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Statistical Identification of Chemotactic Parameter in Bacterial Foraging Algorithm

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Abstract: Optimization is a complex process whose success depends on the formulation of the objective function (OF) and the selection of parameters in the optimization algorithm. This paper presents a statistical approach to select the values of such parameters in the optimization algorithms. It uses statistical tests to determine the stability of the algorithm and identify the best value of the parameter. The approach determines the chemotactic parameter in the Bacterial-Foraging Optimization Algorithm (BFOA). The advantage of the proposed approach is that it does not require (i) complete knowledge of the subject, (ii) mathematical analysis to determine the parameter, and (iii) it is extendible to *n*-dimensional systems. Also, a logical approach, based on the parameter sensitivity and error dynamics, to select the OF for PI controller-tuning problem in Shunt Active Power Filter (SAPF) is presented. The results obtained using the proposed approach are verified analytically.

Keywords: Bacteria Foraging, Chemotaxis, Statistics

1. INTRODUCTION

Bacterial-Foraging Optimization Algorithm (BFOA) is a nature-inspired optimization algorithm. It's effectiveness in solving real parameter optimization problems in terms of convergence speed, and accuracy as compared to GA, PSO, etc. has popularized the algorithm [1]. It eliminates animals with poor foraging strategies and propagates species with successful foraging behavior, which enjoy reproductive success later. Foraging theory tries to maximize energy intake per unit time with physiology and environment constraints. However, better performance of BFOA comes at the cost of selecting the algorithm parameters, which are more in number as compared to PSO, etc. Thus, for its successful implementation, their determination is essential.

Chemotaxis is a significant step lying at the heart of BFOA. It implements a type of optimization process, which is a foraging strategy, where bacteria try to reach nutrient concentration while avoiding harmful substances [2]. The size of the chemotactic step, *C*, is a deciding factor. It guides the algorithm towards the optimal solution. The parameters in the optimization algorithm are selected mathematically or require complete knowledge of the subject. The algorithm presents no approach to select the value of this parameter. The dynamics of *C* have been mathematically analyzed in [5], but the analysis has been limited to a one-dimensional problem, whereas the real-world problems are multi-dimensional.

This paper presents a statistical approach to obtain the values of the chemotactic step, *C(i)* experimentally. The objective is to provide an easy to implement selection criteria without going into detailed mathematical analysis and cumbersome nonlinear dynamics of the problem. The experimental data is obtained by running the algorithm for different values of *C(i)*. This helps in identifying the pattern and realizes the stable region to select the chemotactic step size. The statistical tests, namely frequency deviation, standard deviation, skewness, kurtosis, and box plots **[4],** help to determine the most suitable values from this data. The approach offers the following advantages:

1. Since the approach is statistical; it is extendible to the *n*-dimensional problem, which is not possible with the mathematical approach given in [5].
2. The problem dynamics and stability of the algorithm at a particular value of *C(i)* can be analyzed.
3. It can be used to determine different parameters in different algorithms.

The approach is tested on the practical problem of proportional and Integral (PI) controller tuning for Shunt Active Power Filter (SAPF) application [3]. SAPF is a power electronic device which is used to eliminate the

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harmonic current and voltages occurring in the electrical power distribution systems. PI controller is one of its components, which ensures the device performs its operation successfully [3]. It presents a two-dimensional practical optimization problem. Moreover, a detailed analytical analysis for the OF selection of the PI controller problem based on the sensitivity of the parameters and error dynamics has been presented.

2. BACTERIA FORAGING OPTIMIZATION ALGORITHM

BFOA attempts to maximize per unit energy intake of *E.coli* bacterium using four steps, namely chemotaxis, swarming, reproduction, and elimination–dispersal [2],[5].

A. Chemotaxis

It refers to the locomotory response, i.e., swim or tumbles, of a cell to its environment. The chemotactic movement is given by

$$\theta^i(j+1,k,l)=\theta^i(j,k,l)+C(i)\frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}} \quad (1)$$

where Δ indicates unit length vector in random direction with values [-1, 1], $\theta^i(j,k,l)$ represents the i^{th} bacterium at j^{th} chemotactic step, k^{th} reproductive step, and l^{th} elimination-dispersal step, C(i) is the step-size in the random direction specified by tumble.

B. Swarming

It is a collective behavior possessed by organisms, where they aggregate to approach the best entity. The cell-cell signaling is represented as (2). Here $J_{cc}(\theta, P(j,k,l))$ is the cost at location of bacterium, S is the total number of bacteria, p is the number of variables to be optimized, $d_{attractant}$ is depth of attractant released by cell, $\omega_{attractant}$ is measure of width of the attractant signal, $h_{repellant}$ is height of repellant effect, and $\omega_{repellant}$ is a measure of width of repellant.

$$\begin{aligned} J_{cc}(\theta,P(j,k,l)) &= \sum_{i=1}^{S} J_{cc}(\theta,\theta^i(j,k,l)) \\ &= \sum_{i=1}^{S}\left[-d_{\text{attractant}}\exp\left(-\omega_{\text{attractant}}\sum_{m=1}^{p}\left(\theta_m-\theta_m^i\right)^2\right)\right] \\ &\quad + \sum_{i=1}^{S}\left[h_{\text{repellant}}\exp\left(-\omega_{\text{repellant}}\sum_{m=1}^{p}\left(\theta_m-\theta_m^i\right)^2\right)\right] \end{aligned} \quad (2)$$

C. Reproduction

During it, the healthy bacteria split asexually and remain in the swarm, while the least healthy bacteria die out from the swarm to keep the swarm size constant.

D. Elimination and dispersal

With dynamic surroundings, the generation of some new bacteria and the death of old bacteria are always occurring. It simulates this rapidly changing process.

The detailed pseudo-code of the algorithm is presented, as shown in Figure 1.

Step-1 Initialize parameters p, S, N_c, N_s, N_{re}, N_{ed}, P_{ed}, $C(i)$, θ^i.
Step-2 Elimination – dispersal loop: $l=l+1$
Step-3 Reproduction loop: $k=k+1$
Step-4 Chemotaxis loop: $j=j+1$
  a. For i = 1, 2... S take a chemotactic step for bacterium i as,
  b. Compute $J(i,j,k,l)$. Let $J(i,j,k,l)=J(i,j,k,l) + J_{cc}(\theta^i(j,k,l), P(j,k,l))$.
  c. Let $J_{Last} = J(i,j,k,l)$, *and* save it.
  d. Tumble: Generate $\Delta(i)=R^p$ with $\Delta_m(i)$, m = 1,2,..., p, [-1, 1].
  e. Move using (1).
  f. Compute $J(i,j+1,k,l)$.
    Let $J(i,j+1,k,l)= J(i,j+1,k,l) + J_{cc}(\theta^i(j+1,k,l),P(j+1,k,l))$.
  g. Swim
    (i). Let m = 0 (counter for swim length)
    (ii). While $m < N_s$ (if have not climbed down to long)
      - Let $m=m+1$.
      - If $J(i,j+1,k,l) < J_{Last}$ (if doing better), let $J_{Last} = J(i,j+1,k,l)$ and let $\theta^i(j+1,k,l)=\theta^i(j,k,l)+C(i)\frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}}$ and use this $\theta^i(j+1,k,l)$ to compute new $J(i,j+1,k,l)$ as we did in f.
      - Else let $m = N_s$. His is the end of the while statement.
  h. Go to the next bacterium ($i+1$) if $i \neq S$ (i.e., go to b.).
Step-5 If $j < N_c$, then go to step 4.
Step-6 Reproduction
  a. For the given k and l, and each i = 1,2, ..., S. Let $J^i_{health} = \sum_{j=1}^{N_c+1} J(i,j,k,l)$ Sort bacteria and chemotactic parameters $C(i)$ in order of ascending cost J_{health}.
  b. S_r bacteria with the highest J_{health} value die and remaining S_r bacteria with the best values split.
Step-7 If $k < N_{re}$, go to step 3.
Step-8 Elimination-dispersal: For i = 1, 2... S with probability P_{ed}, eliminate and disperse each bacterium. If $l < N_{ed}$, then go to step 2; otherwise end.

Figure 1. Pseudocode of the BFOA

3. OBJECTIVE FUNCTION (OF) SELECTION

In optimization, the problem is formulated as an OF. Its effective formation requires proper mathematical analysis, as it has a direct effect on the accuracy of the solution. For the PI controller tuning problem in SAPF application considered. Generally, the OFs are either Integral Time Square Error (ITSE) or Integral Time

Absolute Error (ITAE) performance indices **[3].** The advantage of using them as OF is that they weight the large initial error and penalize the small error occurring later in the response heavily [6]**.** However, selection amongst these two is difficult. This section attempts to solve this problem by considering the system dynamics and sensitivity to the model parameters.

A. Shunt Active Power Filter (SAPF)

To enhance the performance of the SAPF, some methods and topologies have been devised **[3].** In this work, the topology shown in Figure 2 and detailed in [1] is considered. This topology utilizes the PI controller, which plays a vital role in improving the dynamics of the SAPF system. The gains of the proportional (K_p) and Integral (K_I) controller decide the operating point of the controller. To obtain these gains, several methods have been used, but tuning using evolutionary methods have performed in recent past **[3].** The problem is a two-dimensional problem. Here, mathematical analysis for the choice of OF is presented, based on which the OF is selected, and the algorithm is implemented on it.

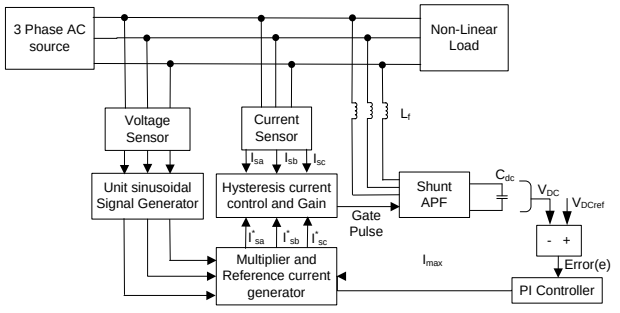


Figure 2. Control and reference current generation in Shunt Active Power Filter (SAPF)

B. Objective Function

To obtain the OF, the controller can be modeled in the transfer function form [1] as

$$G(s)=\frac{3(V_s-L_cI_{co}s-2I_{co}R_c)}{C_{dc}V_{dco}s} \tag{3}$$

where, $V_{s.}$ is the ac side voltage, (230 V), L_c, R_c are the filter inductance and resistance, (3.34mH, 0.4Ω), I_{co}, V_{dco} are the steady-state operating points of I_c and V_{dc} (20 A, 700V), and C_{dc} is the DC link capacitor (2200 µF). Applying unit step $(R(s)=1/s)$ the error in the system can be given as

$$E(s)=\frac{C_{dc}V_{dco}}{3(V_s-2I_{co}R_c)+(C_{dc}V_{dco}-3L_cI_{co})s} \tag{4}$$

Equation (4) can be rewritten in the time domain as

$$e(t)=\frac{C_{dc}V_{dco}}{C_{dc}V_{dco}-3L_cI_{co}}e^{-\left(\frac{3(V_s-2I_{co}R_c)}{C_{dc}V_{dco}-3L_cI_{co}}\right)t} \tag{5}$$

The stability of the system can be easily observed by plotting error against time, as depicted in Figure 3. It shows that the error approaches zero in about 10ms. Thus, an illation can be drawn that the system is stable. Based on the error obtained in (4), ITAE and ITSE are analyzed. Mathematically ITAE is given as (6). It integrates the absolute error of any system with time.

$$ITAE=\int_0^t\left(|e(t)|\times t\right)dt \tag{6}$$

Where *e(t)* is an error in the system, and *t* is time.

Substituting (5) in (6), it becomes, (7). Similarly, ITSE, which is time integral of the square of error for any system is calculated as,

$$ITSE=\int_0^t\left(e(t)^2\times t\right)dt \tag{8}$$

Putting (5) in (8), and Integrating ITSE becomes, (9).

To analyze their variation, (7) and (9) with I_{co}, for a working range of 5 to 20A, is plotted in Figure 4 (a) and Figure 4 (b), respectively, and their slopes are measured at an interval of 2ms (1 cycle). The comparative results are shown in Table I, clearly indicates that the slope of ITAE is more as compared to that of ITSE. Thus, it can be inferred from the analysis, that ITAE being more sensitive than ITSE reacts rapidly to the changes in the loading condition, whereas less sensitive ITSE remains nearly constant. A higher value of sensitivity is essential, as the load in electric power distribution is dynamic. The change in I_{co} also depicts this load change. As (7) and (9) depends on I_{co}, they should also change. Their variation is measured for a 1 A change in I_{co} and plotted in Figure 5. It also shows that ITAE is more sensitivity than ITSE. ITSE remains nearly constant for the whole current range, whereas the sensitivity of ITAE increases for the changes at higher current value.

Thus, the sensitivity of ITAE to load disturbances, and its characteristic confirms the features of ITAE described

$$ITAE = -\frac{(C_{dc}V_{dco})}{2(V_s - 2I_{co}R_c)} e^{-\left(\frac{3(V_s-2I_{co}R_c)}{C_{dc}V_{dco}-3L_cI_{co}}\right)t} - \frac{(C_{dc}V_{dco} - 3L_cI_{co})(C_{dc}V_{dco})}{(2(V_s - 2I_{co}R_c))^2}\left(e^{-\left(\frac{2(V_s-2L_{co}R_c)}{C_{dc}V_{dco}-3L_cI_{co}}\right)t} - 1\right) \quad (7)$$

$$ITSE = -\frac{(C_{dc}V_{dco})t}{4(V_s - 2I_{co}R_c)} e^{-\left(\frac{3(V_s-2I_{co}R_c)}{C_{dc}V_{dco}-3L_cI_{co}}\right)t} - \frac{(C_{dc}V_{dco} - 3L_cI_{co})(C_{dc}V_{dco})}{(4(V_s - 2I_{co}R_c))^2}\left(e^{-\left(\frac{2(V_s-2I_{co}R_c)}{C_{dc}V_{dco}-3L_cI_{co}}\right)t} - 1\right) \quad (9)$$

in **[6].** Based on this, ITAE is selected as the OF. This OF obtained is evaluated using the BFOA algorithm, and the results obtained are analyzed using the data analysis techniques, which are discussed in the next section.

TABLE I. COMPARISON OF SLOPES OF ITAE AND ITSE AT TIME STEP OF 2MS

Slope / Time (ms)	ITAE $(\times 10^{-7})$	ITSE $(\times 10^{-7})$
2	7000	4000
4	8500	1750
6	5000	430
8	2500	70
10	1000	10
12	525	2.5
14	200	≈ 0
16	96	≈ 0
18	35	≈ 0
20	12.5	≈ 0

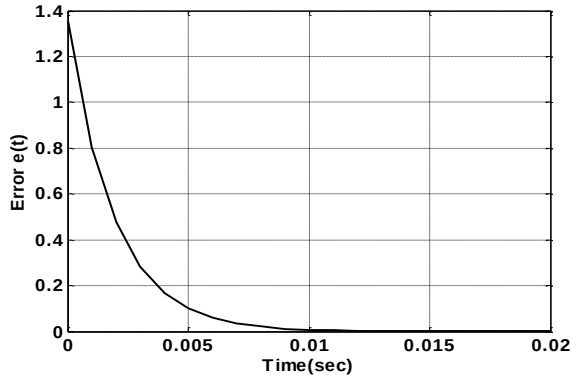
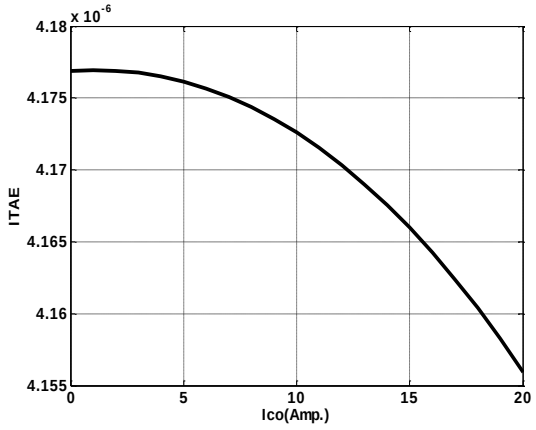
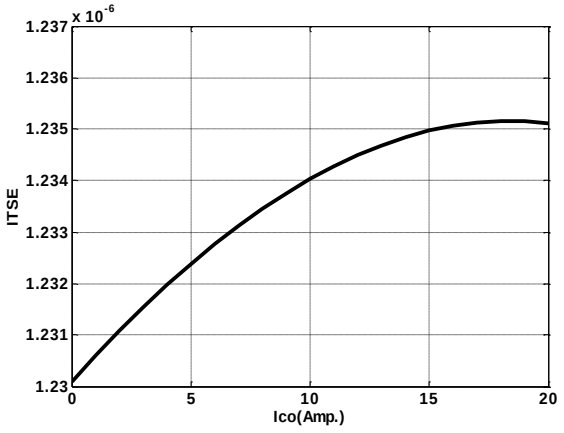


Figure 3. Variation of error *e(t)* against time



(a)



(b)

Figure 4. Sensitivity of performance indexes to I_{co} (a) ITAE, (b) ITS

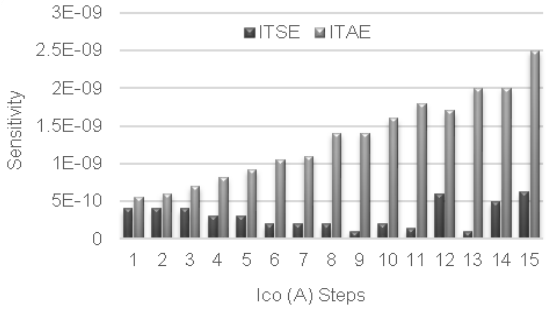


Figure 5. Variation of ITAE and ITSE for step changes of 1 A in I_{co}

4. DATA ANALYSIS

To obtain the value of the chemotactic parameter, the OF is evaluated for different *C(i)* for 100 runs. The results of the experiment are analyzed using data analysis techniques, viz. Frequency Deviation, Standard deviation, skewness, kurtosis, and box plots, based on which the best value is selected. These tests permit to identify the regions with high data concentration to judge the stability. The results obtained are verified mathematically using the previously developed technique [5].

A. Frequency Deviation

It is a way to show unorganized data in tabulated form. Each entry in the table contains the count of occurrences of values within a particular group or interval.

B. Standard Deviation

It describes the spread out of data around the mean in a data set. Data distributed close to the mean have a small SD. Otherwise, the SD value is large. Mathematically, SD (σ) is calculated as, [4]

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2} \tag{10}$$

where $\bar{x} = (1/N)\sum_{i=1}^{N} x_i$ is the mean, N is the sample size, and x_i represents data value from $i=1$ to $i=N$.

C. Skewness

It measures the symmetry of the data about the mean. Symmetric data distribution is divided into has two halves, whereas the skewed data lacks symmetry **[4].** Mathematically,

$$skewness(\gamma_1) = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3}{(n-1)\sigma^3} \tag{11}$$

For $\gamma_1 > 0$, the distribution is right-skewed, i.e., distribution has a tail on the right side, whereas for $\gamma 1 < 0$ the distribution is left-skewed, i.e., having a tail on the left side.

D. Kurtosis

It gives the relative flatness of a distribution compared with a normal distribution. It is defined as the fourth standardized moment about the mean (μ_4). Mathematically, [4],

$$\beta_2 = \frac{\mu_4}{\sigma^4} \tag{12}$$

For the normal distribution, which is taken as the reference. $\beta_2 = 3$. Thus, if $\beta_2 - 3 > 0$, kurtosis is positive, whereas if $\beta_2 - 3 < 0$ kurtosis is negative. Distributions with positive kurtosis have heavier tails, and higher peaks, i.e., data concentrated around the mean with thicker tails, while lighter tails with flatter kurtosis are obtained for negative value, i.e., values have a wider spread.

E. Box plots

Box plots are an efficient way to show the mean, median, quartiles, and extremes graphically. It summarizes the whole data into lower quartile, upper quartile, median, smallest, and largest observation [4]. The advantage of using them is that they display differences between values without making any assumptions about the underlying statistical distribution. They also indicate the degree of dispersion (spread) and skewness in the data.

5. Results and Discussion

To obtain the desired value of chemotactic step, *C*, the OF is evaluated its different values for 100 runs, in each value. The resulting fitness data is collected for 100 runs, which is analyzed using the data analysis techniques discussed above. The other BFOA parameters that are taken are $S = 10$, $p = 2$, $N_c = 10$, $N_s = 4$, $N_{re} = 4$, $N_{ed} = 2$, $p_{ed} = 0.25$, $d_{attractant} = 0.01$, $\omega_{attractant} = 0.04$, $h_{repellant} = 0.01$, $\omega_{repellant} = 10$. The data analysis is performed as tests. The motive of performing these tests is to analyze the stability and convergence of the algorithm towards an equilibrium point. If the system is stable for a value of *C*, then the fitness values for 100 runs should have the same value, i.e., the distribution should be concentrated, while, scattered distribution indicates the premature convergence of BFOA. This analysis is beneficial as the stability of BFOA is identified without mathematical analysis. Mathematical verification [2] of the results obtained validates the suitability/ merits of the proposed method.

A. Test 1: Frequency Deviation

In frequency deviation, the fitness values are redistributed into groups, and their occurrence in a particular group is recorded. Here, the fitness values are divided into ten equal groups of 1.15 x 10^{-11} each, and the frequency of their occurrence in each group is shown in Table II. Being a minimization problem, it is desirable that fitness should lie in the minimum region $\left(1.94\times10^{-11} - 3.09\times10^{-11}\right)$ of the distribution. Table II depicts that for *C = 0.06, 0.04, 0.03,* and *0.02* maximum frequency of fitness values is attained in the minimum region. These values are selected considering the 1% deviation from the maximum value of frequency. However, this test alone does not provide a particular value of *C*. Therefore, some more tests are performed.

TABLE II. Frequency Deviation of the Solutions

C(i) / Fitness value Interval (x 10^{-11})	0.15	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
1.94 - 3.09	37	86	87	92	96	***99***	98	***100***	***100***	***99***	90
3.09 - 4.25	59	13	13	8	4	1	2	0	0	1	10
4.25 - 5.40	2	0	0	0	0	0	0	0	0	0	0
5.40- 6.56	0	0	0	0	0	0	0	0	0	0	0
6.56- 7.71	1	0	0	0	0	0	0	0	0	0	0
7.71- 8.86	0	0	0	0	0	0	0	0	0	0	0
8.86- 10.0	0	0	0	0	0	0	0	0	0	0	0
10.0- 11.2	0	0	0	0	0	0	0	0	0	0	0
11.2- 12.3	0	0	0	0	0	0	0	0	0	0	0
12.3- 13.5	1	0	0	0	0	0	0	0	0	0	0
>13.5	0	1	0	0	0	0	0	0	0	0	0
TOTAL	100	100	100	100	100	100	100	100	100	100	100

B. Test 2: Standard Deviation

For data concentrated around the mean, σ should be minimum. Table III gives the results of this test, shows that for *C = 0.06*, σ has a minimum value of 1.90. Hence, it can be concluded that for *C=0.06*, the data is concentrated around the mean. These two tests show that for *C=0.06*, the fitness values are located in the minimum region as well as concentrated in nature.

C. Test 3: Skewness

For a stable algorithm, the fitness values should repeat, i.e., the data should be concentrated about $\bar{x}$; hence γ_1=0. The results tabulated in Table III depict that for *C=0.06* the skewness (γ_1) attains a value of -0.021, which is nearest to zero. In other words, for *C = 0.06*, the data is concentrated about the mean with slight skewness. So, it can be concluded that for *C=0.06* the fitness values are the most stable, in comparison to the other values of *C*. The results obtained are in accordance with the results of the previous two tests and confirms the stability of BFOA for *C=0.06*.

TABLE III. RESULTS OF THE STANDARD DEVIATION AND SKEWNESS TEST

C(i)	Standard Deviation, σ (x10^{-12})	Skewness (γ_1)
0.15	11.70	7.115
0.1	12.20	9.204
0.09	2.40	-0.047
0.08	2.10	0.267
0.07	2.30	0.479
0.06	**1.90**	**-0.021**
0.05	2.00	0.276
0.04	2.10	0.429
0.03	2.00	0.275
0.02	21.00	0.728
0.01	3.80	1.632

D. Test 4: Kurtosis

After data distribution, the Kurtosis test is performed to analyze the shape of data distribution for *C*. The results tabulated in Table IV shows that the value of $\beta_2 - 3$ for *C=0.08, 0.04,* and *0.03* is *-0.08*, *-0.04* and *-0.22*, respectively. However, being negative ($\beta_2 - 3<0$), i.e., even distribution of data, it is disregarded. The positive nature of the kurtosis about zero is given *C= 0.09, 0.07, 0.06* and *0.05* having corresponding values *0.17, 0.73, 0.61,* and *0.18*. Taking into account the results of the previous tests, the best solution of *C* is *0.06*. Hence, it can be concluded that for *C=0.06*, the data shows desired peakedness, which indicates the stabilization of the BFOA.

E. Test 5: Box Plots

The box plot of the experimental data is shown in Figure 6. It shows that for 0.15, 0.1, and 0.01 data is highly scattered in nature, leading to premature convergence of BFOA with a variety of results. *C=0.03* and *0.06* may be considered best, in that order, as most of the data is located between the interquartile region and the whisker, resulting in convergence towards an optimum solution. Incorporating observations of previous tests, it is found that C= 0.06 provides better results than other values.

TABLE IV. RESULTS OF THE KURTOSIS TEST

Chemotactic Step C(i)	Kurtosis(β_2)	$\beta_2 - 3$
0.15	60.20	57.20
0.1	89.67	86.97
0.09	3.17	0.17
0.08	2.92	-0.08
0.07	3.73	0.73
0.06	**3.61**	**0.61**
0.05	3.18	0.18
0.04	2.96	-0.04
0.03	2.78	-0.22
0.02	4.33	1.33
0.01	5.63	2.63

The results of all the five tests are tabulated in Table VI, where, values qualifying the individual test are presented as a tick (✓), and rejected values as a cross (x). From Table V, it is visible that C= 0.09, 0.05, 0.04, and 0.02 can pass only one test, whereas C=0.03 gave better results with two tests. However, C= 0.06 performed well in all the tests confirming the concentrated and undistorted nature of the fitness values in the BFOA. Hence from Table V, following inference can be drawn for C=0.06, (a) The frequency of data lying in the minimum region of the distribution is high, (b) Minimum valve of SD indicates that distribution of data is around the mean, (c) Skewness shows that the data are symmetrically located around the mean, which verifies the stability of the algorithm, (d) Kurtosis test depicts the peakedness in the data, showing the concentrated nature of data, and (e) Box plots show that data lies between the inter-quartile and whiskers. Thus, confirming the concentrated nature of the data and suitability of the C= 0.06 for PI controller tuning for SAPF application.

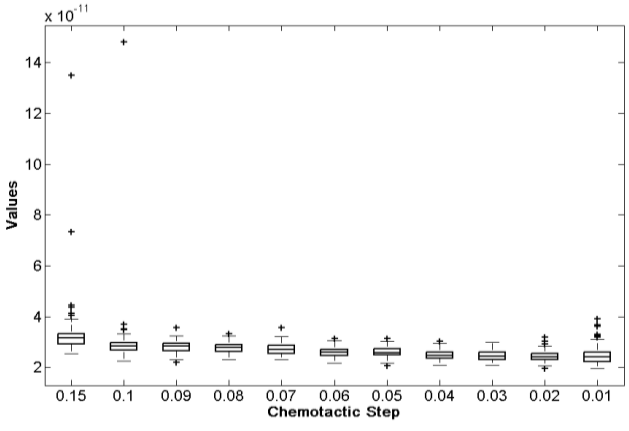


Figure 6. Box plot for different chemotactic steps

6. VERIFICATION OF RESULTS

To verify the result obtained through the proposed approach, mathematical analysis [5] is adopted and extended for the 2-D problem. During the analysis, the bacteria in each dimension are dealt with separately with the only link between them via an OF. The dynamics of gains can be described as,

$$\left.\begin{aligned}\frac{dK_P}{dt} &= -\alpha.G_1 + \beta \\ \frac{dK_I}{dt} &= -\alpha.G_2 + \beta\end{aligned}\right\} \quad (13)$$

where, $\alpha = -kC^2/8$, is learning rate, $G_1 = dJ(K_P)/d(K_p)$, the gradient of an OF, K_p = Proportional Gain, K_I Integral Gain. $\beta = C\Delta/2$ is momentum, and $G_2 = dJ(K_I)/d(K_I)$.

To verify the prediction in the bacteria motion given by (13), the variation in the motion of the predicted and actual the value is plotted in Figure 7. The figure depicts the predicted model that tracks the actual model. The error obtained between the actual model and the predicted model is drawn in Figure 8. It is visible that the errors are well within the range of ±0.2, for the values of K_P and K_I shown in Figure 7. Hence, the extended model is validated. Based on this mathematical model the condition for the stability can be derived as in [5]

$$\begin{aligned}C > \frac{4}{k}\left|\frac{\theta_i - \theta_0}{J(\theta_i)}\right| \quad & if\ \theta_i \neq \theta_0 \\ = 0 \qquad\qquad & if\ \theta_i = \theta_0\end{aligned} \quad (14)$$

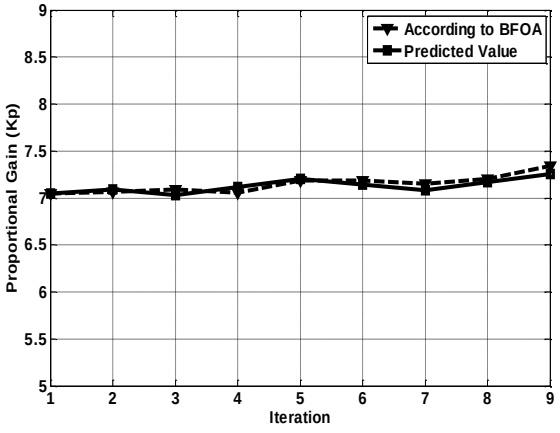
where, $\theta_1 = K_P$, $\theta_2 = K_I$, and θ_0 is the equilibrium point.

This condition derived in (14) is tested for the final result, i.e., C=0.06 taken from the data analysis. The condition is tested for the first 10 iterations of the derived model. Thus, from Table V the approach adopted for solving the problem is verified as the mathematical condition is satisfied.

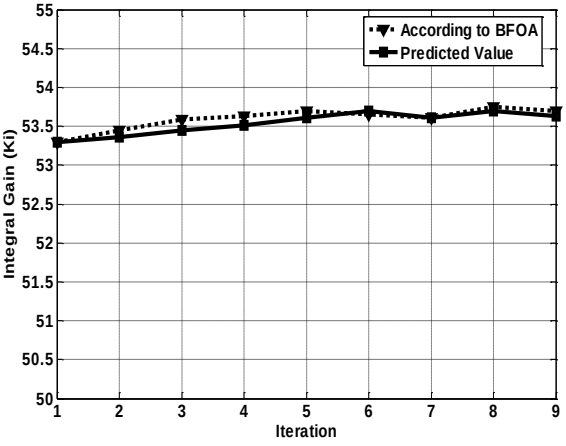
TABLE V. COMPARATIVE TABULATION OF RESULTS OF VARIOUS TESTS

Chemotactic Step C(i)*	0.15	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
Frequency Deviation	x	x	x	x	x	✓	x	✓	✓	✓	x
Standard Deviation	x	x	x	x	x	✓	x	x	x	x	x
Skewness	x	x	x	x	x	✓	x	x	x	x	x
Kurtosis	x	x	✓	x	x	✓	✓	x	x	x	x
Box plot	x	x	x	x	x	✓	x	x	✓	x	x

*✓- Selected value x- Rejected Value

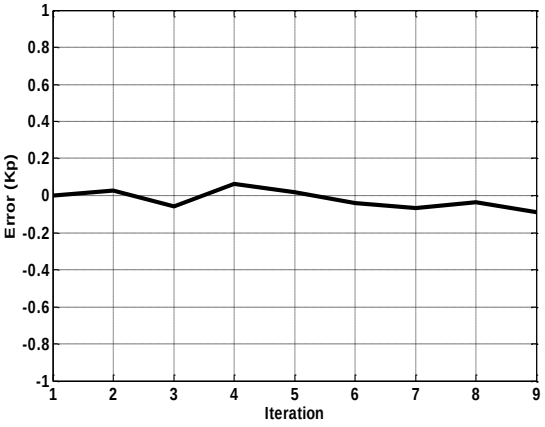


(a)

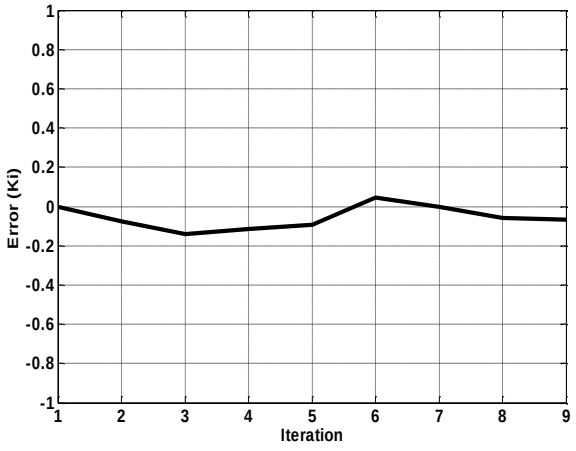


(b)

Figure 7. Actual and predicted motional states of the bacterium for (a) Proportional Gain (K_p) (b) Integral Gain (K_I)



(a)



(b)

Figure 8. Errors between actual and predicted motional states of the bacterium for (a) Proportional Gain (K_p) (b) Integral Gain (K_I);

7. CONCLUSION

The work performed in this paper successfully demonstrates the implementation of statistical methods for obtaining the chemotactic parameters step in BFOA. Here, the importance of chemotactic and how it influences the optimization is discussed first. Then, analytical analysis for the determination of OF presented. After the objective function determination, the statistical techniques used in this paper are described. Finally, the results after the application of the proposed approach and its verification using previously adopted methods are presented. The salient features of the proposed methodology are:

1) An objective function based on a logical approach is presented
2) Dimensionality is not a restriction for the proposed method. It can be used for any dimension problem.
3) By analyzing the pattern of the data obtained, the dynamics of the problem can be analyzed without probing into the mathematical details for the selection of the parameter.
4) The concentrated nature of the data represents a stable system. Thus, the stability of the algorithm for different chemotactic steps can be analyzed by adopting these methods.

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