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طريقة التكرار المتغير لحل مسائل الشروط الحدية ذات الرتبة السابقة باستخدام كثيرات حدود (He)

Shahid S. Siddiqi [a], Muzammal Iftikhar [b,*]

[a] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan
[b] Department of Mathematics, University of Education, Okara Campus, Okara 56300, Pakistan

الملخص:

من أمثلة الشروط الحدية، مسألة سلوك حث المحمول والتي تمثل بمعادلة تفاضلية ذات الرتبة الخامسة. عند إضافة تأثير مسائل عزم الدوران تتحول المعادلة التفاضلية إلى الرتبة السابعة. يقدم في هذا البحث طريقة التكرار المتغير باستخدام كثيرات حدود (He) لإيجاد حلول مسائل الشروط الحدية ذات الرتبة السابعة. الحلول التقريبية المستنتجة تم الحصول عليها بدلالة متسلسلات ذات التقارب السريع، تم تقديم أمثلة عددية لتوضيح كفاءة الطريقة المقدمة.

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ORIGINAL ARTICLE

Variational Iteration Method for the Solution of Seventh Order Boundary Value Problems using He's Polynomials

Shahid S. Siddiqi [a], **Muzammal Iftikhar** [b,*]

[a] *Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan*
[b] *Department of Mathematics, University of Education, Okara Campus, Okara 56300, Pakistan*

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Variational iteration method;
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Abstract The induction motor behaviour is represented by a fifth order differential equation model. Addition of a torque correction factor to the model accurately reproduces the transient torques and instantaneous real and reactive power flows of the full seventh order differential equation model. The variational iteration method using He's polynomials is employed to solve the seventh order boundary value problems. The approximate solutions to the problems are obtained in terms of a rapidly convergent series. Several numerical examples are given to illustrate the implementation and the efficiency of the method.

Introduction

The theory of seventh order boundary value problems is not much available in the numerical analysis literature. These problems generally arise in modelling induction motors with two rotor circuits.

The induction motor behaviour is represented by a fifth order differential equation model. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of the second rotor circuit representing deep bars, a starting cage or rotor distributed parameters.

To avoid the computational burden of additional state variables when additional rotor circuits are required, model is often limited to the fifth order and rotor impedance is algebraically altered as a function of rotor speed under the assumption that the frequency of rotor current depends on the rotor speed. This approach is efficient for the steady state response with sinusoidal voltage, but it does not hold up during the transient conditions, when rotor frequency is not a single value. So the behaviour of such models show up in the seventh order (Richards and Sarma, 1994).

J. He was first to propose a new kind of analytical method for a non-linear problem called the variational iteration method. In (He, 1999a) J. He used variational iteration method to give approximate solutions for some well-known non-linear problems. In variational iteration method, the problems are initially approximated with possible unknown. Then a correction functional is constructed by a general Lagrange

* Corresponding author. Tel.: +923334401730.
E-mail addresses: shahidsiddiqiprof@yahoo.co.uk (S.S. Siddiqi), miftikhar@hotmail.com (M. Iftikhar).

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multiplier, which can be identified optimally via the variational theory.

Homotopy perturbation method was also proposed by He (1999b, 2000b, 2006). To investigate the given problem with the help of homotopy perturbation method, firstly a homotopy equation is constructed. It is assumed that the solution of the problem is $u = \sum_{i=0}^{\infty} u_i c^i$. Substituting the value of u in the homotopy equation and equating the like powers of c, a system of differential equations is obtained. The corresponding solution of the system provides a series solution. The results revealed that the homotopy perturbation method is a powerful and accurate method for finding solutions for BVPs in the form of analytical expressions and presents a rapid convergence for the solutions.

Jafari et al. (Jafari et al., 2011) proposed the homotopy analysis method to solve an evolution equation. The authors compared the results obtained with the help of homotopy analysis method and the results obtained with the help of Adomian decomposition method. Siddiqi and Iftikhar (Siddiqi and Iftikhar, 2013a) used the variation of parameter method to solve the seventh order boundary value problems. In (Siddiqi and Iftikhar, 2013b) the authors used the homotopy analysis method, an approximating technique for solving linear and nonlinear higher order boundary value problems.

Odibat discussed the convergence of variational iteration method in (Odibat, 2010). Tatari and Dehghan presented the sufficient conditions to guarantee the convergence of the variational iteration method (Tatari and Dehghan, 2007).

The aim of this study is to solve the seventh order boundary value problems and the variational iteration method using He's polynomials is used for this purpose.

Variational Iteration Method using He's Polynomials (He, 1999a)

The boundary value problem is considered as under

$$L[u(x)] + N[u(x)] = g(x), \tag{1}$$

where L and N are linear and nonlinear operators respectively and $g(x)$ is a forcing term. Following the variational iteration method used by J. He (He, 1998, 1999a,b, 2000a, 2001) the correct functional for the problem (1) can be written as follows

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(t)\{Lu_n(t)) + N\tilde{u}_n(t) - g(t)\}dt, \tag{2}$$

where λ is a Lagrange multiplier, that can be identified optimally *via* variational iteration method. Here, $\widetilde{u}_n$ is considered to be a restricted variation which shows that $\delta\widetilde{u}_n = 0$. Making the correct functional (2) stationary, yields

$$\begin{aligned}\delta v_{n+1}(x) &= \delta v_n(x) + \delta \int_0^x \lambda(t)\{Lv_n(t)) + N\tilde{v}_n(t) - g(t)\}dt \\ &= \delta v_n(x) + \int_0^x \delta\{\lambda(t)Lv_n(t))\}dt. \end{aligned}\tag{3}$$

Its stationary conditions can be obtained using integration by parts in Eq. (3). Therefore, the Lagrange multiplier can be written as

$$\lambda = \frac{(-1)^m (t-x)^{m-1}}{(m-1)!}. \tag{4}$$

Applying the homotopy perturbation method (He, 1999b, 2000b, 2006), the following relation is obtained as follows

$$\begin{aligned}\sum_{i=0}^{\infty} p^i u_i(x) = u_0(x) + \int_0^x \lambda(t)\left\{L\left(\sum_{i=0}^{\infty} p^i u_i\right) + N\left(\sum_{i=0}^{\infty} p^i \widetilde{u}_i\right)\right\}dt \\ - \int_0^x \lambda(t)g(t)dt, \end{aligned}\tag{5}$$

Equating the like powers of p gives $u_0, u_1, \cdots$. The embedding parameter $p \in [0,1]$ can be used as an expanding parameter. The nonlinear term can be expanded into He's polynomials (Ghorbani, 2009). The approximate solution of the problem (1); therefore, can be expressed as follows

$$u = \lim_{p\to 1} \sum_{i=0}^{\infty} p^i u_i = u_0 + u_1 + u_2 + \cdots \tag{6}$$

The series (6) is convergent for most of the cases. It is assumed that (6) has a unique solution.

In fact, the solution of the problem (1) is considered as the fixed point of the following functional under the suitable choice of the initial term $v_0(x)$.

$$v_{n+1}(x) = v_n(x) + \int_0^x \lambda(t)\{\mathbf{L}v_n(t)) + \mathbf{N}v_n(t) - g(t)\}dt. \tag{7}$$

Convergence

In this section, we will present Banach's theorem about the convergence of the variational iteration method using He's polynomials. The method changes the given differential equation into a recurrence sequence of functions. The limit of this sequence is considered as the solution of the given differential equation.

Theorem 1. *(Banach's fixed point theorem) (*Tatari and Dehghan, 2007*) Suppose that X is a Banach space and $B : X \longrightarrow X$ is a nonlinear mapping, and assume that*

$$\|B[u] - B[\bar{u}]\| \leqslant \gamma\|u - \bar{u}\|, \quad \forall \quad u, \bar{u} \in X. \tag{8}$$

for some constant $\gamma < 1$. Then B has a unique fixed point. Moreover, the sequence

$$u_{n+1} = B[u_n] \tag{9}$$

with an arbitrary choice of $u_0 \in X$ converges to the fixed point of B and

$$\begin{aligned}\|u_k - u_l\| &\leqslant \|u_k - u_{k-1}\| + \cdots + \|u_{l+1} - u_l\| = \|B(u_{k-1}) \\ &\quad - B(u_{k-2})\| + \cdots + \|B(u_l) - B(u_{l-1})\| \\ &\leqslant \gamma\|u_{k-1} - u_{k-2}\| + \cdots + \gamma\|u_l - u_{l-1}\| \leqslant \cdots \leqslant \\ &\leqslant (\gamma^{k-2} + \gamma^{k-3} + \cdots + \gamma^{l-1})\|u_1 - u_0\| \\ &\leqslant \frac{\gamma^{l-1}}{1-\gamma}\|u_1 - u_0\|, \end{aligned}\tag{10}$$

where, $\gamma < 1$, it can be assumed that $k > l \geqslant 1$. This yields $\|u_k - u_l\| \to 0$ as $k, l \to \infty$ and hence $(u_k)_{k=1}^{\infty}$ is a Cauchy sequence. Since X is a Banach space the sequence converges to a fixed point.

According to Theorem 1, for the nonlinear mapping

$$B[u] = u(x) + \int_0^x \lambda(t)\{Lu_n(t)) + N\widetilde{u}_n(t) - g(t)\}dt, \tag{11}$$

is a sufficient condition for convergence of the variational iteration method using He's polynomials is strictly contraction of B. Furthermore, the sequence (9) converges to the fixed point of B which also is the solution of the problem (1).

To implement the method, some numerical examples are considered in the following section.

Numerical Examples

Example 1. The following seventh order linear boundary value problem is considered

$$\left.\begin{array}{l} u^{(7)}(x) = -u(x) - e^x(35 + 12x + 2x^2), 0 \leqslant x \leq 1, \\ u(0) = 0,\ u(1) = 0, \\ u^{(1)}(0) = 1,\ u^{(1)}(1) = -e, \\ u^{(2)}(0) = 0,\ u^{(2)}(1) = -4e, \\ u^{(3)}(0) = -3. \end{array}\right\} \tag{12}$$

The exact solution of the Example 1 is $u(x) = x(1-x)e^x$, (Siddiqi and Iftikhar, 2013a).

The correct functional for the problem (12) can be written as follows

$$v_{n+1}(x) = v_n(x) + \int_0^x \lambda(t)\{v_n^{(7)}(t)) - v_n(t) + e^t(35 + 12t + 2t^2)\}dt, \tag{13}$$

Making the correct functional (13) stationary, yields

$$\delta v_{n+1}(x) = \delta v_n(x) + \delta \int_0^x \lambda(t)\{v_n^{(7)}(t)) - v_n(t) + e^t(35 + 12t + 2t^2)\}dt = \delta v_n(x) + \int_0^x \delta\{\lambda(t)v_n^{(7)}(t))t\}dt, \tag{14}$$

Hence, the following stationary conditions can be determined

$$\begin{array}{l} \lambda^{(7)}(t) = 0, \\ \lambda(t)|_{t=x} = 0, \\ \lambda'(t)|_{t=x} = 0, \\ \quad\vdots \\ \lambda^{(5)}(t)|_{t=x} = 0, \\ 1 + \lambda^{(6)}(t)|_{t=x} = 0, \end{array}$$

which yields

$$\lambda = \frac{(-1)^7(t-x)^6}{(6)!}. \tag{15}$$

The Lagrange multiplier can be identified as follows

$$\lambda = \frac{(-1)^m(t-x)^{m-1}}{(m-1)!}. \tag{16}$$

According to (5), the following iteration formulation is obtained

$$\sum_{i=0}^{\infty} p^i u_i(x) = u_0(x) + \int_0^x \frac{(-1)^7(t-x)^6}{6!}\left\{\left(\sum_{i=0}^{\infty} p^i u_i\right)^{(7)} + \sum_{i=0}^{\infty} p^i u_i + e^t(35 + 12t + 2t^2))\right\}dt. \tag{17}$$

Now, assume that an initial approximation has the form

$$u_0(x) = x - \frac{x^3}{2} + Ax^4 + Bx^5 + Cx^6. \tag{18}$$

Comparing the coefficient of like powers of p

$$p^0: \quad u_0(x) = x - \frac{x^3}{2} + Ax^4 + Bx^5 + Cx^6,$$

$$p^1: \quad u_1(x) = -\frac{x^7}{144} - \frac{x^8}{840} - \frac{x^9}{5760} - \frac{x^{10}}{45360} + \left(\frac{-107}{39916800} - \frac{A}{1663200}\right)x^{11} + \left(\frac{-1}{3548160} - \frac{B}{3991680}\right)x^{12} + O(x)^{13},$$

$$\vdots$$

where A, B and C are unknown constants to be determined later.

Using the first two approximations the series solution can be written as follows

$$u(x) = x - \frac{x^3}{2} + Ax^4 + Bx^5 + Cx^6 - \frac{x^7}{144} - \frac{x^8}{840} - \frac{x^9}{5760} - \frac{x^{10}}{45360} + \left(\frac{-107}{39916800} - \frac{A}{1663200}\right)x^{11} + \left(\frac{-1}{3548160} - \frac{B}{3991680}\right)x^{12} + O(x)^{13}.$$

Using the boundary conditions (12), the values of the unknown constants can be determined as follows

$A = -0.3333333170467781$, $B = -0.12500003614813987$, $C = -0.03333331303032349$.

Finally, the series solution is

$$u(x) = x - (0.5)x^3 - 0.333333x^4 - 0.125x^5 - 0.0333333x^6 - 0.00694444x^7 - 0.00119048x^8 - 0.000173611x^9 - 0.0000220459x^{10} - (2.48016 \times 10^{-6})x^{11} - (2.50521 \times 10^{-7})x^{12} + O(x)^{13}.$$

The comparison of the values of maximum absolute errors of the present method with the variation of parameter method (Siddiqi and Iftikhar, 2013a) for the Example 1 is given in Table 1, which shows that the present method is more accurate. In Fig. 1, the comparison of exact and approximate solutions is given and absolute errors are plotted in Fig. 2 for Example 1. Fig. 3.

Example 2. The following seventh order nonlinear boundary value problem is considered

$$\left.\begin{array}{l} u^{(7)}(x) = e^{-x}u^2(x),\ 0 < x < 1, \\ u(0) = u^{(1)}(0) = u^{(2)}(0) = u^{(3)}(0) = 1, \\ u(1) = u^{(1)}(1) = u^{(2)}(1) = e. \end{array}\right\} \tag{19}$$

Table 1 Comparison of maximum absolute errors for Example 1.

Present method	Variation of Parameter method (Siddiqi and Iftikhar, 2013a)
2.46782×10^{-10}	2.1729×10^{-09}

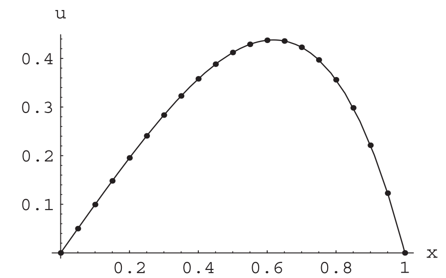

Fig. 1 Comparison between the exact solution and the approximate solution for Example (1). Dotted line: approximate solution, solid line: the exact solution.

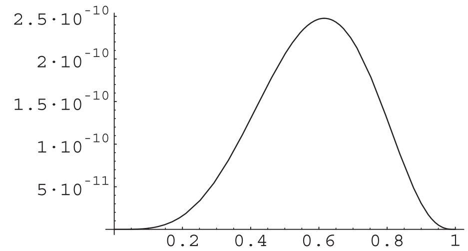

Fig. 2 Absolute errors for Example (1).

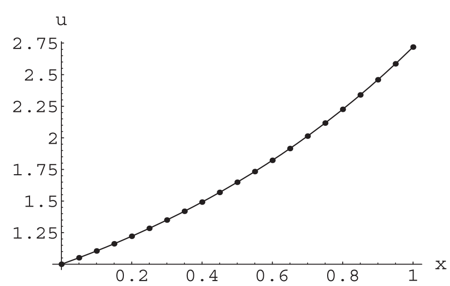

Fig. 3 Comparison between the exact solution and the approximate solution for Example (2). Dotted line: approximate solution, solid line: the exact solution.

The exact solution of the Example 2 is $u(x) = e^x$, (Siddiqi and Iftikhar, 2013a).

The Lagrange multiplier can be identified as follows

$$\lambda = \frac{(-1)^m (t-x)^{m-1}}{(m-1)!}. \tag{20}$$

According to (5), the following iteration formulation is obtained

$$\sum_{i=0}^{\infty} p^i u_i(x) = u_0(x) + \int_0^x \frac{(-1)^7 (t-x)^6}{6!} \left\{ \left(\sum_{i=0}^{\infty} p^i u_i \right)^{(7)} - \left(\sum_{i=0}^{\infty} p^i u_i \right)^2 e^{-t} \right\} dt. \tag{21}$$

Now, assume that an initial approximation has the form

$$u_0(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + Ax^4 + Bx^5 + Cx^6. \tag{22}$$

The nonlinear term $N(u)$ in the Eq. (21) can be expressed as follows

$$N(u) = N(u_0) + cN(u_0, u_1) + c^2 N(u_0, u_1, u_2) + \cdots, \tag{23}$$

where

$$N(u_0, u_1, u_2, \cdots, u_n) = \frac{1}{n!} \frac{d^n}{dc^n} \left[N\left(\sum_{k=0}^{n} c^k u_k \right) \right]_{c=0}, \quad n = 0, 1, 2, \cdots \tag{24}$$

is called He's polynomial (Ghorbani, 2009). Comparing the coefficient of like powers of p

$$p^0: \quad u_0(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + Ax^4 + Bx^5 + Cx^6,$$

$$p^1: \quad u_1(x) = \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \frac{x^{10}}{3628800} + \left(\frac{-1}{39916800} + \frac{A}{831600} \right) x^{11} + \left(\frac{-1}{479001600} + \frac{B}{1995840} \right) x^{12} + O(x)^{13},$$

$$\vdots$$

where A, B and C are unknown constants to be determined later.

Using the first two approximations the series solution can be written as follows

$$u(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + Ax^4 + Bx^5 + Cx^6 + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \frac{x^{10}}{3628800} + \left(\frac{-1}{39916800} + \frac{A}{831600} \right) x^{11} + \left(\frac{-1}{479001600} + \frac{B}{1995840} \right) x^{12} + O(x)^{13}.$$

Using the boundary conditions (19), the values of the unknown constants can be determined as follows

$A = 0.041666667529862395$, $B = 0.008333331197193119$, $C = 0.001388890268167299$.

Finally, the series solution is

$$u(x) = 1. + x + (0.5)x^2 + 0.166667x^3 + 0.0416667x^4 + 0.00833333x^5 + 0.00138889x^6 + 0.000198413x^7 + 0.0000248016x^8 + (2.75573 \times 10^{-6})x^9 + (2.75573 \times 10^{-7})x^{10} + (2.50521 \times 10^{-8})x^{11} + (2.08768 \times 10^{-9})x^{12} + O(x)^{13}.$$

In Table 2, errors obtained by the present method are compared with errors obtained using the variation of parameters method (Siddiqi and Iftikhar, 2013a) for the Example 2. It is observed that the maximum absolute error value for the present method is 4.5614×10^{-9} which is better than the maximum absolute error value, 7.7176×10^{-7}, of the variation of parameters method (Siddiqi and Iftikhar, 2013a). Fig. 2 shows the comparison of exact and approximate solutions and absolute errors are plotted in Fig. 4 for Example 2. The results reveal that the present method is more accurate.

Example 3. The following seventh order nonlinear boundary value problem is considered

Table 2 Comparison of numerical results for Example 2.

x	Exact solution	Approximate solution	Absolute Error present method	Absolute Error (Siddiqi and Iftikhar, 2013a)
0.0	1.0000	1.0000	1.32455E-09	0.0000
0.1	1.1051	1.1051	5.26137E-10	2.26257E-07
0.2	1.2214	1.2214	1.64015E-09	4.38942E-07
0.3	1.3498	1.3498	4.56139E-09	6.1274E-07
0.4	1.4918	1.4918	2.9619E-09	7.71759E-07
0.5	1.6487	1.6487	7.54889E-10	7.71759E-07
0.6	1.8221	1.8221	2.67612E-09	7.37682E-07
0.7	2.0137	2.0137	8.42306E-10	6.25932E-07
0.8	2.2255	2.2255	1.16866E-09	4.68244E-07
0.9	2.4596	2.4596	4.4716E-09	2.95852E-07
1.0	2.7183	2.7183	1.02746E-09	1.25922E-07

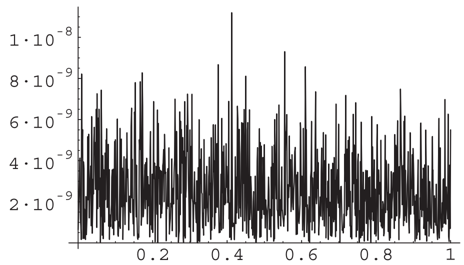

Fig. 4 Absolute errors for Example (2).

$$\left.\begin{array}{l} u^{(7)}(x) = -u(x)u'(x) + g(x),\ 0 \leqslant x \leq 1, \\ u(0) = 0, \quad u(1) = 0, \\ u^{(1)}(0) = 1, \quad u^{(1)}(1) = -e, \\ u^{(2)}(0) = 0, \quad u^{(2)}(1) = -4e, \\ u^{(3)}(0) = -3. \end{array}\right\} \quad (25)$$

where $g(x) = e^x(-35 + (-13 + e^x)x - (1 + 2e^x)x^2 + e^x x^4)$.

The exact solution of the Example 3.3 is $u(x) = x(1 - x)e^x$.

Following the procedure of the previous examples the series solution, using the first two approximations, can be written as follows

$$\begin{aligned} u(x) = x &- (0.5)x^3 - 0.333333x^4 - 0.125x^5 - 0.0333332x^6 \\ &- 0.00694444x^7 - 0.00119048x^8 - 0.000173611x^9 \\ &- 0.0000220459x^{10} - (2.48016 \times 10^{-6})x^{11} \\ &- (2.50521 \times 10^{-7})x^{12} + O(x)^{13}. \end{aligned}$$

The comparison of the exact solution with the series solution of the Example 3 is given in Table 3. In Fig. 5 absolute errors are plotted.

Example 4. The following seventh order nonlinear three point boundary value problem is considered

$$\left.\begin{array}{l} u^{(7)}(x) = u(x)u'(x) - e^x(6 + x - e^x x + e^x x^2), \quad 0 \leqslant x \leq 1, \\ u(0) = 1, \quad u(\frac{1}{2}) = \frac{e^{\frac{1}{2}}}{2}, \\ u^{(1)}(0) = 0, \quad u^{(1)}(\frac{1}{2}) = -\frac{e^{\frac{1}{2}}}{2}, \\ u^{(2)}(0) = -1, \quad u^{(2)}(1) = -2e, \\ u(1) = 0. \end{array}\right\} \quad (26)$$

Table 3 Comparison of numerical results for Example 3.

x	Exact solution	Approximate solution	Absolute Error
0.0	0.0000	0.0000	0.00000
0.1	0.0994654	0.0994654	5.28944E-12
0.2	0.195424	0.195424	6.44606E-11
0.3	0.28347	0.28347	2.38427E-10
0.4	0.358038	0.358038	5.20559E-10
0.5	0.41218	0.41218	8.11431E-10
0.6	0.437309	0.437309	9.55209E-10
0.7	0.422888	0.422888	8.30543E-10
0.8	0.356087	0.356087	4.67351E-10
0.9	0.221364	0.221364	1.04882E-10
1.0	0.0000	3.90259E-12	3.90259E-12

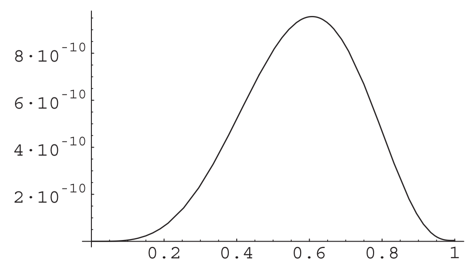

Fig. 5 Absolute errors for Example (3).

Table 4 Comparison of numerical results for Example 4.

x	Exact solution	Approximate series solution	Absolute Error
0.0	1.0000	1.0000	0.0000
0.1	0.9946	0.9946	9.48615E-11
0.2	0.9771	0.9771	3.7371E-10
0.3	0.9449	0.9449	4.8626E-10
0.4	0.8950	0.8950	2.46565E-10
0.5	0.8243	0.8243	8.16711E-11
0.6	0.7288	0.7288	8.67514E-11
0.7	0.6041	0.6041	9.51461E-11
0.8	0.4451	0.4451	2.63398E-09
0.9	0.2459	0.2459	1.44494E-08
1.0	0.0000	−4.90417E-08	4.90417E-08

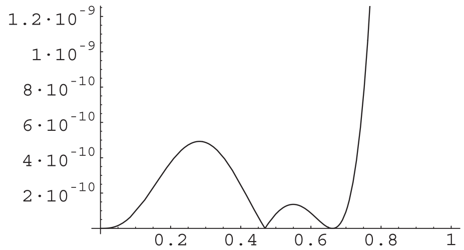

Fig. 6 Absolute errors for Example (4).

The exact solution of the Example 4 is $u(x) = (1 - x)e^x$.

Following the previous examples the series solution, using the first two approximations, can be written as follows

$$\begin{aligned} u(x) = 1. &- (0.5)x^2 - 0.333333x^3 - 0.125001x^4 - 0.0333319x^5 \\ &- 0.00694445x^6 - 0.00119048x^7 - 0.000173611x^8 \\ &- 0.0000220459x^9 - (2.48016 \times 10^{-6})x^{10} \\ &- (2.50517 \times 10^{-7})x^{11} - (2.29651 \times 10^{-8})x^{12} + O(x)^{13}. \end{aligned}$$

The comparison of the exact solution with the series solution of the Example 4 is given in Table 4. Absolute errors are plotted in Fig. 6.

Conclusion

In this paper, variational iteration method using He's polynomials has been applied to obtain the numerical solutions of linear and nonlinear seventh order boundary value problems. The method solves nonlinear problems using He's polynomials. The method gives rapidly converging series solutions in both linear and nonlinear cases. The numerical results revealed that the present method is a powerful mathematical tool for the solution of seventh order boundary value problems. Numerical examples also show the accuracy of the method.

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