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Graphical Analysis of Single Sideband Modulation

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Abstract: In any undergraduate program in Electrical engineering or related fields, such as Electrical, Electronics, Computer Engineering or Telecommunications Engineering, at least one basic course on communication systems is given. Communication systems can be either analogue or digital. Analogue communication techniques are a pre-requisite to understanding digital communication systems that affects all aspects of our life. In analogue communication a key topic is Amplitude Modulation (AM) which includes Single Sideband (SSB) modulation. This paper presents a new technique of analyzing SSB graphically in frequency-domain in contrast to most of standard textbooks which normally analyze SSB in time-domain using trigonometric identities. This technique is a general one that can be applied for both single tones (sine or cosine) as well as complex signals. This makes it easy to apply, understand and visualize by undergraduate students encountering this topic for the first time. Effectiveness of the graphical technique was tested at Ahlia University, College of Engineering which shows this technique is preferred by students over the classical one.

Keywords: Single-side band (SSB), Hilbert Transform (HT), Phase-shift method, Modulation Property.

1. Introduction

In any undergraduate program in Electrical engineering or related fields, such as Electrical, Electronics, Computer Engineering or Telecommunications Engineering, a basic course on communication systems is given. Teaching analogue communication techniques is a pre-requisite to understanding digital communication systems that affects all aspects of our life. Typically, the course starts with analogue communications that covers basic modulation techniques such as amplitude and angle modulation. Amplitude modulation covers a range of topics: double sideband suppressed carrier (DSB-SC), double sideband total (full) carrier (DSB-TC) and single sideband (SSB). DSB-SC and DSB-TC require double bandwidth of the information message signal in the form of Lower Side Band (LSB) and Upper Side Band (USB). USB and LSB carry the same information and hence only one sideband need to be sent. Such modulation is defined as Single Side Band (SSB) modulation. In this technique, the carrier is not transmitted and hence it is called SSB–suppressed carrier (SSB-SC) which is simply called SSB modulation.

Normally analyzing SSB is done through trigonometric identities analysis in time-domain and Hilbert transform (HT) in frequency-domain. Using this technique is normally carried out as a mechanical exercise by students without an overall understanding of their work. It is normally prone to error and student face difficulty in applying different input message signals.

In this paper, we present a graphical technique for analyzing frequency-domain of an SSB system that is easy to understand and gives more insight to the students. This technique is a general one that can be applied for both single tones (sine or cosine) as well as complex signals. This technique is not covered in standard textbooks [1-5] but it is only found in University of California (Berkeley) archives [6] in a way that it may not be accessible to most students. Furthermore, the topic is mentioned very briefly such that it is difficult to understand by most students. This paper explains the topic in more details by showing the steps needed to achieve the results and gives more examples that makes the topic more comprehendible by students. The paper starts with a brief theory of SSB in section 2. Section 2 presents classical SSB theory (using trigonometric identities) in section 2.A, followed by HT technique in 2.B. Section 3 presents our method of graphical analysis of SSB modulation. Section 4 represents the demodulation of SSB message using the Graphical method. Section 5 analyses the students' performance using Graphical and

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Classical method, while the conclusion is given in section 6.

2. SINGLE SIDE BAND (SSB) THEORY

In SSB, only one of the sidebands is transmitted, thus saving half of the bandwidth – which is necessary to preserve frequency spectrum. In general, an SSB signal [1-5] can be represented by (1) and a mathematical derivation of this definition is given in [1].

$$\phi_{SSB\pm}(t) = m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t$$

$$= m(t)\,c(t) \mp \hat{m}(t)\hat{c}(t) \qquad (1)$$

where:

$m(t) \Rightarrow$ Message signal

$c(t) \Rightarrow$ Carrier signal

$\hat{m}(t) \Rightarrow$ Hilbert transform of $m(t)$, and it is the signal phase shifted by $\frac{\pi}{2}$

$\hat{c}(t) \Rightarrow$ Hilbert transform of $c(t)$

A block diagram of a Single-sideband modulation using phase-shift method is shown in fig. 1, which is direct application of the basic definition of SSB given in (1).

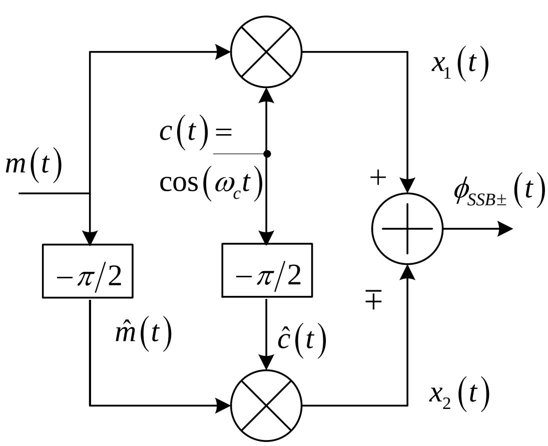

Figure 1. SSB generation – phase shift method

Following the block diagram of SSB modulation shown in fig. 1 is straightforward in time-domain with trigonometric identities but can be more difficult for students in frequency-domain, especially for the sine wave and generalized signals. Analyzing this circuit normally consider single tone (often cosine) to illustrate its principles of operation, using trigonometric identities, which is tedious and is carried out by students as a mechanical exercise, without any insight. Dealing with more complex signals is normally avoided due to its complexity.

A. SSB using trigonometric identities

Normally in teaching communications systems where analogue modulation schemes forms the main part of the course; a single tone is taken as an input message signal and then general case is considered to simplify the concepts for students. Quite often, the message signal is taken as a cosine wave as it is a real signal to avoid complication of the sine wave, which can involve (j) factor. In this section, we apply single tone messages to the SSB modulator, both sine and cosine functions.

For a single tone cosine wave:

$$m(t) = cos(2\pi f_m t)$$
$$c(t) = cos(2\pi f_c t)$$
$$\hat{m}(t) = \cos\left(2\pi f_m t - \frac{\pi}{2}\right) = \sin(2\pi f_m t)$$
$$\hat{c}(t) = \cos\left(2\pi f_c t - \frac{\pi}{2}\right) = \sin(2\pi f_c t)$$
$$\hat{m}(t)\hat{c}(t) = \sin(2\pi f_m t)\sin(2\pi f_c t)$$
$$= \frac{1}{2}\{\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t\}$$
$$m(t)\,c(t) = \cos(2\pi f_m t)\cos(2\pi f_c t)$$
$$= \frac{1}{2}\{\cos 2\pi(f_m - f_c)t + \cos 2\pi(f_m + f_c)t\}$$
$$\phi_{SSB+}(t) = m(t)\,c(t) - \hat{m}(t)\hat{c}(t)$$
$$= \begin{bmatrix} \frac{1}{2}\{\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t\} \\ -\frac{1}{2}\{\cos 2\pi(f_m - f_c)t - \cos 2\pi(f_m + f_c)t\} \end{bmatrix}$$
$$= \cos 2\pi(f_c + f_m)t$$

For a single tone sine wave:

$$m(t) = \sin(2\pi f_m t)$$
$$c(t) = \cos(2\pi f_c t)$$
$$\hat{m}(t) = \sin\left(2\pi f_m t - \frac{\pi}{2}\right) = -\cos(2\pi f_m t)$$
$$\hat{c}(t) = \cos\left(2\pi f_c t - \frac{\pi}{2}\right) = \sin(2\pi f_c t)$$
$$\hat{m}(t)\hat{c}(t) = -\cos(2\pi f_m t)\sin(2\pi f_c t)$$
$$= -\frac{1}{2}\{\sin 2\pi(f_c - f_m)t + \sin 2\pi(f_c + f_m)t\}$$
$$m(t)\,c(t) = \sin(2\pi f_m t)\cos(2\pi f_c t)$$
$$= \frac{1}{2}\{\sin 2\pi(f_m - f_c)t + \sin 2\pi(f_c + f_m)t\}$$
$$= \frac{1}{2}\{-\sin 2\pi(f_c - f_m)t + \sin 2\pi(f_c + f_m)t\}$$

$$\phi_{SSB+}(t) = m(t)\,c(t) - \hat{m}(t)\hat{c}(t)$$

$$= \begin{bmatrix} \frac{1}{2}\{-\sin 2\pi(f_c - f_m)t + \sin 2\pi(f_c + f_m)t\} \\ +\frac{1}{2}\{\sin 2\pi(f_c - f_m)t + \sin 2\pi(f_c + f_m)t\} \end{bmatrix}$$

$$= \sin 2\pi(f_c + f_m)t$$

The frequency content of the SSB output can be found by applying the Fourier Transform (FT). Alternatively, plotting frequency spectrum follows directly from mathematical representations, noting that the amplitudes are divided equally between positive and negative frequencies, see fig. 4 and fig. 5.

B. Using Hilbert Transform (HT)

HT technique is very important tool when dealing with narrow-band signals such as modulated waveforms, where a small band of frequencies is centered around a high carrier frequency.

HT is defined as follows:

$$\hat{f}(t) \rightleftharpoons \hat{F}(f) = -jF(f)\,\mathrm{sgn}(f) = \begin{cases} -jF(f) & f \geq 0 \\ +jF(f) & f < 0 \end{cases} \quad (2)$$

Based on (2), a Hilbert Transform operation can be represented as shown in fig. 2.

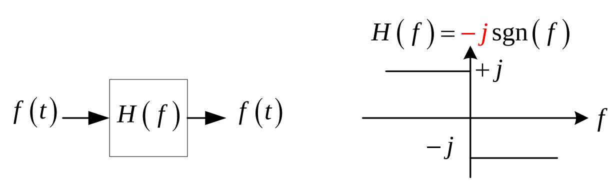

Figure 2. Hilbert transformer

To find $\hat{f}(t)$ mathematically:

$$\hat{f}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau}\,d\tau$$

The inverse Hilbert transform is found by:

$$f(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{f}(\tau)}{t-\tau}\,d\tau$$

Mathematical solution is tedious, and alternatively, based on basic definition of Hilbert transform, HT of a signal is found by applying the following three steps:

1. Finding the FT of the function $F(f)$
2. Apply basic definition of the HT:

$$\hat{F}(f) = -jF(f) \cdot \mathrm{sgn}(f)$$

3. Find the inverse FT which will be the required

$$HT{:}\, \hat{f}(t) \rightleftharpoons \hat{F}(f)\,.$$

An example illustrating this procedure follows.

Example:
Find the HT of the following function:

$$g(t) = \cos(2\pi f_c t)$$

Solution:
Step 1: Finding the FT of the function

$$G(f) = \tfrac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

Step 2: Applying basic definition of HT in (b) above:

$$\hat{G}(f) = -j\,\mathrm{sgn}(f)\,G(f)$$

$$= -j\frac{1}{2}\{\delta(f - f_c) + \delta(f + f_c)\}\,\mathrm{sgn}(f)$$
$$= -j\frac{1}{2}\{\delta(f - f_c)(+1) + \delta(f + f_c)(-1)\}$$
$$= j\frac{1}{2}\{\delta(f + f_c) - \delta(f - f_c)\}$$

Step 3: Taking the inverse FT:

$$\hat{G}(f) \rightleftharpoons \hat{g}(t) = \sin(2\pi f_c t)$$

Therefore, HT of $\cos(2\pi f_c t)$ is: $\sin(2\pi f_c t)$, i.e.

$$\cos(2\pi f_c t) \Leftrightarrow \sin(2\pi f_c t)$$

3. SSB MODULATOR – GRAPHICAL SOLUTION TECHNIQUE

To prepare the ground for this suggested technique, the HT example depicted in section 2B can be solved graphically in frequency-domain. After finding the FT of the signal (step 1), its HT (step 2) is found directly from HT definition, which is multiplying the HT frequency response with that of the message function, fig. 3. Taking the inverse FT of the resultant frequency-domain (step 3) gives the Hilbert Transform of the message. Fig. 3 presents the way HT is found for two single tone signals $[\cos(2\pi f_o t)]$ and $[\sin(2\pi f_o t)]$.

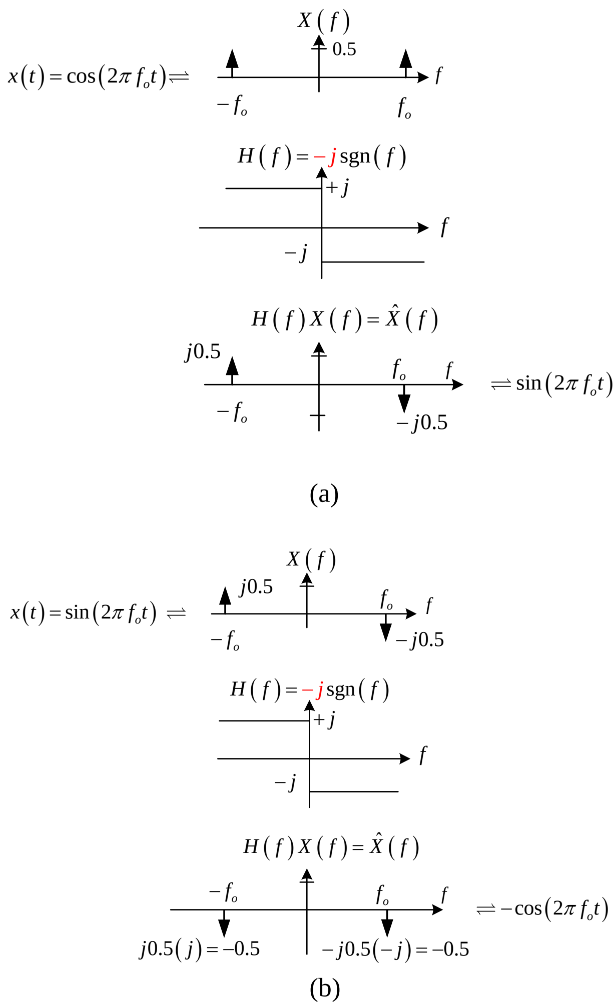

Figure 3. HT example for two single tone signals (a) $[\cos(2\pi f_o t)]$ and (b) $[\sin(2\pi f_o t)]$

It is clear how easy it is to apply HT using graphical technique. This technique applied to SSB generation shown in fig. 1. We will first consider single tone modulating signals, $f(t) = \cos(2\pi f_m t)$ and $f(t) = \sin(2\pi f_m t)$, then we present a general case of modulating signal, $f(t)$, fig. 4, 5 and 6 respectively. The results presented are for USB SSB generation $[\Phi_{SSB+}(t)]$ but the process is the same for LSB SSB $[\Phi_{SSB-}(t)]$, the only difference is the addition sign at the last (summing) stage of the modulator. Note that we have used the modulation property given in the appendix to plot some parts of our figures. The carrier has two possibilities, and hence:

$$m(t)\cdot\cos(2\pi f_o t) \rightleftharpoons \frac{1}{2}\{M(f+f_o)+M(f-f_o)\}$$
$$m(t)\cdot\sin(2\pi f_o t) \rightleftharpoons \frac{j}{2}\{M(f+f_o)-M(f-f_o)\}$$

The spectrum of the modulating signal is shifted around the carrier, with the resulting amplitude being equal to the multiplication of the signal spectrum amplitude with that of the carrier (in case of cosine it is 0.5 in both positive and negative frequencies and for a sine carrier it is $(j0.5)$ in negative frequency and $(-j0.5)$ in the positive frequency). When shifting spectrum, it is recommended for the students to keep the original amplitudes levels of the signal spectrum without inverting in case of sine carrier. Then multiply the signal and carrier amplitudes where the multiplication result will determine the sign of the spectrum.

4. SSB DEMODULATOR – GRAPHICAL SOLUTION TECHNIQUE

For SSB detection, the synchronous detector is used, fig. 7.

Mathematically:

$$\phi_{SSB\pm}(t) = m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t$$

At receiver (Rx):

$$\phi_{SSB\pm}(t).\cos\omega_c t = m(t)\cos^2\omega_c t \mp \hat{m}(t)\sin\omega_c t \cdot \cos\omega_c t$$

$$= \overbrace{m(t)}^{Signal} \cdot \frac{1}{2}\left(1 + \overbrace{\cos 2\omega_c t}^{high\ freq.}\right)$$

$$\mp \hat{m}(t) \cdot \frac{1}{2}[\underbrace{\sin 2\omega_c t}_{high\ frequency} + \cancel{\sin 0}]$$

Applying low pass filter (LPF) operation, then:

$$e_o(t) = \frac{1}{2}m(t)$$

Graphically, SSB demodulator operation is illustrated in fig. 8 for the upper sideband signal, which is easy and straightforward.

5. STUDENTS PERFORMANCE

Effectiveness of the graphical technique was tested at Ahlia University, College of Engineering and to have a fair comparison, only semesters where students attempted both classical and graphical methods are considered in our analysis. Semesters where students attempted solely Graphical or solely Classical were excluded from the analysis. A total number of 56 samples were considered where 24 of 56 opted to use Classical method while 32 of 56 opted to use Graphical method to solve the SSB question i.e., $P(Classical) = 0.4286$ and $P(Graphical) = 0.5714$ respectivelty. The grades of the SSB question were normally distributed as shown in fig. 9 with a mean of $\mu_{Classical} = 65.42\%$ and $\mu_{Graphical} = 75.16\%$ and a standard deviation of $\sigma_{Classical} = 22.01$ and $\sigma_{Graphical} = 22.23$. Intuitively, the students who opted to use Graphical method performed better than the others based on the mean analysis. This intuition can be further supported using conditional probability and Bayes' rule shown in (3) which reflects the preference of choice to succeed where the success rate given they choose Classical and Graphical are $P(Success \mid Classical) = 0.6542$ and $P(Success \mid Graphical) = 0.7516$ respectively. The statistical and conditional analysis showed a clear intuition to the preference of choice to succeed with a rate of 60.51% and a success rate of 75.16% for the Graphical method. A summary is given in Table I.

TABLE I. STATISTICAL COMPARISON BETWEEN GRAPHICAL AND CLASSICAL METHODS

	Graphical	Classical
Method attempted by students (%)	57.14	42.86
Average grade (%)	75.16	65.42
Preference rate (%)	60.51	39.49

$$P(Graphical \mid Success) = \frac{P(Success \mid Graphical).P(Graphical)}{P(Success)} \quad (3)$$

$$= \frac{P(Success \mid Graphical).P(Graphical)}{P(Success \mid Graphical).P(Graphical) + P(Success \mid Classical).P(Classical)}$$

$$= \frac{0.7516 \times 0.5714}{0.7516 \times 0.5714 + 0.6542 \times 0.4286} = 0.6051$$

$$P(Classical \mid Success) = 1 - P(Graphical \mid Success)$$

$$= 1 - 0.6051 = 0.3949$$

6. CONCLUSIONS

This paper presented a new way of analyzing SSB graphically in frequency-domain in contrast to most of standard textbooks which normally analyze SSB in time-domain using trigonometric identities. This technique is easy to apply and understand by students. It gives the student an insight of the frequency content of the SSB modulator at each stage. Furthermore, it is straightforward application for various modulating signals whether it is single tone (sine or cosine) or a general signal. This technique has been utilized at Ahlia University in a first undergraduate course on Communication Systems, where the statistical and conditional analysis showed a clear intuition to the preference of choice to succeed with a rate of 60.51% and a success rate of 75.16% for the Graphical method, while for the Classical method, these figures are only 39.49% and 65.42% respectively.

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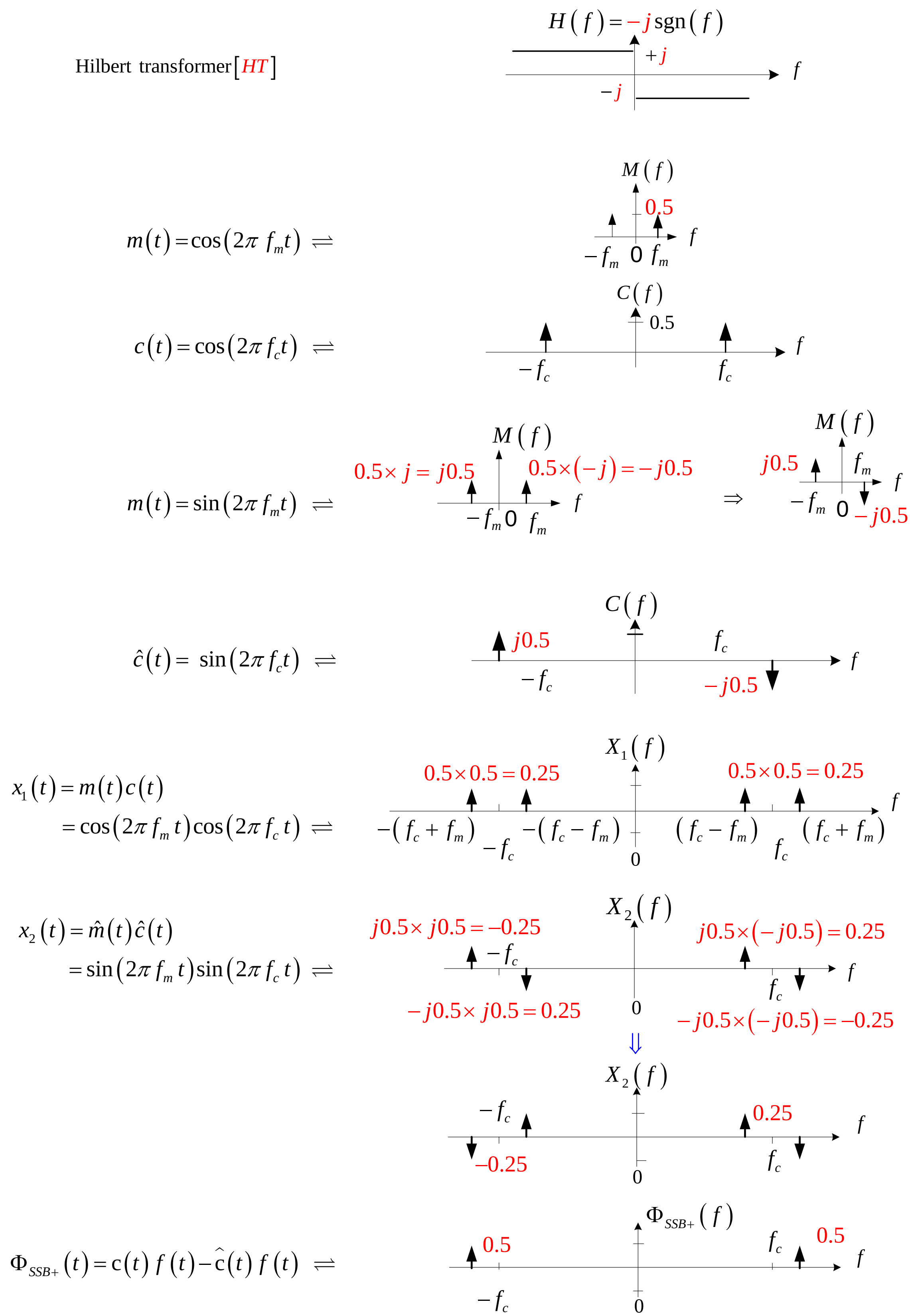

Figure 4. SSB for a single tone $[\cos(2\pi f_m t)]$

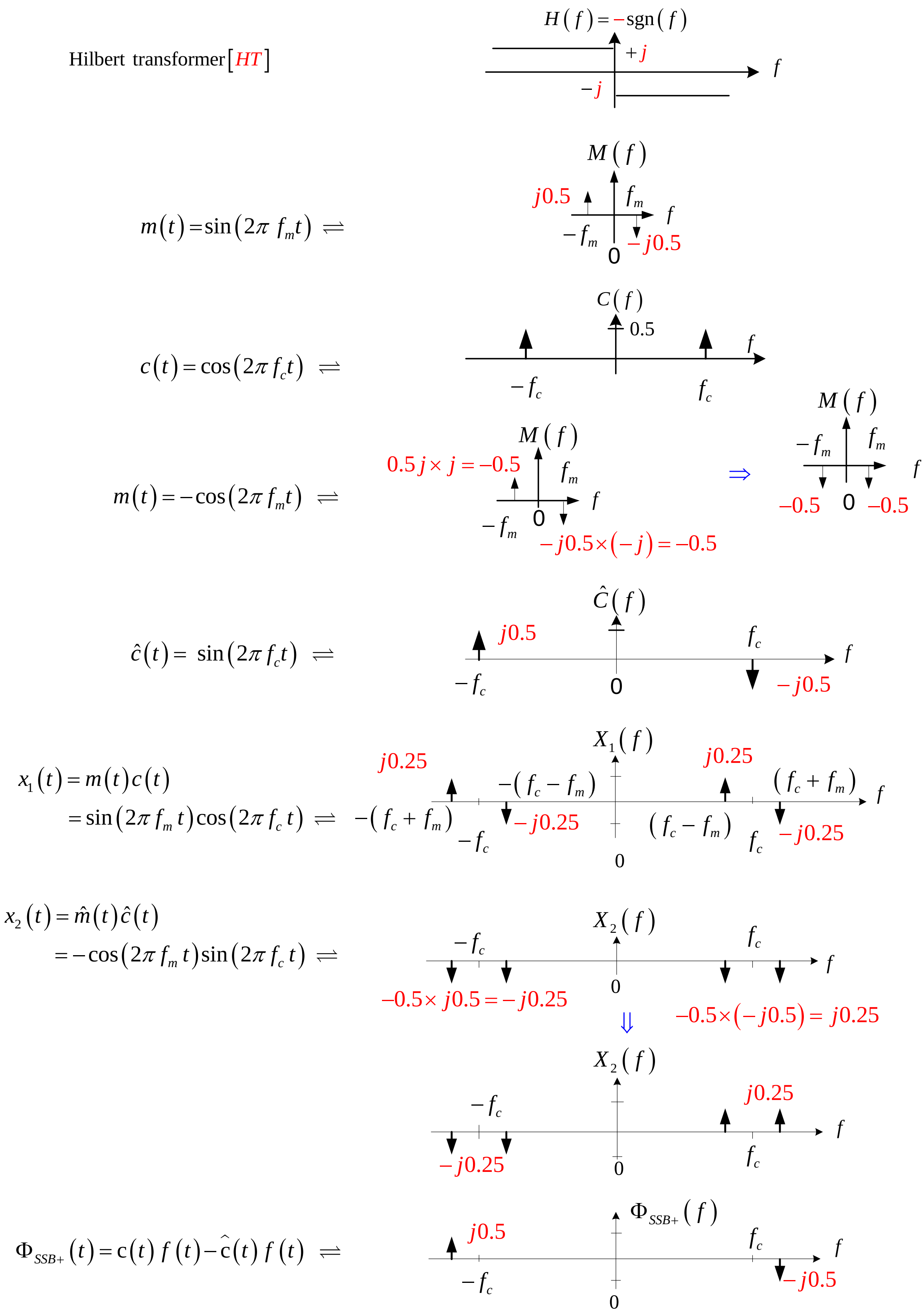

Figure 5. SSB for a single tone [$\sin(2\pi f_m t)$]

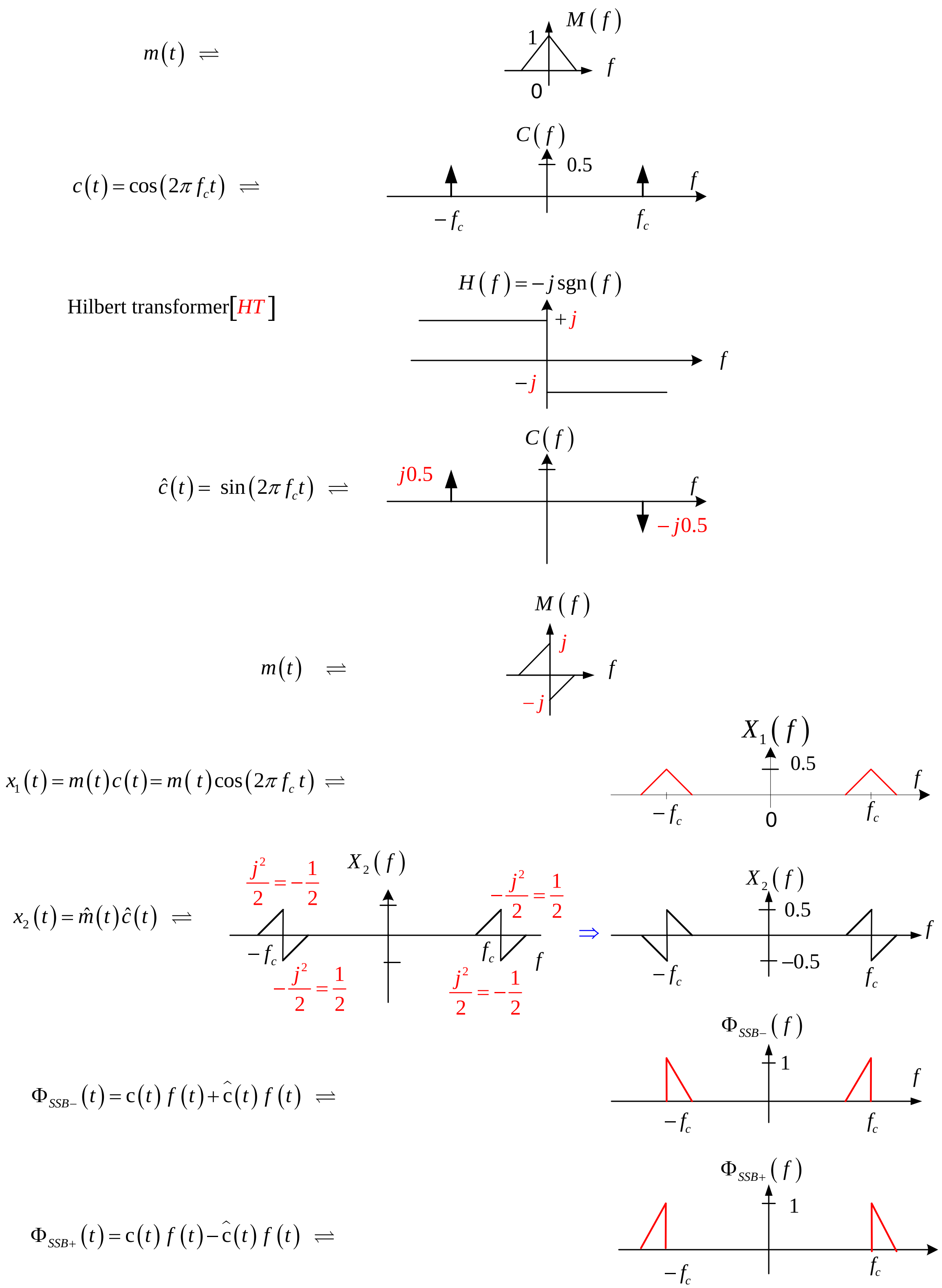

Figure 6. SSB for a generic signal $m(t)$

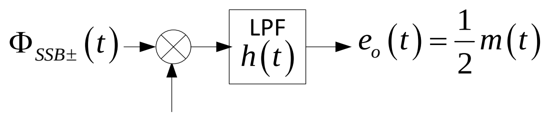

Figure 7. SSB demodulator

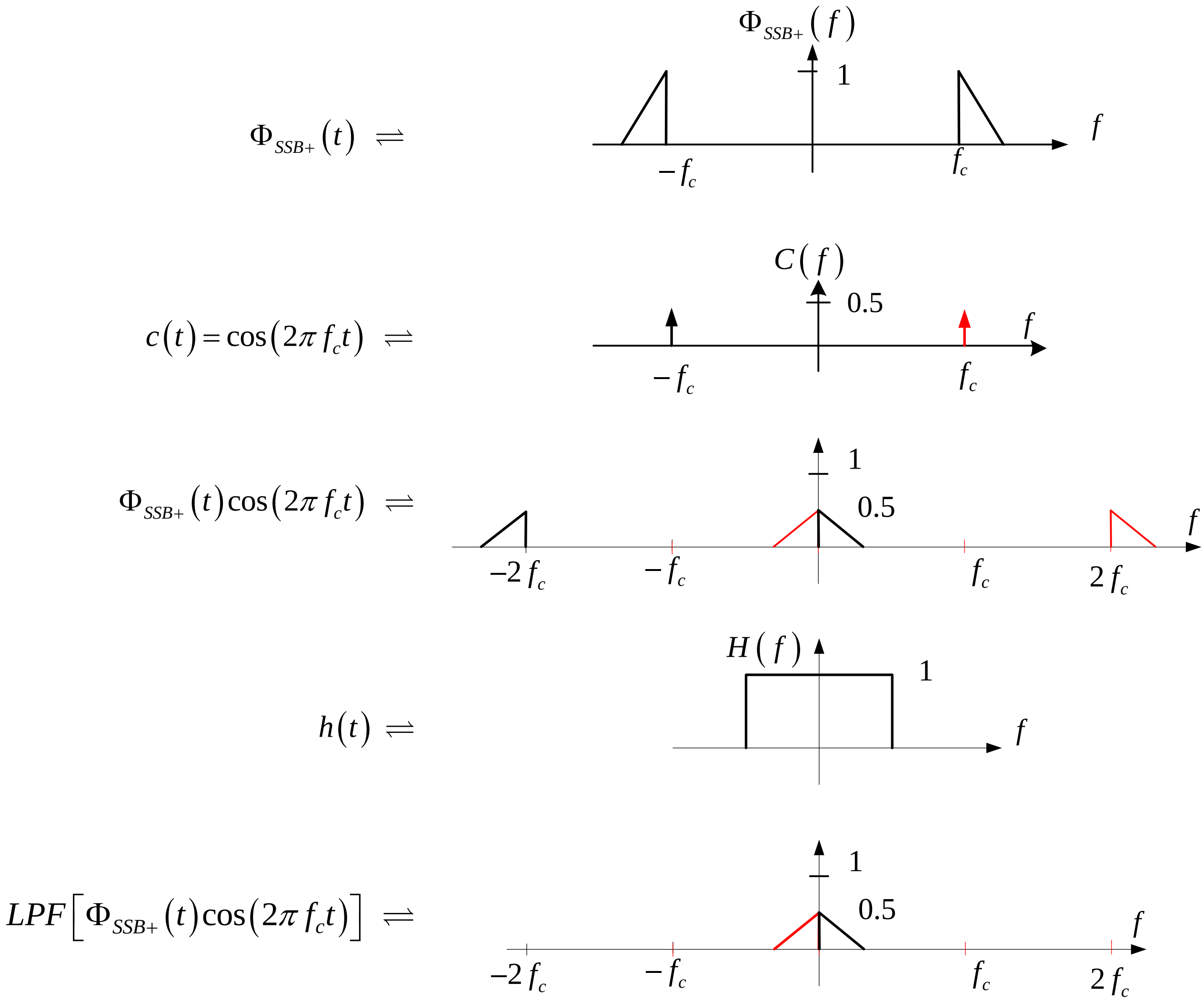

Figure 8. SSB demodulator operation in frequency-domain

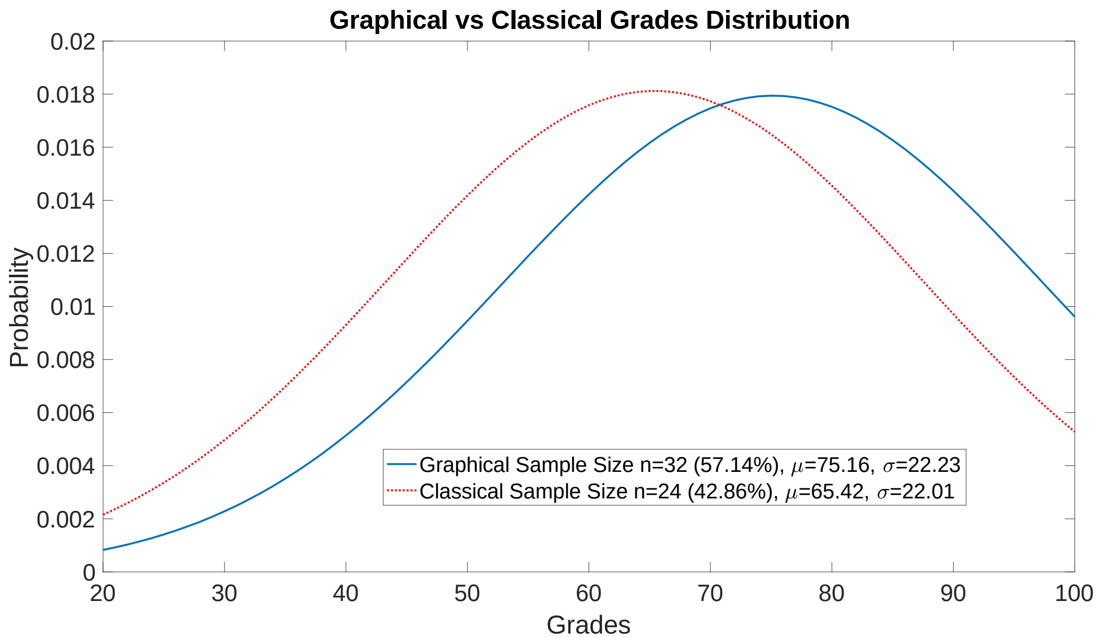

Figure 9. Graphical vs classical method comparison

APPENDIX: MODULATION PROPERTY

Modulation property is defined as follows:

$$m(t)\cdot\cos(2\pi f_o t) \rightleftharpoons \frac{1}{2}\{M(f+f_\circ)+M(f-f_\circ)\}$$

$$m(t)\cdot\sin(2\pi f_o t) \rightleftharpoons \frac{j}{2}\{M(f+f_\circ)-M(f-f_\circ)\}$$

Pictorially, the spectrum of the modulating signal is shifted around the carrier. In case of a sine carrier, the spectrum is multiplied by a factor of (j) and that of the positive frequency side is inverted.

The amplitude of the signal is multiplied by that of the carrier to determine the resultant amplitude of the frequency spectrum. When shifting spectrum, it is recommended for the students to keep the original amplitudes levels of the signal spectrum without inverting in case of sine carrier. Then multiply the signal and carrier amplitudes where the multiplication result will determine the sign of the spectrum.

We will take some special cases:

$$m(t)=\cos(2\pi f_m t) \rightleftharpoons \frac{1}{2}\{\delta(f+f_m)+\delta(f-f_m)\}$$

$$m(t)=\sin(2\pi f_m t) \rightleftharpoons \frac{j}{2}\{\delta(f+f_m)-\delta(f-f_m)\}$$

$$c(t)=\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\{\delta(f+f_c)+\delta(f-f_c)\}$$

$$c(t)=\sin(2\pi f_c t) \rightleftharpoons \frac{j}{2}\{\delta(f+f_c)-\delta(f-f_c)\}$$

The four possible outcomes are presented in fig. A.

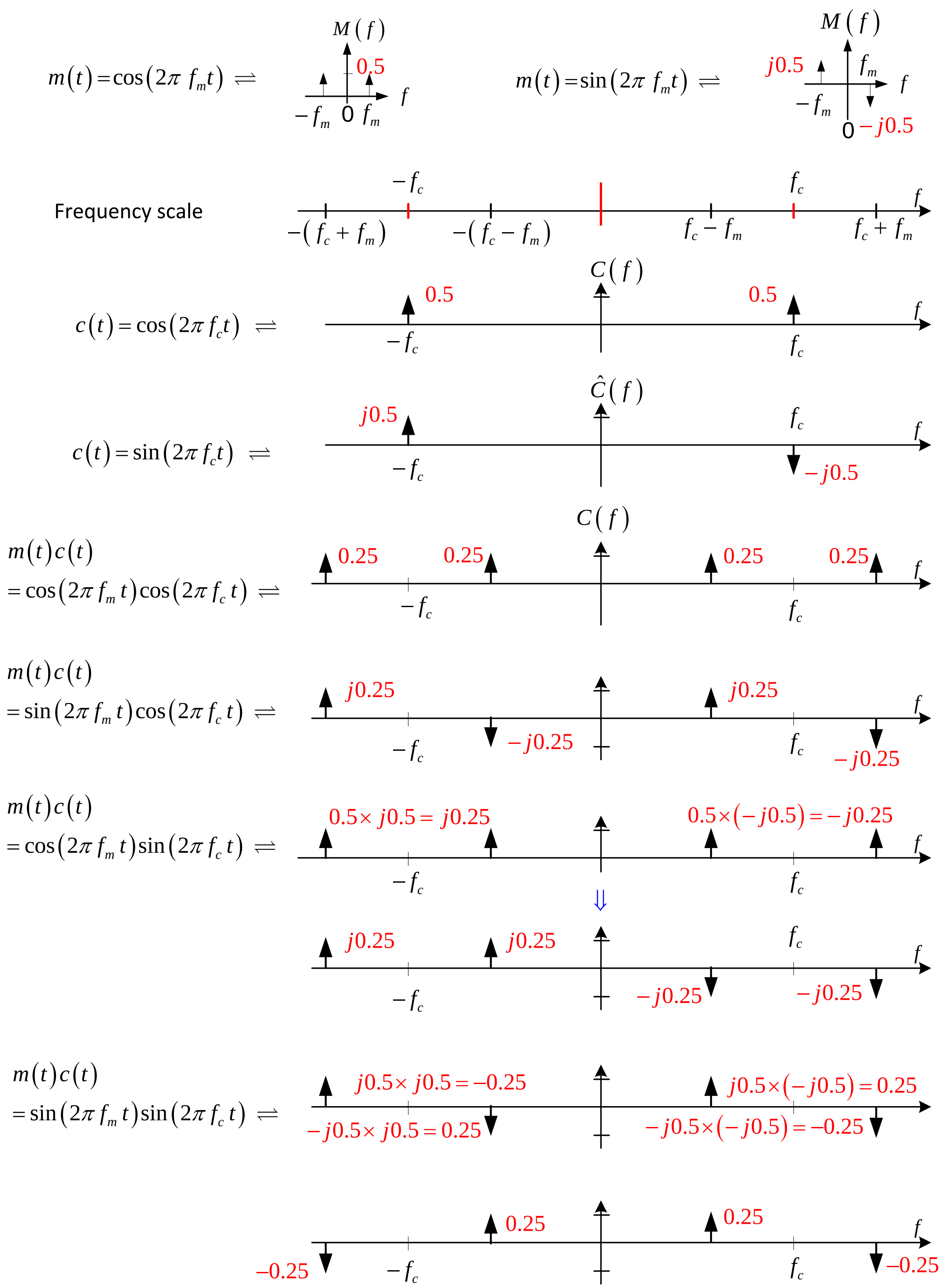

Figure A: Modulation property

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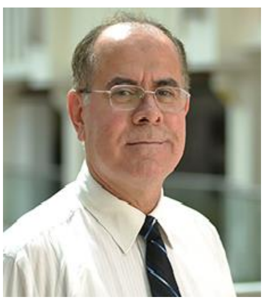

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