figure 4 depicts the estimation performance of the vehicular speeds versus different E_b/N_0. The estimation procedure is carried out over a time interval of 1.5s. The estimated value approaches the true value of the vehicular speed when E_b/N_0 increases to 5dB. As E_b/N_0 reaches up to 10dB, very good estimation performance can be obtained.

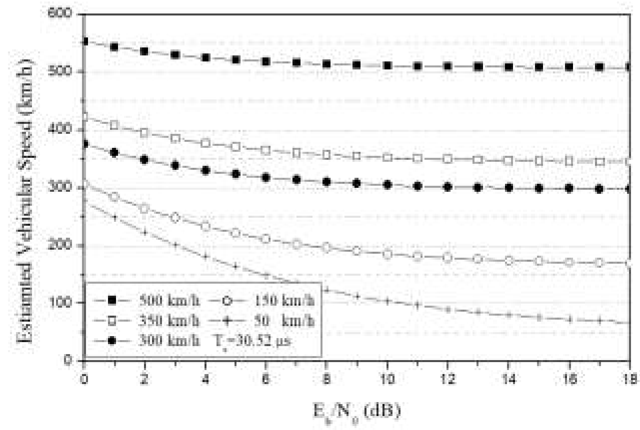

Figure 4. Performance of the vehicular speed estimator ($T_s = 30.52\mu s$).

For comparison, Figure 5 illustrates the effect of the modified estimate of $\hat{f}_d$ given in equation (25). It is found that the modified estimate $\hat{f}_d$ is acceptable even as E_b/N_0 is as low as 0dB in the cases of the mobile speeds taking the values of 300km/h and 350km/h. However, the modification is over-adjusted for the mobile speeds higher than 350km/h, whereas it is under-adjusted for the speeds lower than 300km/h. It attributes to the modification based on the linear approach of the hypergeometric function and the lack of an analytic relation between $F[-1/2, -1/2; 1; x]$ and $F[-1/2, -1/2; 1; \alpha x]$ when x is given arbitrarily.

To verify the mean-square value of the tracking error of the MA technique given in Equation (32), computer simulations are also conducted for the noise-free case ($\sigma_z^2 = 0$). The corresponding numerical results are provided in Figure 6 for various normalized maximum channel fading rates $\omega_d T_s$. As it is shown, the theoretical results are the same as the simulation results. Although the deviation of the theoretical results from the simulation results occurs when the averaging interval is extremely large, it will not limit the practical applications of our MA technique with the optimal averaging interval.

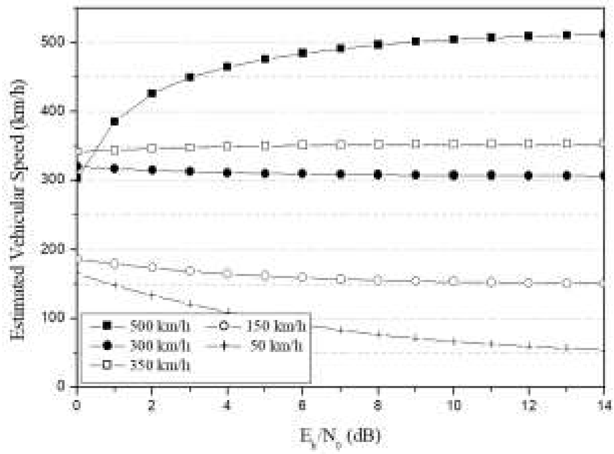

Figure 5. Performance of the modified vehicular speed estimator.

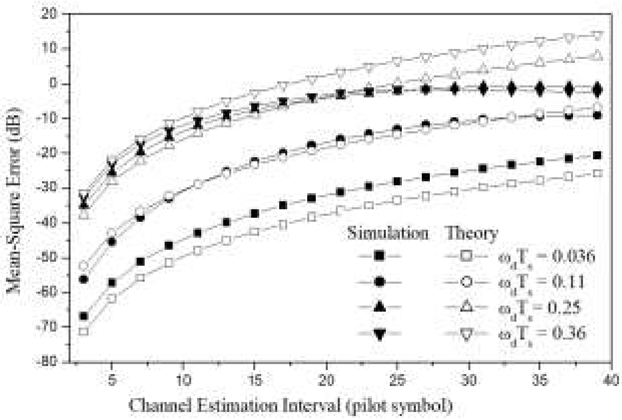

Figure 6. The mean-squared tracking error versus the MA interval.

The mean-square value of the total error of the MA channel technique given in Equation (33) is also investigated via computer simulations. Assuming that $E_0 = 1$ and $\frac{\sigma_Z^2}{\varepsilon_p} = 1$, the corresponding numerical results are provided in Figure 7. It is evident that for various mobile speeds, the optimal averaging interval can provide the best performance in that the total mean-squared error function is minimized, which coincides with our theoretical results.

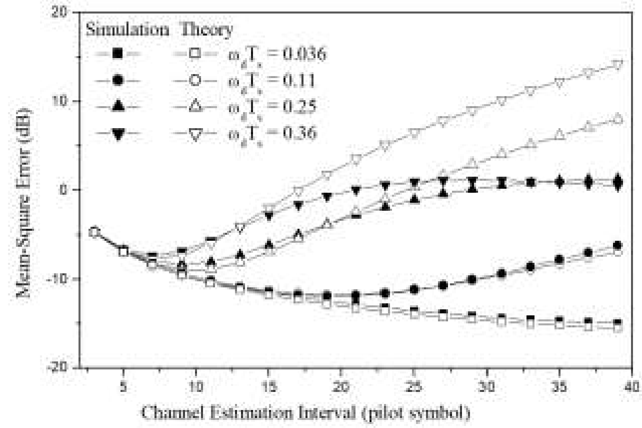

Figure 7. The total mean-squared estimation error ($\frac{\sigma_Z^2}{\varepsilon_p} = 1$) versus the MA interval.

Figure 8 and figure 9 show the coded BER performance of a RAKE receiver employing the MA technique with the averaging interval varying from 1 to 100 pilot symbols. The cases of $\omega_d T_s = 0.11$ and $\omega_d T_s = 0.25$ are respectively considered. The convolutional code of rate 1/3 and constraint length of 9 is employed with the interleaver depth equal to 32×96. It is evident from the Figures that there is a best averaging interval that can provide better receiver performance than other intervals for each specific normalized maximum fading rate. As can be seen, the averaging intervals leading to the best receiver performance coincide with the optimal averaging interval based on the criterion that the mean square error plus noise is minimized.

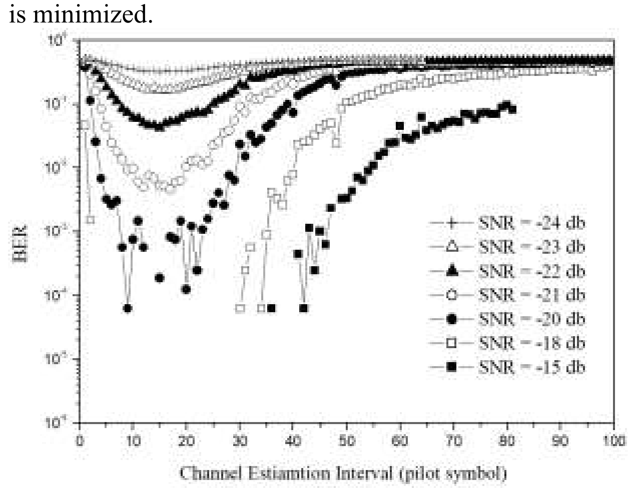

Figure 8. Coded BER of RAKE receiver versus the MA interval with $\omega_d T_s = 0.11$

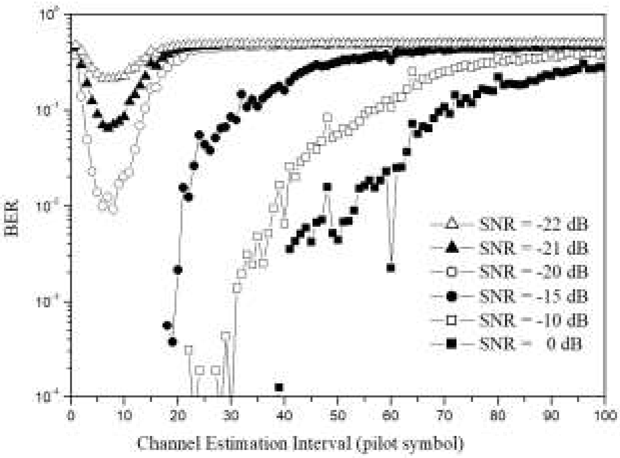

Figure 9. Coded BER of RAKE receiver versus the MA interval with $\omega_d T_s = 0.25$

5. CONCLUSION

As compared with the MMSE and MAXSNR techniques, the MA technique yields very good channel estimation performance with lower computational burden, provided that the optimal averaging interval is employed. However, the optimal averaging interval varies with the channel fading rate and the mobile speed. Hence the adaptive channel estimation is needed to get improved performance. The proposed estimate for channel maximum Doppler-shift is accurate at high SNR and the modified estimate enables a more accurate estimate at lower SNR. The optimal averaging interval of the MA technique, which leads to the channel estimates with the minimum mean-squared error, is presented in a closed-form equation and verified by simulation results. Consequently, the developed adaptive channel estimator selects the optimum averaging interval adaptively according to the proposed estimate of maximum Doppler-shift and enables the RAKE receiver to perform with simple implementation almost as well as those employing MMSE and MAXSNR channel estimation with high complexity.

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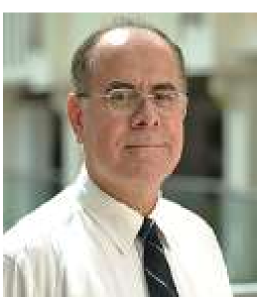

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