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A Queuing Model for Hospital Bed Occupancy Management: A Case Study

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Abstract: The quality of Medical Service in the Orthopaedic clinic of the NKST Rehabilitation Hospital Mkar is constantly threatened by the inadequacy of hospital beds across wards. This is due to the high influx of patients needing this specialized healthcare service. The optimal number of beds required in each ward to ensure patients are not turned away and beds are not underutilized has not been determined scientifically. It is important to mention that the consequence of patients being turned away is the loss of revenue to this hospital and frustration on the part of patients. This paper has been able to address this problem via the analytical queuing modeling approach. The queuing model has been able to determine the optimal number of beds required in the wards, successfully ensured that no patient is turned away from the wards and no revenue is lost. Though the model has being able to solve the problem of patients been turned away from the wards and that of revenue loss, it is not without the challenges of coping with the holding cost of empty beds across as the wards. The optimal number of beds for the wards (Private, Alam, Chile and Dooase) are respectively 85, 99, 86 and 105, while the mean number of empty beds is respectively 22, 24, 30 and 25.

Keywords: Queue, Model, Bed, Occupancy

1. Introduction

According to Gorunescu *et al* [9], "an under- provision of hospital beds lead to patients in need of hospital care being turned away. Consequently, patient dissatisfaction, build-up of waiting list and stress characterize the hospital system. For example, when insufficient medical beds are provided to meet demands, emergency medical patients spill over into surgical beds; therefore, surgical waiting list increases as planned admissions are postponed. On the other hand, an over provision of hospital beds is a waste of scarce resources."

The aforementioned scenario is a picture of the problems currently facing the orthopaedic clinic arm of the NKST Rehabilitation Hospital Mkar, Benue State, Nigeria. The hospital was established in 1930 by the Dutch Reformed Church Mission of South Africa. The main objective of the hospital is the treatment and rehabilitation of orthopaedic patients. It is the referral hospital for community based rehabilitation in the State. It receives patient for orthopedic rehabilitation from neighboring states; Adamawa, Plateau, Taraba, Cross River and Akwa Ibom. The quality of medical service in the orthopaedic clinic is constantly threatened by the inadequacy of hospital beds across wards. This is due to the high influx of patients needing this specialized healthcare service. The optimal number of beds required in each ward to ensure patients are not turned away and beds are not underutilized has not been determined scientifically. It is important to mention that the consequence of patients being turned away is the loss of revenue to the hospital and frustration on the part of patients.

The followings are indicative of some applications of queuing models in solving problems associated with hospital bed occupancy optimization. Steve *et al.* [13] via a mathematical modeling approach based on probability theory studied booked inpatient admissions and hospital bed capacity of an intensive care unit after cardiac surgery. A queuing model for bed occupancy management and planning of hospitals was developed by Gorunesco *et al.* [9]. The model was used to describe the movements of patients through a hospital department and to determine the main characteristics of the access of patients to hospital such as; mean bed occupancy, and probability that a demand for hospital care is lost because all beds are occupied. They present a technique for optimizing the number of beds in order to maintain an

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acceptable delay probability at a sufficiently low level and finally they provide a way of optimizing the average cost per day of empty beds against costs of delayed patients. They established that 10-15% bed emptiness is necessary to maintain service efficiency and provide more responsive and cost effective services.

Arnoud *et al.* [3] modeled an emergency cardiac in-patient flow using queuing theory while, three bed prediction models to aid hospital bed planners in anticipating bed demands so as to manage resources efficiently were proposed by Arun *et al.* [1]. A queuing approach in determining optimal number of beds in a hospital serving Urgent and Non- urgent patients was used by Abolnikov and Zachariah [2]. Bhavin and Pravin [5] describe the movement of patients in a hospital by using queuing model with exponential arrival and service time distributions (M/M/1). Bagust *et al.* [4] studied the dynamics of bed use in accommodating emergency admissions via a stochastic simulation modeling approach. The result showed that the risk of failure to admit occurs at occupancy rate above 85%.

Jean *et al.* [10] developed a method not based on queuing theory. Their method is based on the simultaneous maximization of the mean and standard deviation of three (3) parameters namely; assessing accessibility, clinical effectiveness and productive efficiency. This was compared to the target ratio method using simulated data. The result demonstrated that in all the situations, the method is more appropriate since unlike the target ratio method, it considered the fluctuation of demands for bed over time. De Bruin *et al.* [8] reported the dimensioning of hospital wards using the Erlang loss model. The authors argue that though most hospitals use the same target occupancy rate for all wards, often 85%, sometimes an exception need be made for critical care and intensive units. They pointed out that this equity assumption is un realistic and that it might result in an excessive number of refused admissions particularly for smaller units. They used queuing theory to assess the impact of this assumption and also to develop a decision support system to evaluate the current size of the nursing unit.

Christoper *et al.* [7] in a work titled Myths of ideal hospital occupancy argued the result of Bagust *et al* [4] which state specifically that; risks are discernable when average bed occupancy rates exceed 85% and that an acute hospital can expect regular bed shortages and periodic bed crises if average bed occupancy rises to 90% or more. They contended by emphasizing that this conclusion can only apply to the particular queuing system the authors investigated and that generalization and application of this result can be misleading and not justified. They buttress their point by stating that there is a more fundamental problem with making general statements that relate blocking probability with steady-state mean occupancy. It was concluded that a better way is to relate queue performance measures to inputs such as the arrival, service process and the number of beds.

Asaduzzaman and Chaussalet [15], developed a model framework to solve capacity planning problems that are faced by many perinatal networks in the UK. They proposed a loss network model with overflow based on a continuous time Markov chain for a perinatal network with specific application to a network in London. Steady state expressions for overflow and rejection probabilities for each neonatal unit of the network were derived on the basis of a decomposition approach. Results obtained from the model were very close to observed values. Using the model, decisions on numbers of cots were made for specific levels of admission acceptance probabilities, for each level of care at each neonatal unit of the network and specific levels of overflow to temporary care.

Griffths, *et al.* [16], proposed a mathematical model that shows how improvements in bed management may be achieved by distinguishing between two categories of patients; unplanned (emergency) and planned (elective). This was done for a Critical Care Unit (CCU), where inability to provide adequate facilities on demand can lead to serious consequences. They explained that the vast majority of previous literature in this field is concerned only with steady state conditions, whereas in reality, activities in virtually all hospital environments are very much time dependent. They made considerable efforts in addressing this problem.

Diwas and Singh [17], explored the rationing of bed capacity in a cardiac intensive care unit (ICU). They found that the length of stay for patients admitted to the ICU is influenced by the occupancy level of the ICU. In particular, they stated that a patient is likely to be discharged early when the occupancy in the ICU is high which in turn lead to an increased likelihood of the patient having to be readmitted to the ICU at a later time. This capacity implication was analyzed by comparing the total capacity usage for patients who were discharged early versus those who were not. They also showed that an aggressive discharge policy applied to patients with lower clinical severity levels frees up capacity in the ICU and that an increased number of readmissions of patients with high clinical severity levels occur when the ICU capacity is constrained, thereby effectively reducing peak bed capacity.

Bower [18], examined the balance between operating theatres and beds in a specialist facility providing elective heart and lung surgery. It was state that without both operating theatre time and an Intensive Care bed a patient's surgery has to be postponed and that while admissions can be managed; there are significant stochastic features, notably the cancellation of theatre procedures and patients' length of stay on the Intensive Care Unit. In collaboration with the clinical and management staff, a simulation model was developed to explore the interdependencies of resource availabilities and the daily demand. The model was used to examine options for expanding the capacity of the whole facility. It was stated that ideally the bed and theatre capacity should be well balanced but unmatched increases in either resource can still be beneficial. The study provided an example of a capacity planning problem in which there is uncertainty in the demand for two symbiotic resources.

The robust application of queuing models in solving problems associated with bed occupancy in hospitals, guided the researchers in selecting it as the analytical tool for this work.

2. AIM AND OBJECTIVES OF THE STUDY

The aim of this study is to improve on the quality of orthopaedic ward service in the NKST Rehabilitation Hospital Mkar.

The specific objectives include:

(i) To compute patients' delay probabilities to admission in each ward
(ii) To determine the mean number of beds occupied across wards
(iii) To determine the percentage utilization of beds across wards
(iv) To determine the optimal number of beds for delay probability zero across wards
(v) To determine the mean number of patients turned away across wards
(vi) To determine the average loss of revenue over the period of average length of stay across the wards
(vii) To determine the mean number of empty beds across wards.
(viii) To contribute to the efficient management of the orthopedic clinic of the NKST Rehabilitation Hospital Mkar.

2.1 MODEL DESCRIPTION AND EQUATIONS

We consider the $M/E_k/c$, k- Erlarge queuing model with fixed 'c' number of beds where a patient who finds that all beds are occupied is considered lost. In practical situation these patients wait elsewhere or go back home.

It is assumed here that the patient arrivals follow a poison process with rate λ and the service time is the phase type which according to Gorunescu *et al.*, (2002) has probability density function

$$f(t) = \sum_{i=1}^{k} \alpha_i p_i e^{\alpha_i t} \qquad (2.1)$$

where $\sum_{i=1}^{k} p_i = 1$,

$k =$ the number of phases/compantments,

$\propto_i' \, s = mixing$ proportion

$p_i' s =$ transistion rate and the corresponding mean

$$\tau = \sum_{i=1}^{k} \frac{p_i}{\alpha_i} \qquad (2.2)$$

According to them, the parameters $k, \alpha_i \, and \, p_i$ may be estimated using likelihood ratio test.

The average number of arrivals occurring during an interval t is λt therefore, the average number $'a'$ of arrivals during an average length of stay $'\tau'$ is

$$a = \lambda t \qquad (2.3)$$

this is called the offered load.

In this work, the arrival rate $'\lambda'$ is estimated from data as the average number of arrivals per day, while the average length of stay $'\tau'$ is also estimated from data on the length of stay of patients in a ward over the period of study.

According to Cooper [6] and Tijims [14], the probability of having j occupied beds is given by

$$p_j = \frac{a^j / j!}{\sum_{k=0}^{c} a^k / k!} \qquad (2.4)$$

where $c =$ number of beds and a is as defined above.

The statistical equilibrium of p_j depends on the service time distribution only through its mean. This occurs if after a sufficiently long period of time, the state probabilities are independent of the initial conditions.

Gorunescu *et al.*, [9] deduced that the probability that all $'c'$beds are occupied or the fraction of arrivals that is lost (patient delay probability) is given by the Erlang's loss formula:

$$\mathrm{B}(c, a) = \frac{a^c / c!}{\sum_{k=0}^{c} a^k / k!} \qquad (2.5)$$

They added that;

$$a' = a[1 - \mathrm{B}(c, a)] \qquad (2.6)$$

is the mean number of occupied beds called the carried load and we add that the mean number of patients turned away called the lost demand is given as:

$$a'' = a[\mathrm{B}(c, a)] \qquad (2.7)$$

and

$$\mathrm{r}_0 = a'/c * 100\% \qquad (2.8)$$

is the bed occupancy in percentage (%)

Four (4) wards in the orthopaedic clinic of the Mkar Rehabilitation centre were studied. Each has a fixed charge per day in the ward. This is shown in Table 1.

The revenue lost by the clinic over a period of average length of stay in the ward is given as:

$$R = a'' * fixed\ charge\ of\ bed\ per\ day \qquad (2.9)$$

This information provides an idea of the amount of money the clinic will lost when patients are turned away.

The mean number of empty beds is computed as the optimal number of beds c_o less the carried load a'

3.THEORITICAL BASIS FOR SENSITIVITY ANALYSIS

Here, the theoretical basis for sensitivity analysis carried out in the work is shown. Gorunescu *et al.*, [9] obtained optimal number of beds for delay probabilities $0.1\%, 1\%, 5\%$ *and* 10%. In this work, we were able to determine optimal number of beds across wards with delay probability zero

Recall from equation 2.5 that,

$$\mathrm{B}(c,a) = \frac{a^c/_{c!}}{\sum_{k=0}^{c} a^k/_{k!}}$$

$$= \frac{1}{c!/_{a^c} \sum_{k=0}^{c} (a^k/_{k!})} \qquad (3.1)$$

There is no guarantee that $\mathrm{B}(c,a) = 0$ by simply letting $= a$, but

$$\lim_{c\to\infty} B(c,a) = \lim_{c\to\infty} \frac{1}{c!/_{a^c} \sum_{k=0}^{c} (a^k/_{k!})} = 0$$

This means that as the number of beds becomes large, we are guaranteed that the delay probability tends to zero.

From equation (2.6), the mean number of occupied beds is $a' = a[1 - \mathrm{B}(c,a)]$. If we let $\mathrm{B}(c,a) = 0$ as earlier infered, then

$$a = a' \qquad (3.2)$$

Showing that the offered load $=$ the carried load.

Hence from the result of the sensitivity of the number of beds to delay probability $\mathrm{B}(c,a)$, the optimal number of bed c_o is easily determined when $a' = a$. We mention here that due to the enormous task of computing $\mathrm{B}(c,a)$ in equation 2.5 and for the purpose of accuracy, the Microsoft Excel package (2003) was programmed and used.

3.1 SOURCE OF DATA AND DISTRIBUTION FIT

Data on patients' admission and discharge dates were sourced from the orthopedic clinic records across four (4) wards; Private, Chile, Alam and Dooashe wards of the NKST Rehabilitation Hospital Mkar. The period covered is six months (January to June). The distribution fit to data on number of arrival of patients to the wards and their length of stay was done using the Predictive Analytical Software (PASW) and the Easy Fit version 5.5 professional distribution fitting software respectively. Table 2 and table 3 show the distribution fits and the parameter details respectively for number of arrival of patients and their length of stay in each ward.

4. RESULT

This section presents the distribution of number of beds on ground and charge per day across wards, the results of the distribution fits to data on the number of arrival of patients and their length of stay in each ward. Furthermore, the results on the relationship between the number of beds and each of the system performance measures (delay probability, mean number of occupied beds, bed occupancy, mean number of patients turned away and the revenue lost over an average length of stay) across wards is presented.

4.1 Distribution of beds, charge per day and distribution fit

Table 1 below, shows the distribution of beds and charge per day across wards , while, tables 2 and 3 show respectively, the distribution fits and the parameter details for number of arrival of patients and their length of stay in each ward using the chi-square (χ^2) and the Kolmogorov Smirnov (KS) statistics.

TABLE 1. DISTRIBUTION OF BEDS AND CHARGE PER DAY ACROSS WARDS

Ward	Number of beds on ground	Bed charge per day (Naira)
Private	10	600.00
Alam	24	283.33
Chile	42	283.33
Dooashe	24	283.33

TABLE 2. GOODNESS OF FIT SUMMARY FOR DISTRIBUTION OF NUMBER OF ARRIVAL OF PATIENTS TO EACH WARD

			Goodness of fit test		
Ward	***Distribution***	***Parameter***	***Test***	***Statistic***	***P-value***
Private	Poisson	$\lambda = 1.6875$	KS	0.2850	1.0000
Alam	Poisson	$\lambda = 1.4285$	KS	0.1060	1.0000
Chile	Poisson	$\lambda = 1.4634$	KS	0.3720	0.9990
Dooshe	Poisson	$\lambda = 1.8750$	KS	0.2770	1.0000

TABLE 3. GOODNESS OF FIT SUMMARY FOR DISTRIBUTION OF LENGTH OF STAY OF PATIENTS IN EACH WARD

			Goodness of fit test		
Ward	*Distribution*	*Parameter*	*Test*	*Statistic*	*P-value*
Private	K-Erlang	$k = 5,\ \beta = 7.0327$	KS	0.1374	0.6388
Alam	K-Erlang	$\mathrm{k} = 4,\ \beta = 10.8800$	χ^2	4.4314	0.1091
Chile	K-Erlang	$\mathrm{k} = 4,\ \beta = 7.9426$	KS	0.1413	0.1652
Dooshe	K-Erlang	$\mathrm{k} = 4,\ \beta = 9.4396$	KS	0.1760	0.2767

4.2 The relationship between the number of beds and system performance measures

In this section, the relationship between the number of beds and the system performance measures are tabulated and presented graphically across the wards.

TABLE 4. DISTRIBUTION OF THE NUMBER OF BEDS AND SYSTEM PERFOMANCE MEARSURES FOR PRIVATE WARD

Number of beds	Delay probability	Mean number of occupied beds	Bed occupancy (%)	Mean number of patients turned away	Revenue lost
10	0.84	10	100.00	53	31800.00
15	0.77	15	100.00	48	28800.00
20	0.69	20	100.00	43	25800.00
25	0.61	24	96.00	39	23400.00
30	0.54	29	96.67	34	20400.00
35	0.46	34	97.14	29	17400.00
40	0.39	39	97.50	24	14400.00
45	0.32	43	95.56	20	12000.00
50	0.25	47	94.00	16	9600.00
55	0.18	52	94.55	11	6600.00
60	0.12	55	91.67	8	4800.00
63	0.09	57	90.48	6	0.00
70	0.04	61	87.14	2	0.00
75	0.02	62	82.67	1	0.00
80	0.01	63	78.75	0	0.00
85	0.00	63	74.12	0	0.00
90	0.00	63	70.00	0	0.00
95	0.00	63	66.32	0	0.00
100	0.00	63	63.00	0	0.00

Offered load $(\lambda\tau) = 62.94$ where $\lambda =$ arrival rate and $\tau =$ average length of stay (days)

TABLE 5. DISTRIBUTION OF THE NUMBER OF BEDS AND SYSTEM PERFORMANCE MEASURES FOR ALAM WARD

Number of bed	Delay probability	Mean number of occupied beds	Bed occupancy (%)	Mean number of patients turned away	Revenue lost
24	0.69	23.55	98.14	51	14449.83
30	0.61	29.38	97.92	46	13033.18
36	0.53	35.15	97.65	40	11333.2
42	0.46	40.87	97.30	34	9633.22
48	0.38	46.49	96.85	29	8216.57
54	0.31	51.97	96.24	23	6516.59
60	0.24	57.23	95.39	18	5099.94
66	0.17	62.17	94.19	13	3683.29
75	0.09	68.50	91.33	7	1983.31
81	0.05	71.61	88.41	3	849.99
87	0.02	73.61	84.60	1	283.33
93	0.01	74.57	80.19	0	0
99	0.00	74.91	75.66	0	0
105	0.00	74.99	71.42	0	0
111	0.00	75.00	67.57	0	0
117	0.00	75.00	64.10	0	0
123	0.00	75.00	60.98	0	0
129	0.00	75.00	58.14	0	0
135	0.00	75.00	55.56	0	0

Offered load $(\lambda\tau) = 75.34$ where $\lambda =$ arrival rate and $\tau =$ average length of stay (days

TABLE 6. DISTRIBUTION OF THE NUMBER OF BEDS AND SYSTEM PERFORMANCE MEASURES

Number of beds	Delay probability	Mean number of occupied beds	Bed occupancy (%)	Mean number of patients turned away	Revenue lost
42	0.2885	40	94.91	16	4533.28
45	0.2432	42	94.22	14	3966.62
50	0.1725	46	92.72	10	2833.30
56	0.0997	50	90.07	6	1699.98
61	0.0534	53	86.94	3	849.99
66	0.0231	55	82.92	1	283.33
71	0.0077	56	78.30	0	0.00
76	0.0019	56	73.58	0	0.00
81	0.0003	56	69.14	0	0.00
86	0.0000	56	65.14	0	0.00
91	0.0000	56	61.56	0	0.00
96	0.0000	56	58.36	0	0.00
101	0.0000	56	55.47	0	0.00
106	0.0000	56	52.85	0	0.00
111	0.0000	56	50.47	0	0.00
116	0.0000	56	48.30	0	0.00
121	0.0000	56	46.30	0	0.00
126	0.0000	56	44.46	0	0.00
132	0.0000	56	42.44	0	0.00

Offered load $(\lambda\tau) = 56.02$, where $\lambda =$ arrival rate and $\tau =$ average length of stay (days)

TABLE 7. DISTRIBUTION OF THE NUMBER OF BEDS AND SYSTEM PERFOMANCE MEASURES FOR DOOASHE WARD

Number of beds	Delay probability	Mean number of occupied beds	Bed occupancy (%)	Mean number of patients turned away	Revenue lost
24	0.71	23.59	98.30	56	15982.22
30	0.63	29.43	98.11	51	14326.74
36	0.56	35.24	97.89	45	12681.70
42	0.49	41.00	97.61	39	11050.95
48	0.42	46.68	97.25	33	9440.26
54	0.35	52.26	96.78	28	7858.55
60	0.28	57.69	96.16	22	6319.97
66	0.21	62.89	95.29	17	4847.24
75	0.13	69.95	93.27	10	2846.24
80	0.08	73.27	91.59	7	1906.67
87	0.04	76.83	88.31	3	898.13
93	0.02	78.70	84.62	1	368.74
99	0.01	79.60	80.40	0	113.56
105	0.00	79.91	76.11	0	25.33
111	0.00	79.99	72.06	0	4.06
117	0.00	80.00	68.37	0	0.47
123	0.00	80.00	65.04	0	0.04
129	0.00	80.00	62.02	0	0.00
135	0.00	80.00	59.26	0	0.00

Offered load $(\lambda\tau) = 80.04$ where $\lambda =$ arrival rate and $\tau =$ average length of stay (days)

TABLE 8. DISTRIBUTION OF NUMBER OF BEDS ON GROUND AND SYSTEM PERFORMANCE MEASURES

Ward	Number of beds on grc	Offered loa	Carried loa	Delay probal	Mean number of patie turned away	Revenue lost (#)
Private	10	62.94	10.00	0.84	53	31,800.00
Alam	24	75.34	23.55	0.69	51	14,449.83
Chile	42	56.02	40.00	0.29	16	4,533.28
Dooash	24	80.34	23.59	0.71	56	15,982.22

TABLE 9. DISTRIBUTION OF OPTIMAL NUMBER OF BEDS AND SYSTEM PERFORMANCE MEASURES ACROSS WARDS

Ward	Optimal number of beds	Offered load	Carried load	Delay probability	Mean number of patients turned away	Revenue lost (#)	Mean number of empty beds
Private	85	63	63	0.00	0	0.00	22
Alam	99	75	75	0.00	0	0.00	24
Chile	86	56	56	0.00	0	0.00	30
Dooashe	105	80	80	0.00	0	0.00	25

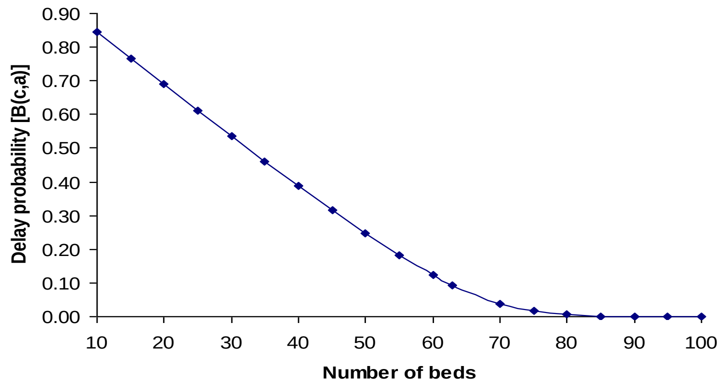

Figure 1: Delay probability of patient's admission to Private ward

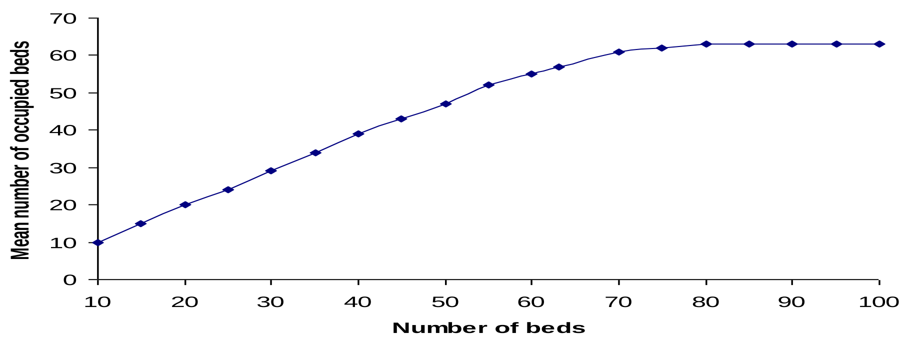

Figure 2: Mean number of occupied beds in Private ward

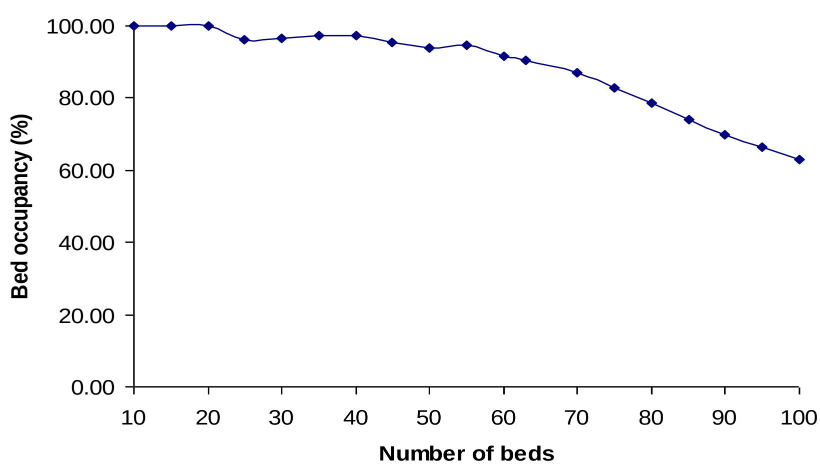

Figure 3: Bed occupancy in Private ward

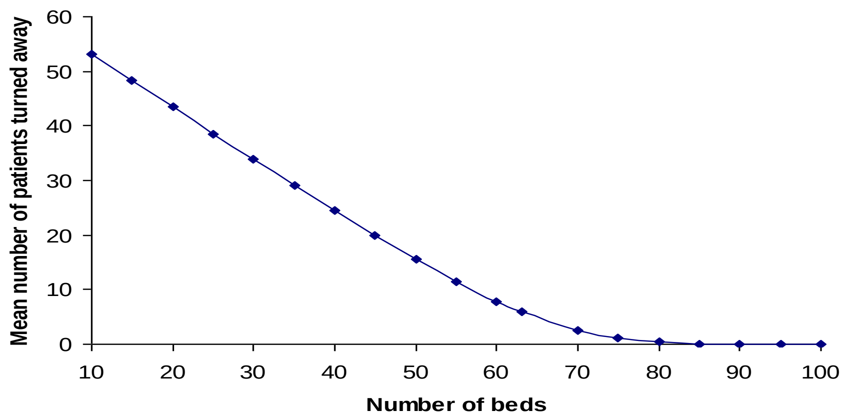

Figure 4: Mean number of patients turned away in Private ward

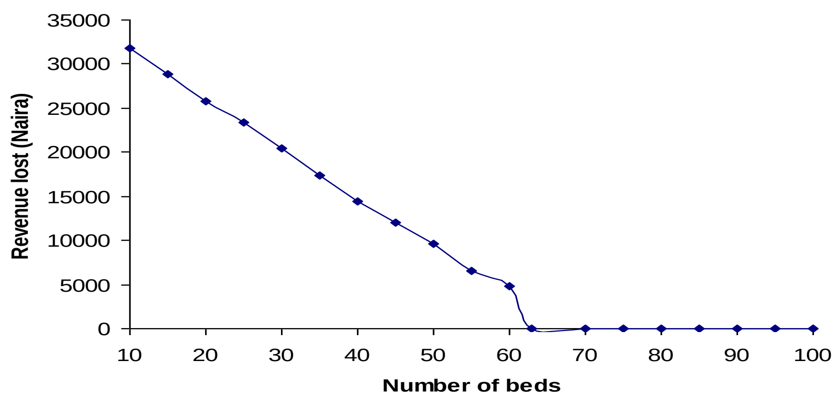

Figure 5: Revenue lost in Private ward

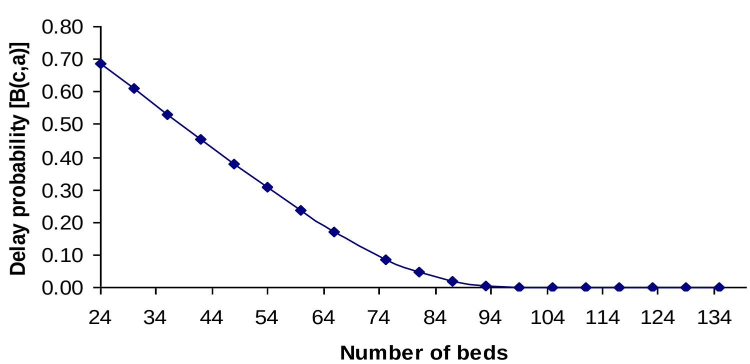

Figure 6: Delay probability of patient's admission into Alam ward

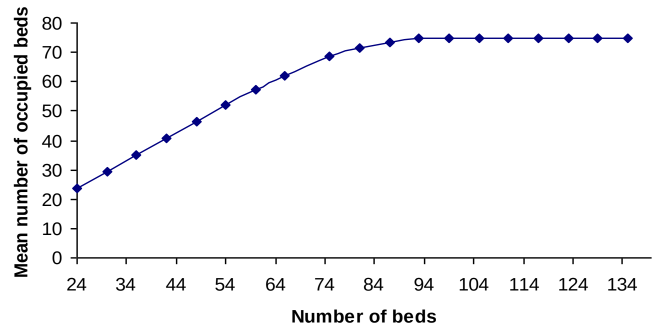

Figure 7: Mean number of occupied beds in Alam ward

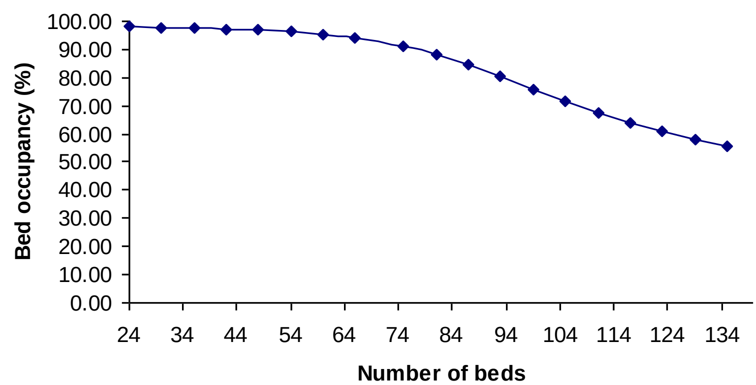

Figure 8: Bed occupancy in Alam ward

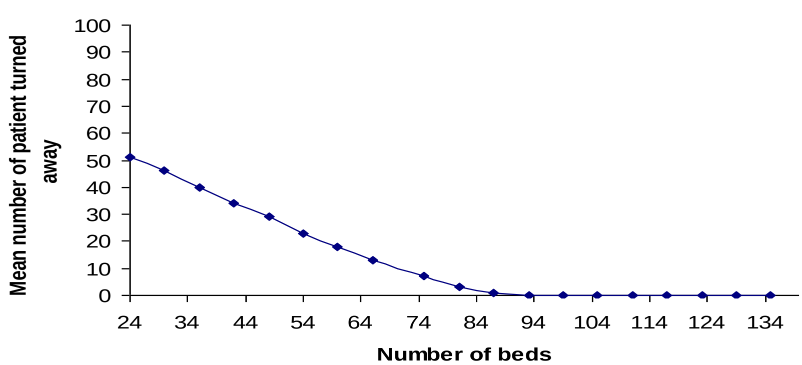

Figure 9: Mean number of patients turned away in Alam ward

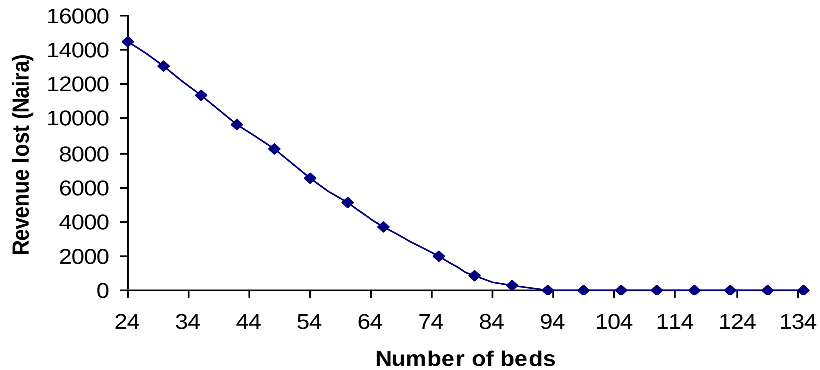

Figure 10: Revenue lost in Alam ward

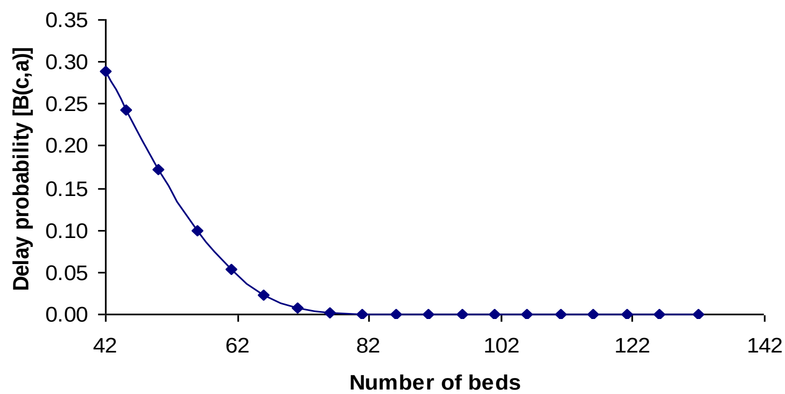

Figure 11: Delay probability of patient's admission into Chile ward

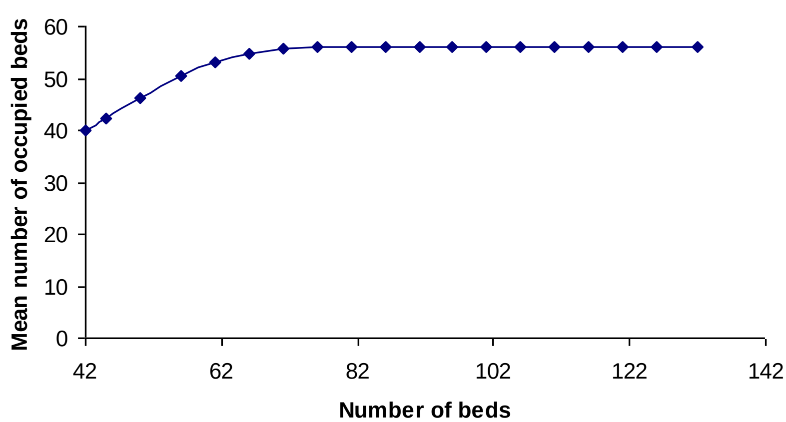

Figure 12: Mean number of occupied beds in Chile ward

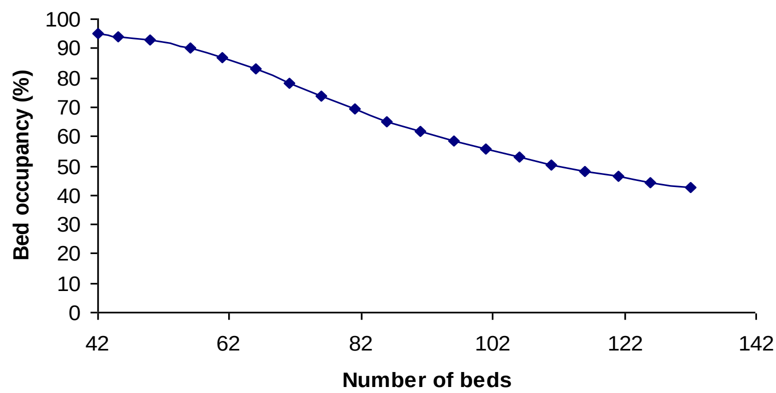

Figure 13: Bed occupancy in Chile ward

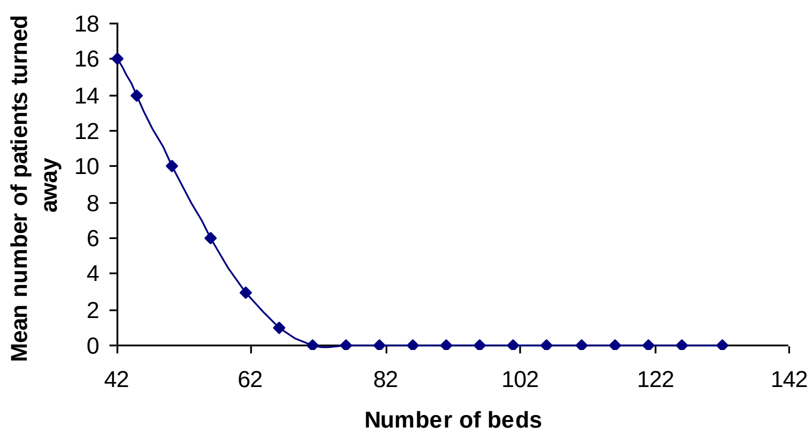

Figure 14: Mean number of patients turned away in Chile ward

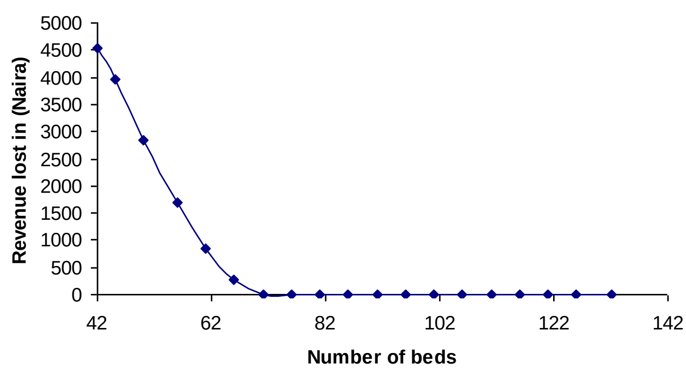

Figure 15: Revenue lost in Chile ward

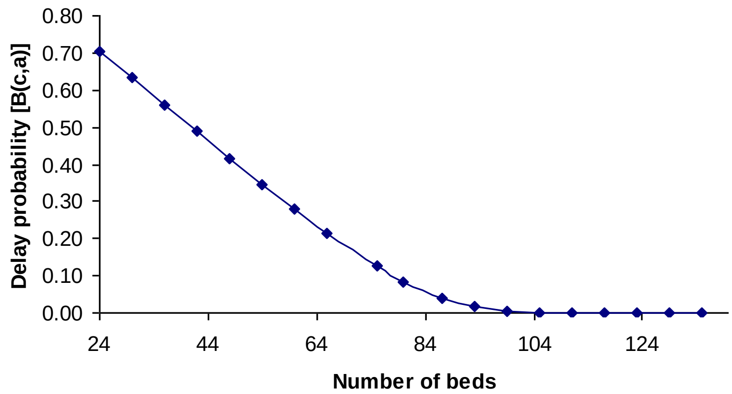

Figure 16: Delay probability of patient's admission into Dooashe ward

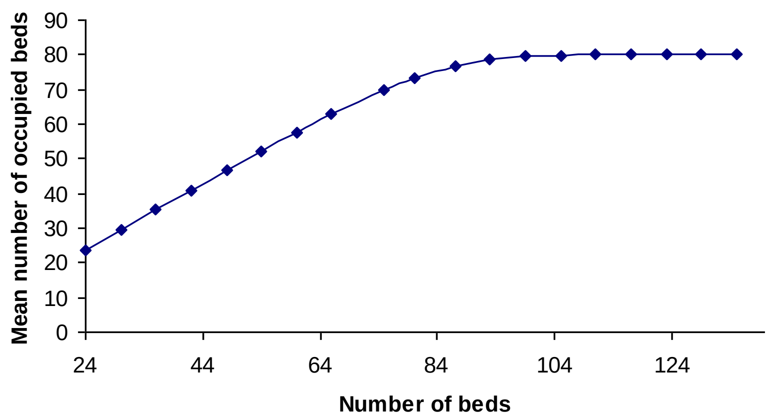

Figure 17: Mean number of occupied beds in Dooashe ward

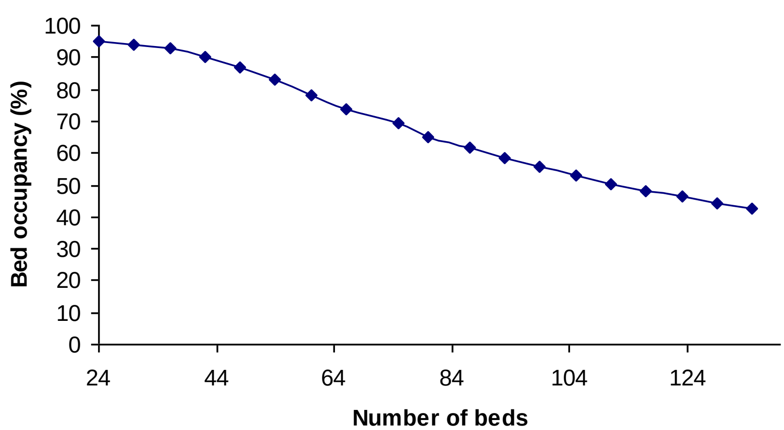

Figure 18: Bed occupancy in Dooashe ward

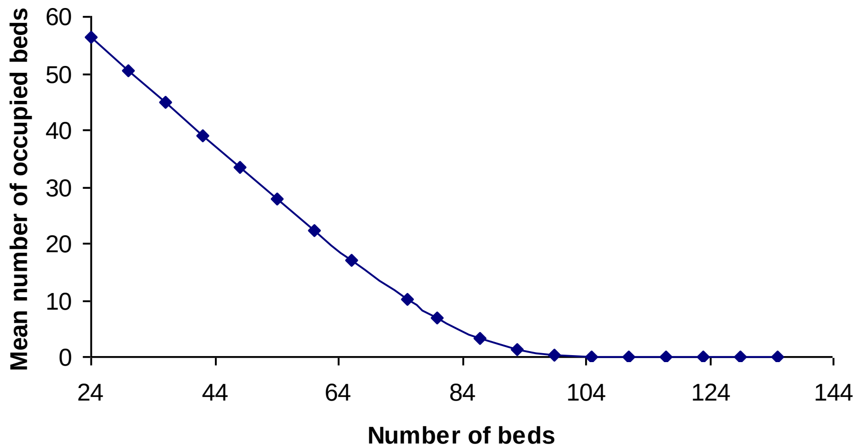

Figure 19: Mean number of patients turned away in Dooashe ward

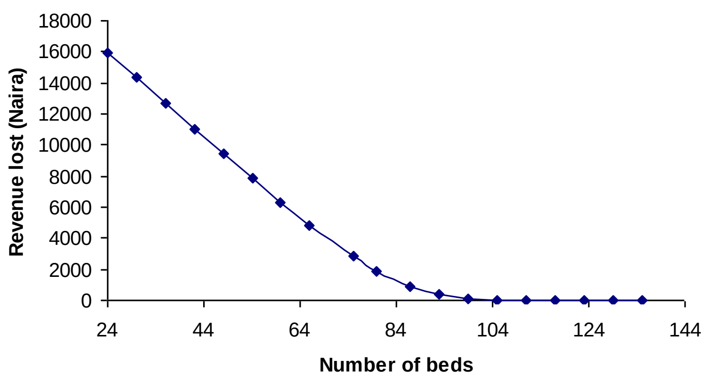

Figure 20: Revenue lost in Dooashe ward

5. DISCUSSION

This section is concerned with the discussions on the analysis of the bed occupancy problem confronting the clinic, the optimal number of beds determined for each ward, the relationship between number of beds and system performance measures and the distribution fits to admission and length of stay data across the wards.

5.1 Discussion on the analysis of the bed occupancy problem across wards

The sensitivity analysis of the system performance measures to changes in the number of beds began with the actual number of beds in each ward; 10, 24, 42 and 24 beds for the Private, Alam, Chile and Dooashe wards respectively (table 1). The inadequacy of these beds across the wards has translated into high delay probability of patient's admission into the wards, a high number of patients being turned away from the wards and a high loss of revenue over the period of average length of stay in the wards. The Private ward shows an offered load of 63 patients, a carried load of 10 patients and a delay probability of 0.84. This translates into 53 patients being turned away on the average and a loss of ₦31,800.00 of revenue over the period of average length of stay.

The Alam ward paints a similar picture with an offered load of 75 patients, carried load of 24 patients with a delay probability of 0.69. This translates into an average of 51 patients being turned away as well as a loss of ₦14,449.83 of revenue over the period of average length of stay. In the Chile ward, the current situation revealed an offered load of 56 patients, a carried load of 40 patients with a delay probability of 0.29. This results into 16 patients being turned away on the average. A loss of revenue of ₦4,533.28 over a period of average length of stay is also incurred in this ward. Dooashe ward shows that the situation on ground reflects an offered load of 80 patients, a carried load of 24 patients. The delay probability is 0.71 translating into 56 patients being turned away on the average and a loss of revenue of ₦15,982.22 over a period of average length of stay. See table 8 for the above details.

It is this unpleasant situation across the wards that necessitated this research in order to determine the optimal number of beds across wards. The constraint for optimality is that patient delay probability must be zero which means no patient is turned away.

5.2 Discussion on the optimal number of beds determined for each ward

From the results of the sensitivity analysis across wards in tables 4-7, observe that the optimal number of beds is determined when the delay probability is zero and that occurs as soon as the offered load equals the carried load as earlier proved (equation 3.2). The result of this optimal number of beds is shown in table 9 for each ward. The table further revealed that the optimal number of beds is 85, 99, 86 and 105 for Private, Alam, Chile and Dooashe wards respectively. Since 10, 24, 42 and 24 beds are already on ground respectively for the wards; only 75, 75, 44 and 81 additional beds are respectively needed in each ward. This will ensure that no patient is turned away and no revenue is lost in the ward. This optimal result is not without a price as the average number of empty beds is 22, 24, 30 and 25 respectively for the aforementioned wards. Gorunesco *et al.* [9] in their case study affirmed that 10-15% bed emptiness is necessary to maintain service efficiency and provide more responsive and cost effective services. In this work, we see this as a price to be paid for improved service as the hospital management must learn to cope with the holding cost of these empty beds.

5.3 Discussion on the relationship between the number of beds and the system performance measures across the wards

Observing across the wards shows that their exist an inverse relationship between the number of beds and the delay probability, the bed occupancy, the mean number of patients turned away and the revenue lost in each wards. The mean number of occupied beds is the only performance measure that revealed a direct variation with the number of beds. This is because it increases as the number of beds increases and decreases as the number of beds decreases. It is important to mention that despite the direct and inverse relationships established, the graphs depict the steady state nature of the results. The steady state sets in when the offered load equals the carried loads. On the graphs, it is seen at the point of optimality; where the graph begins to straighten out despite further increase in the number of beds. See figures 1- 20 for details.

5.4 Discussion on the fitted distribution of number of arrivals and length of stay of patients in the wards

Data on patient's admission and discharge dates were used for the distribution fit. The Poisson distribution was ascertained to fit the number of arrival of patients to the wards while the k-Erlang distribution fit their length of stay in the wards. Table 2 and table 3 show the distribution fits and the parameter details. These distribution fits agree with those of Gorunescu *et al.* [9] in their $M/E_K/c$ queuing model for bed occupancy management and planning of hospitals

6. CONCLUSION AND RECOMMENDATIONS

From the result of the sensitivity analysis of the variation of the number of beds with the systems performance measures, the following conclusions were drawn.

(i) That the queuing model has been able to determine the optimal number of beds required in each ward.

(ii) That the queuing model has successfully ensured that no patient is turned away from the ward and no revenue is lost.

(iii) That though the model has being able to solve the problems of patients been turned away from the wards and that of revenue loss, it is not without the challenges of coping with the holding cost of empty beds across the wards.

6.1 Recommendation

(i) That this model should be used in the bed occupancy management and planning of hospitals and the results implemented by the management of the N.K.S.T Rehabilitation Centre Mkar, Benue State.

(ii) As part of future research efforts, the cost of empty beds should be balanced with the cost of delayed patients in determining the optimal number of beds in each ward of the clinic.

Acknowledgment

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