Journal of the Association of Arab Universities for Basic and Applied Sciences (2014) **16**, 11–15

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Journal of the Association of Arab Universities for Basic and Applied Sciences
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طريقة تحليلية معدلة لسريان جيفري-هامل باستخدام تحويل سومودا

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الملخص:

إن الهدف الاساس من هذه الدراسة هو تقديم طريقة موثوقة لحساب حل تقريبي لسريان جيفري–هامل (Hamel–Jeffery) وذلك ياستخدام طريقة اضطراب هوموتوبي (homotopy) المعدلة الى جانب تحويل سومودا (Sumudu). الطريقة المعتمدة تعمل على ايجاد الحلول بدون تفرد او تقريبات مقيدة وتتجنب تقريب الخطأ العددي. في الحقيقة فإن هذه الطريقة تعمل على حل مسائل لاخطية دون استخدام متعددة حدود ادومين (Adomian) والذي يمكن اعتباره احد ميزات هذه الطريقة على طريقة التحلل. إن الحلول العددية التي تم الحصول عليها من الطريقة المقترحة تدل على ان النهج سهل التنفيذ وحسابيا جذاب.

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ORIGINAL ARTICLE

A modified analytical technique for Jeffery–Hamel flow using sumudu transform

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Received 30 May 2013; revised 19 August 2013; accepted 19 October 2013
Available online 18 November 2013

KEYWORDS

Jeffery–Hamel flow;
Homotopy perturbation method;
Sumudu transform;
Fluid mechanics;
Nonlinear equation

Abstract The main objective of this paper is to present a reliable approach to compute an approximate solution of Jeffery–Hamel flow by using the modified homotopy perturbation method coupled with sumudu transform. The method finds the solution without any discretization or restrictive assumptions and avoids the round-off errors. The fact that this technique solves nonlinear problems without using Adomian's polynomials can be considered as a clear advantage of this algorithm over the decomposition method. The numerical solutions obtained by the proposed method indicate that the approach is easy to implement and computationally very attractive.

1. Introduction

Internal flow between two plates is one of the most applicable cases in mechanics, civil and environmental engineering. In simple cases, the one-dimensional flow through tube and parallel plates, this is known as Couette–Poisseuille flow, has an exact solution, but in general, like most of fluid mechanic equations, a set of nonlinear equations must be solved which make some problems for analytical solution.

The flow between two planes that meet at an angle was first analyzed by Jeffery (1915) and Hamel et al. (1916) and so, it is known as Jeffery–Hamel flow, too. They worked mathematically on incompressible viscous fluid flow through convergent-divergent channels. They presented an exact similarity solution of the Navier–Stokes equations. In the special case of two-dimensional flow through a channel with inclined plane walls meeting at a vertex and with a source or sink at the vertex and have been studied extensively by several authors and discussed in many textbooks e.g. (Rosenhead, 1940; White, 1991; Esmali et al., 2008; Joneidi et al., 2010; Ganji et al., 2009; Inc et al., 2013). Sadri (1997) has denoted that Jeffery–Hamel is used as a asymptotic boundary condition to examine a steady two-dimensional flow of a viscous fluid in a channel. But, here some symmetric solutions of the flow have been considered, although asymmetric solutions are both possible and of physical interest (Sobey and Drazin, 1986).

Most of the scientific problems such as Jeffery–Hamel flow and other fluid mechanic problems are inherently nonlinear. Except a limited number of these problems, most of them do not have an exact solution. There exists a wide class of literature dealing with the problems of approximate solutions to nonlinear equations with various different methodologies,

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Peer review under responsibility of University of Bahrain.

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http://dx.doi.org/10.1016/j.jaubas.2013.10.001

called perturbation methods. But, the perturbation methods have some limitations e.g., the approximate solution involves a series of small parameters which poses difficulty since majority of nonlinear problems have no small parameters at all. Although appropriate choices of small parameters some times lead to an ideal solution, in most of the cases unsuitable choices lead to serious effects in the solutions. Therefore, an analytical method is welcome which does not require a small parameter in the equation modeling the phenomenon. The homotopy perturbation method (HPM) was first introduced and developed by He (1999, 2005, 2006a, 2006b, 2012). It was shown by many authors that this method provides improvements over existing numerical techniques (Ganji and Ganji, 2008; Ganji et al., 2008, 2009; Rashidi et al., 2009; Yildirim and Sezer, 2010; Noor et al., 2013; Mirzabeigy et al., 2013). In recent years, many authors have paid attention to study the solutions of linear and nonlinear partial differential equations by using various methods combined with the Laplace transform (Khuri, 2001; Khan et al., 2012; Gondal and Khan, 2010; Singh et al., 2013a) and sumudu transform (Singh et al., 2011, 2013b).

In this paper, we present a modified analytical technique namely the modified homotopy perturbation method (MHPM) coupled with sumudu transform to obtain the approximate solution of nonlinear equation governing Jeffery–Hamel flow. The MHPM coupled with sumudu transform provides the solution in a rapid convergent series which may lead to the solution in a closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact and approximate solutions for nonlinear equations.

2. Sumudu transform

In early 90's, Watugala (1993) introduced a new integral transform, named the sumudu transform and applied it to the solution of ordinary differential equation in control engineering problems. The sumudu transform is defined over the set of functions

$$A = \{f(t) | \exists M, \tau_1, \tau_2 > 0, |f(t)| < M\, e^{|t|/\tau_j}, \text{if } t \in (-1)^j \times [0,\infty)\}$$

by the following formula

$$\bar{f}(u) = S[f(t)] = \int_0^\infty f(ut)e^{-t}dt, \; u \in (-\tau_1, \tau_2). \tag{1}$$

Some of the properties of the sumudu transform were established by Asiru (2001). Further, fundamental properties of this transform were established by Belgacem et al. (2003), Belgacem and Karaballi (2006), Belgacem (2006). In fact it was shown that there is a strong relationship between sumudu and other integral transform, see Kilicman et al. (2011). In particular the relation between sumudu transform and Laplace transforms was proved in Kilicman and Eltayeb (2010). The sumudu transform has scale and unit preserving properties, so it can be used to solve problems without resorting to a new frequency domain.

3. Mathematical model

Consider the steady unidirectional flow of an incompressible viscous fluid flow from a source or sink at the intersection between two rigid plane walls that the angle between them is 2α as it is shown in Fig. 1.

The velocity is assumed only along radial direction and depends on r and θ. Conservation of mass and momentum for two-dimensional flow in the cylindrical coordinate can be expressed as the following (Schlichting, 2000)

$$\frac{1}{r}\frac{\partial}{\partial r}(rU_r) + \frac{1}{r}\frac{\partial}{\partial r}(rU_\theta) = 0, \tag{2a}$$

$$\rho\left(U_r\frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta^2}{r}\right) = -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial r} - \frac{\tau_{r\theta}}{r}, \tag{2b}$$

$$\rho\left(U_r\frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} - \frac{U_r U_\theta}{r}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \frac{1}{r^2}\frac{\partial(r\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} - \frac{\tau_{r\theta}}{r}, \tag{2c}$$

where P is the pressure term, U_r and U_θ are the velocities in r and θ directions, respectively. Stress components are defined as follows:

$$\tau_{rr} = \mu\left(2\frac{\partial U_r}{\partial r} - \frac{2}{3}div(\vec{U})\right), \tag{3a}$$

$$\tau_{\theta\theta} = \mu\left(2\left(\frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r}\right) - \frac{2}{3}div(\vec{U})\right), \tag{3b}$$

$$\tau_{r\theta} = \mu\left(2\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial U_r}{\partial \theta}\right)\right). \tag{3c}$$

Considering $U_\theta = 0$ for purely radial flow leads to continuity and Navier–Stokes equations in polar coordinates become

$$\frac{\rho}{r}\frac{\partial}{\partial r}(rU_r) = 0, \tag{4a}$$

$$U_r\frac{\partial U_r}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \upsilon\left[\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \theta^2} - \frac{U_r}{r^2}\right], \tag{4b}$$

$$-\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \frac{2\upsilon}{r^2}\frac{\partial U_r}{\partial \theta} = 0. \tag{4c}$$

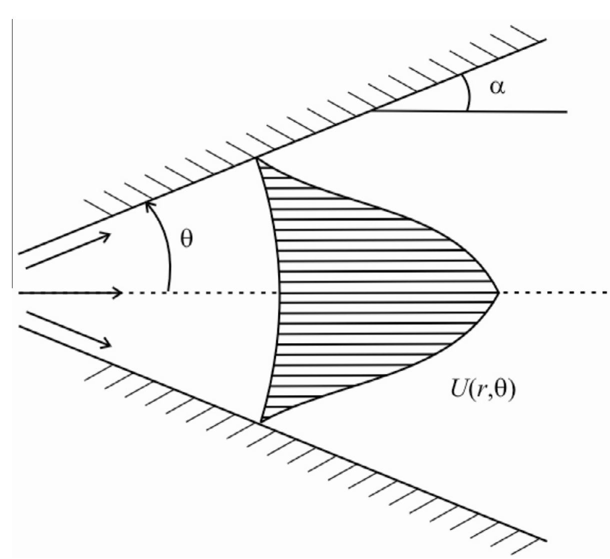

Figure 1 Schematic figure of the problem.

The boundary conditions are

At centerline of the channel: $\frac{\partial U_r}{\partial \theta} = 0$,

On the wall of the channel : $U_r = 0$. (5)

From Eq. (4a)

$$g(\theta) \equiv rU_r, \tag{6}$$

using dimensionless parameter

$$f(x) \equiv \frac{g(\theta)}{g_{\max}}, \quad x \equiv \frac{\theta}{\alpha}, \tag{7}$$

and with eliminating P from Eqs. 4b and 4c, an ordinary differential equation is obtained for the normalized function profile $f(x)$:

$$f''' + 2\alpha \mathrm{Re} f(x) f'(x) + 4\alpha^2 f'(x) = 0. \tag{8}$$

According to the relation Eqs. (5)–(7), the boundary conditions will be

$$f(0) = 1, \quad f'(0) = 0, \quad f(1) = 0. \tag{9}$$

The Reynolds number is

$$\mathrm{Re} = \frac{g_{\max}\alpha}{\upsilon} = \frac{U_{\max} r\alpha}{\upsilon}\left(\begin{array}{l} Divergent\,Channel : \alpha > 0 \\ Convergent\,Channel : \alpha < 0 \end{array}\right). \tag{10}$$

where $U_{\max}$ is the velocity at the center of the channel ($r = 0$).

4. Basic idea of MHPM coupled with sumudu transform

To illustrate the basic idea of this method, we consider a general nonlinear non-homogenous partial differential equation of the form:

$$LU + RU + NU = g(x), \tag{11}$$

where L is the highest order linear differential operator, R is the linear differential operator of less order than L, N represents the general nonlinear differential operator and $g(x)$ is the source term. By applying the sumudu transform on both sides of Eq. (11), we get

$$S[U] = u^n \sum_{k=0}^{n-1} \frac{U^{(k)}(0)}{u^{(n-k)}} + u^n S[g(x)] - u^n S[RU + NU] = 0. \tag{12}$$

Now applying the inverse sumudu transform on both sides of Eq. (12), we get

$$U = G(x) - S^{-1}[u^n S[RU + NU]], \tag{13}$$

where $G(x)$ represents the term arising from the source term and the prescribed initial conditions. Now we construct the following homotopy

$$U = G(x) - p(S^{-1}[u^n S[RU + NU]]), \tag{14}$$

In view of the HPM, we use the homotopy parameter p to expand solution

$$U = \sum_{m=0}^{\infty} p^m U_m \tag{15}$$

and the nonlinear term can be decomposed as

$$NU = \sum_{m=0}^{\infty} p^m H_m, \tag{16}$$

for some He's polynomials (Ghorbani, 2009; Mohyud-Din et al., 2009) that are given by

$$H_m(U_0, U_1, ..., U_m) = \frac{1}{m!}\frac{\partial^m}{\partial p^m}\left[N\left(\sum_{i=0}^{\infty} p^i U_i\right)\right]_{p=0},$$

$$m = 0, 1, 2, 3, \ldots \tag{17}$$

Substituting Eqs. 15 and 16 in Eq. (14), we get

$$\sum_{m=0}^{\infty} p^m\ U_m = G(x) - p\left(S^{-1}\left[u^n S\left[R\sum_{m=0}^{\infty} p^m\ U_m + \sum_{m=0}^{\infty} p^m H_m\right]\right]\right), \tag{18}$$

Comparing the coefficient of like powers of p, the following approximations are obtained

$$\begin{aligned} p^0 &: U_0(x) = G(x), \\ p^1 &: U_1(x) = -S^{-1}[u^n S[RU_0(x) + H_0(U)]], \\ p^2 &: U_2(x) = -S^{-1}[u^n S[RU_1(x) + H_1(U)]], \\ p^3 &: U_3(x) = -S^{-1}[u^n S[RU_2(x) + H_2(U)]]. \end{aligned} \tag{19}$$

Proceeding in this same manner, the rest of the components U_m can be completely obtained and the series solution is thus entirely determined. Finally, we approximate the analytical solution U by truncated series

$$U = \lim_{N\to\infty} \sum_{m=0}^{N} U_m. \tag{20}$$

The above series solutions generally converge very rapidly.

5. Solution of the problem

In this section, we apply the MHPM coupled with sumudu transform to obtain an approximate analytical solution of Eq. (8). By applying the sumudu transform on the both sides of Eq. (8), we have

$$S[f(x)] = 1 + au^2 - u^3 S[2\alpha \mathrm{Re} ff' + 4\alpha^2 f']. \tag{21}$$

Taking inverse sumudu transform on both sides of Eq. (21), we get

$$f(x) = 1 + \frac{1}{2}ax^2 - S^{-1}[u^3 S[2\alpha \mathrm{Re} ff' + 4\alpha^2 f']]. \tag{22}$$

Now applying the HPM, we get

$$\sum_{m=0}^{\infty} p^m f_m(x) = 1 + \frac{1}{2}ax^2 - p\left(S^{-1}\left[u^3 S\left[2\alpha \mathrm{Re}\left(\sum_{m=0}^{\infty} p^m H_m(x)\right) + 4\alpha^2\left(\sum_{m=0}^{\infty} p^m f'_m x\right)\right]\right]\right), \tag{23}$$

where H_m, is He's polynomials that represent the nonlinear terms. So He's polynomials are given by

$$\sum_{m=0}^{\infty} p^m H_m(x) = f(x)f'(x). \tag{24}$$

The first few components of He's polynomials, are given by

$$\begin{aligned} H_0(x) &= f_0(x)f'_0(x), \\ H_1(x) &= f_0(x)f'_1(x) + f_1(x)f'_0(x), \\ H_2(x) &= f_0(x)f'_2(x) + f_1(x)f'_1(x) + f_2(x)f'_0(x), \\ &\vdots \end{aligned} \tag{25}$$

Table 1 The comparison between the RKHSM (with $H = 0$) (Inc et al., 2013) and MHPM coupled with sumudu transform for $f(x)$ when Re = 80 and $\alpha = -5°$.

x	RKHSM Inc et al. (2013)	MHPM coupled with sumudu transform
0	1	1
0.1	0.99595999	0.9962165196
0.2	0.983275	0.9843230775
0.3	0.96017	0.9625668179
0.4	0.923519	0.9276517677
0.5	0.86845826	0.8743082951
0.6	0.78809	0.7949430464
0.7	0.67314	0.6795990006
0.8	0.5119873503	0.5164879998
0.9	0.2915582665	0.2933661078
1	2.851385×10^{-9}	0.0000000005

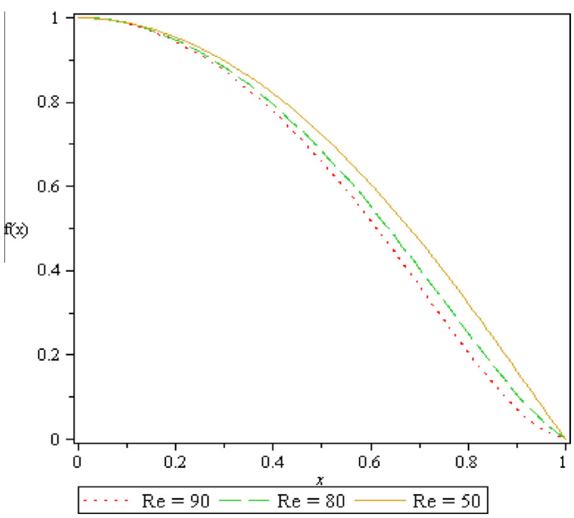

Figure 2 Velocity diagram via MHPM coupled with sumudu transform for different values of Re when $\alpha = 3°$.

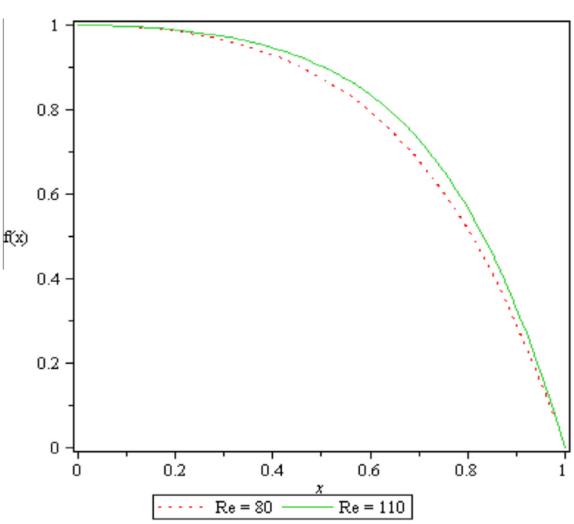

Figure 3 Velocity diagram via MHPM coupled with sumudu transform for different values of Re when $\alpha = -5°$.

Comparing the coefficients of like powers of p, we have

$$p^0 : f_0(x) = 1 + \frac{1}{2}ax^2, \tag{26}$$

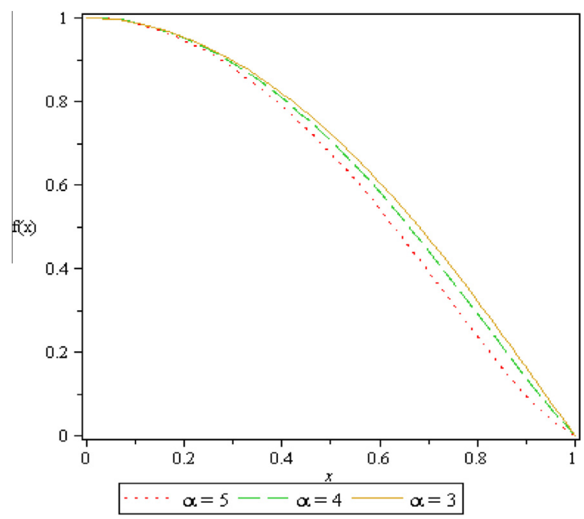

Figure 4 Velocity diagram via MHPM coupled with sumudu transform for different values of α when Re = 50.

$$p^1 : f_1(x) = -\alpha\left[\frac{a^2}{120}\mathrm{Re}x^6 + \frac{1}{12}(\mathrm{Re}a + 2\alpha a)x^4\right], \tag{27}$$

$$p^2 : f_2(x) = -\frac{4}{15}\alpha^2\left[-\frac{a^3}{2880}\mathrm{Re}^2x^{10} - \frac{a^2}{1344}(9\mathrm{Re}^2 + 18\alpha\mathrm{Re})x^8 - \frac{a}{240}(5\mathrm{Re}^2 + 20\alpha\mathrm{Re} + 20\alpha^2)x^6\right], \tag{28}$$

$\vdots$

where $a = f''(0)$ to be determined from the boundary conditions. The solutions of the Eq. (8), when $p \to 1$, will be as follows:

$$f(x) = f_0(x) + f_1(x) + f_2(x) + \cdots. \tag{29}$$

6. Results and discussion

Eq. (8) is solved analytically using the MHPM coupled with sumudu transform. Table 1 shows comparison between the RKHSM (Inc et al., 2013) and MHPM coupled with sumudu transform for $f(x)$ when Re = 80 and $\alpha = -5°$. Figs. 2–4 illustrate the effects of Reynolds number and steep angle of the channel on velocity profile.

7. Conclusions

In this paper, the MHPM coupled with sumudu transform is applied successfully to find the analytical solution of Jeffery–Hamel flow. The results of the present method are in excellent agreement with the RKHSM (Inc et al., 2013) and the obtained solutions are revealed graphically. In this paper, we use Maple Package to calculate the He's polynomials. Also from figures, we can find some results as follows:

(1) When $\alpha > 0$ and steep of the channel is divergent, an increase in the values of Reynolds number decreases the velocity as shown in Fig. 2 when $\alpha = 3°$.
(2) When $\alpha < 0$ and the steep of the channel is convergent, the velocity increases with the increase in Reynolds number as depicted in Fig. 3 when $\alpha = -5°$.
(3) When Reynolds number is fixed, there is an inverse relation between divergence angle of the channel and the velocity of the fluid as shown in Fig. 4.

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