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# Capacity Analysis for Cooperative Amplify and Forward Relay MIMO in Correlated Channels

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**Abstract**: In this paper, various channel models including outdoor Stochastic Spatial Clustered (OSSC) Channel Model have been implemented to investigate and optimize the capacity of different case studies of Amplify and Forward Relay MIMO systems. Hata Model has been included in the model to consider the power dissipation due to large scale fading. Generalized analytical capacity and diversity gains formulas have been presented for different scenarios and various antenna configurations. Comparative studies and analysis for optimum cases are introduced for different MIMO antenna configurations with uniform linear arrays and uniform circular arrays with antenna spacing and the antenna structure. Changing antenna parameters on system capacity was explored on the base stations and Relays. The effect of Comparative studies were carried out where all the three systems were compared against each other keeping the upper and lower bounds, i.e. a MIMO and SISO systems. Moreover, Relay nodes location and source power allocations (The power allocation ratio between the direct link and the relay links) are optimized to obtain the best capacity under different cooperative setups and total transmission power constrain.

**Keywords:** MIMO, Relay,Correlated Channel, Amplify and Forward

## 1. INTRODUCTION

Utilizing MIMO in wireless relaying systems has attracted high interest recently. Relaying is a means of improving the performance of infrastructure- based networks by increasing their coverage. Most of previous work in MIMO Relay system used a simple Rayleigh channel that overestimates system performance compared with realistic models. Therefore, a more realistic channel model [1] and [2] are applied in this paper. In [3] and [4], various amplify and forward (AF) relaying schemes investigated with Rayleigh channel models achieve full diversity in the number of antennas; their relative performance merits are determined by noise amplification at the relays and the exact configuration of the distributed array. It was shown that scattering relays could improve MIMO system performance if these are constrained by correlated propagation. In [5], it was shown that double-directional geometry- based stochastic channel model based on ITU generic model has good accordance with the measurement data in both AF and decode and forward (DF) relay modes. A typical urban micro-cell environment is assumed with 2.35GHz and bandwidth up to 50MHz. RS was fixed on the top of a travel trailer, Base station (BS) was located on the rooftop of a 5-floor high. Antenna heights of MS, RS and BS are 1.8m, 7m and 22m respectively. Buildings on both sides of the routes are mostly 5 or 6 floors high. Line-of-sight propagation condition is the case in the backhaul link, and BS was clearly below the rooftops of the surrounding buildings. In [6], another study on the effect of various factors on the capacity of MIMO relay system. The simulation results show that system capacity is greatly expanded by adding relay nodes. However, spacing between antenna arrays and direction angles at the transmitting and receiving ends, as well as relay node location, Rice K-factor and other factors all can produce certain effect on the capacity of the MIMO relay system. Therefore the correlation analysis in this paper is of application significance. However, the fading correlation model used is not the exact model with multi clusters scattering. In [7], It is shown that the capacity of polarized MIMO relay system outperforms that of co-polarized MIMO system with constant signal to noise ratio (SNR). The channel model for arbitrary polarized antennas can be derived from the model for co- polarized antennas by element wise multiplication with a matrix containing the polarization mismatch loss between the

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transmit and receive antenna pairs as well as the effect of azimuthal direction of the terminal in the cell. Less attention has been paid to the effect of varying the channel model of source, relay and destination links on capacity and outage probability numerical results.

The Rayleigh channel model has been the most accepted channel model for the relay systems. This model however, holds in the case where the relay lies in the non-line-of-sight (NLOS) scenario of both the source and the destination. The channels between the source and relay, and the relay and destination are considered as the full rank multiple-input multiple-output (MIMO) channels. Maintaining a full rank for these channels is a goal by itself in MIMO communications, as the lower the rank leads to a lower capacity of the given system. In [8], the power and location optimization for decode-and-forward relay network were investigated, but the fading coefficient of the channel is still assumed to be distributed according to Gaussian distribution. However, to maximize the coverage, the relay is typically placed in the LOS scenario of the source. Therefore, the Rayleigh channel model is no longer suitable to analyze practical relay channels. In addition, since there is LOS component propagating from the source to relay, in the case of the MIMO communication, the channel between the relay and the source is correlated. Hence the capacity performance of the whole system degrades, and it is necessary to use techniques such as cross-polarized antenna or optimal power allocation to improve the performance in the channel. These previous efforts show that simulation of such systems and investigating its performance requires utilizing a more accurate channel model, which is a main objective of this paper. Therefore previous work is extended in further investigating to the location of the relay, LOS existence between the relay and the source or the destination and the scattering clusters and fading in the channel influence on the capacity and data rates at the receiver using a more accurate and realistic channel model. In this paper the realistic fading correlation model presented in [1] was included in the channel model. This model is an enhanced modified version of the SCM channel model with capability of including different array types and polarization antennas. The results are compared with the published work in [3-4] for validation. The paper is organized as follows. Generalized system model with two hops MIMO AF relay system will be described. Detailed mathematical models for different case studies and channel capacity equations are derived for different propagation scenarios are presented in section III. In section IV, the realistic channel model that is used in the simulations of this paper is presented. In section IV numerical results are obtained pointing out the influence of the performance of MIMO relay system by relay location and LOS existence and channel parameters. Finally, the paper is concluded in section V.

## 2. Two hops MIMO AF Relay system model Description

In this paper, two-hop MIMO AF Relay system is considered. In the first hop, the source transmits signals intended for the destination in a broadcast manner. All relays and the destination receive faded noisy versions of these signals. In the second hop, relays retransmit a processed version of their received signals to the destination, and the destination combines the signals received in the two phases. The processing in this MIMO AF Relay case is just a linear amplification added to the received signal at relay node (RN). In this model, the source transmits K symbols in a period T to the destination. For relay transmission that achieves the same end-to-end delay, the source must transmit its K symbols followed by the relay's transmission in the second period. As a consequence, the spectral efficiency of the individual links must at least be doubled compared to that of the direct system. A general cooperative scheme is depicted in **Fig. 1**. All terminals are equipped with multiple antennas. Realistic channels are considered with scatterers are included in all the signal paths.

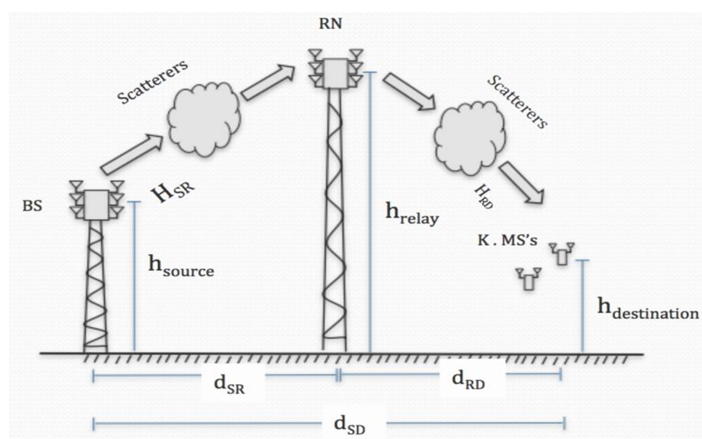



**Figure 1. A general Cooperative MIMO AF relay system model**

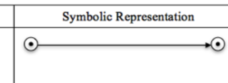
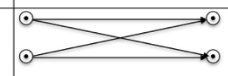
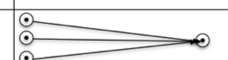
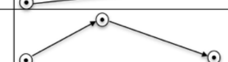
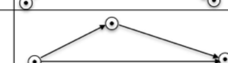
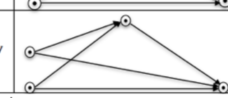
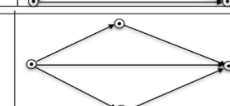
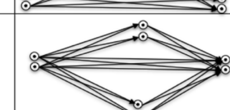
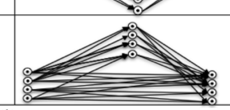
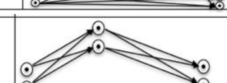
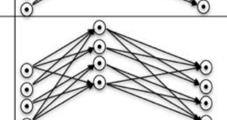
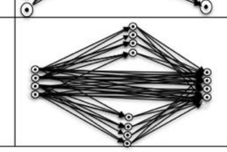
Following are notations, definitions and parameters values that are considered in this paper. (BS) is the Base Station equipped with the ($M_{Tx}$) transmitting antennas. (RN) is the relay node equipped with ($M_{Rel}$) relay antennas. (MS) is the Mobile Station acting as destination with ($M_{Rx}$) defined as the total number of receiving antennas for all the mobile stations. ($H_{SR}$) is the channel matrix between the source and the relay node. ($H_{RD}$) is the channel matrix between RN and MS. (Scatterers) are the surrounding obstacles in the path of the transmitted signal causing it to scatter, diffract and refract. ($d_{SR}$) is the direct distance between BS and the

RN in meters. ($d_{RD}$) is the average direct distance between RN and MS in meters. ($d_{SD}$) is the average direct distance between BS and MS in meters and it is assumed to be 1000 m. ($h_{SOURCE}$) is the height of the BS in meters, set at 12 m. ($h_{RELAY}$) is the height of the RN in meters, set at 32 m. ($h_{DESTINATION}$) is the average height of the MS in meters, set at 2 m. ($f_c$) is the system's center frequency in MHz, set at 2530 MHz. (K) is the number of users, i.e. MSs ($P_s$) is the transmitter power at the source in Watts, set at 10 W. ($V_o$) is the average velocity of the Mobile Stations, set at 1.2 $ms^{-1}$ . (spacing_s) is the source's inter antenna spacing relative to the system's frequency wavelength set at 4λ. (spacing_r) is the relay node's inter antenna λ spacing relative to the system's frequency wavelength λ set at 0.5λ. (spacing_d) is the mobile stations' inter antenna spacing relative to the system's frequency wavelength λ set at 0.5λ.

## 3. Mathematical modeling of MIMO AF Relay case studies

In this paper, different MIMO AF Relay system configurations will be considered and compared. A generalized mathematical analysis and capacity evaluations are presented in this section. Starting with a unified system description to be used throughput the different configurations. The system will be referred to as $p_n$. Four parameters will be used to describe the subject system. The number of relays $M$ will indicate how many distinct relay nodes are being implemented in the system. These relays will be equipped with $m_{rel}$ antennas. The Base station will be equipped with $m_{Tx}$ antennas and the Mobile stations total number of antennas will be referred to as $m_{Rx}$. This shown system description method is being elaborate in describing most of the system parameters. However it has few shortcomings. For instance, it assumes that all the relay nodes do have the same number of antennas, which is a fair assumption but should not always be true. Another example is that it assumes that the direct link between the BS and MS's is always there. Our first model violates this assumption. But aside from these two points, the system description used here is sufficient for our analysis. So the system description form used is $p = \{M, m_{Tx}, m_{rel}, m_{Rx}\}$. Based on the Antenna number and Relay configurations, ten different variations of the MIMO Relay AF systems were studied. As a benchmark; three none Relay systems were used to compare the system performance against. Hence, the full set of systems comprises of thirteen systems. The systems were named $P_1$ through to $P_{13}$. **Table 1** lists all the case studies investigated in this paper with a brief description. The systems will be represented symbolically later. The total number of antennas used in any given system was the criteria for paring them for comparison. This and the number of relays as well were the two basic selection conditions for comparing any two systems. The first three systems were used as a benchmark to provide a base line for all the comparisons.

**TABEL 1** List of case studies for MIMO AF relay system configurations

| System | $\{M, m_{Tx}, m_{rel}, m_{Rx}\}$ | Remark | Symbolic Representation |
|---|---|---|---|
| *P1* | {0,1,0,1} | SISO |  |
| *P2* | {0,2,0,2} | MIMO |  |
| *P3* | {0,3,0,1} | MISO |  |
| *P4* | {1,1,1,1} | No Direct Link between Source and Destination |  |
| *P5* | {1,1,1,1} | SRD and SD links |  |
| *P6* | {1,2,1,1} | Two transmitting antennas and one Relay |  |
| *P7* | {2,1,1,1} | Two relays |  |
| *P8* | {1,2,2,2} | Based on P5 with More Antennas |  |
| *P9* | {2,2,2,2} | Based on P7 with More Antennas |  |
| *P10* | {1,4,4,4} | Based on P5 with More Antennas |  |
| *P11* | {1,2,2,2} | No Direct Link between Source and Destination |  |
| *P12* | {1,4,4,4} | No Direct Link between Source and Destination |  |
| *P13* | {2,4,4,4} | Based on P7 with More Antennas |  |

The Bench Mark systems $P_1$, $P_2$ and $P_3$ were selected to present the upper capacity bounds of the studied systems with a similar number of antennas. The first MIMO Relay AF system model, $P_4 = \{1,1,1,1\}$, is a two hop MIMO Relay AF system without a direct link between the BS and the MS. Adding the direct link leads to the second model, $P_5 = \{1,1,1,1\}$. The no SD subscript for p1 indicates the lack of the direct link between the source and the destination. Adding a second relay node leads to the third studied model, $P_7 = \{2,1,1,1\}$. Another variation to the basic MIMO Relay AF systems is to add more antennas at the transmitter. This resulted in $P_6 = \{1,2,1,1\}$. All the subsequent MIMO Relay AF systems are based on the three basic systems, namely $P_4, P_5$ *and* $P_7$ illustrated in Fig.2. More antennas were simply added to these systems to account for correlation and different antenna structure analysis. The rest of the models should follow the same steps. The final capacity formulas will be presented afterwards. All the MIMO AF Relay systems consist of the Source Node, (S), the Relay node, (R), and the Destination Node, (D). the nodes are communicating over their respected channels. $H_{SD}$ is the channel between the source and the destination, $H_{SR}$ is the channel between the source and the Relay and $H_{RD}$ stands for the channel between the Relay and the destination. The relay introduces a relay gain to the system; G this gain is the ratio of the power transmitted by the relay to that received at the relay. The model parameters are indicated in **Fig. 2** for the three root models.

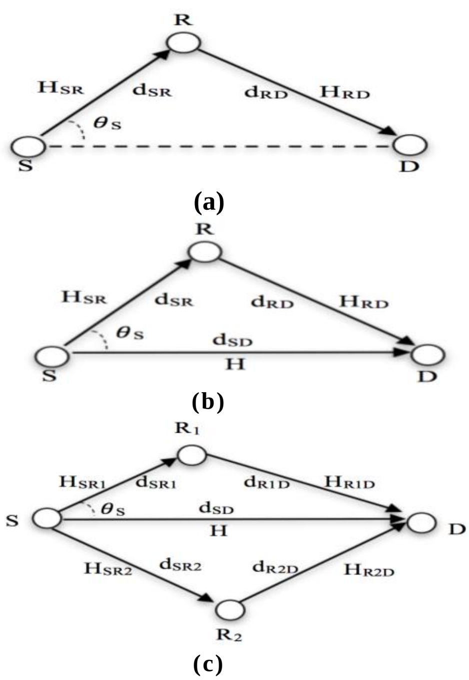



**Figure 2. (a) P4 model (b) P5 model (c) P7 model**

*A. Capacity Analysis of MIMO Relay AF systems*

Before we start analyzing individual systems let us agree on the common ground between all the systems and on a common capacity calculation method for them. As shown previously, the MIMO Relay AF network can be described using the form below.

$$Y = Ax + Bn \tag{1}$$

Where "A" and "B" are system Matrices, x stands for the transmitted data vector and n stands for the noise. The strength of the formula in **equation 1** is realized when using this form to express the system capacity as shown in **equation 2** and the task of finding out any system capacity reduces to finding out the system matrices A and B and the correlation matrices $R_x$ and $R_n$.

$$C = log_2 det(I + (AR_X A^H)(BR_n B^H)^{-1}) \tag{2}$$

Where; A is a system matrix describing the attenuation and changes affecting the transmitted data. Rx is the auto correlation of the transmitted data signal. B is the second system matrix acting on the noise. Rn is the noise power at the receiver and $(.)^H$ is the conjugate transpose operator. **Equation 2** is the standard formula in all literature reviewed being used to calculate the capacity of any given MIMO Relay AF system. In comparing different systems, the task reduces to putting the system in a form similar to equation 1. Using **equation 2** can use this result to estimate the system capacity. The next part works on finding out the system matrixes for each of the three models used to compare their performance.

*B. Case 1: One Relay with no direct link between source and destination (P4)*

So let us derive the A and B system Matrices for the MIMO Relay AF system displayed in **Fig. 2(a).** The analysis will be based on the receiver's prospective of the transmitted data. That is, standing at the receiver's end, how does the transmitted data get through the RN and channel to the transmitter. As the model in hand is the two hop MIMO Relay AF model, then two points need to be clarified here. The first point is that each of the MS's will receive one stream of data coming from the RN only. And the second point is that the data is received over two time slots as it hops from the BS to the RN in the first time slot and from the RN to the MS's in the second time slot. The received data at the RN can be modeled as **equation 3**,

$$y' = H_{SR}x + n_1 \tag{3}$$

Where $y'$ is the noise at the RN, the gain introduced by the Amplify and Forward relay node has a positive real value named G. This amplification factor acts on the received signal at the RN and gets carried to the destination MS's as shown below.

$$y = H_{RD}Gy' + n_2 \tag{4}$$

Rearranging;

$$y = H_{RD} G (H_{SR} x + n_1) + n_2 \quad (5)$$

Expanding;

$$y = H_{RD} G H_{SR} x + H_{RD} G n_1 + n_2 \quad (6)$$

So at the receiver, i.e. MS's, there is one stream of data being received which is described in **equation 6**. Reshaping it into the standard form as in **equation** 1, 6 becomes as shown next.

$$y = [H_{RD} G H_{SR}] x + [H_{RD} G \quad I] \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (7)$$

So the system matrices A and B are easily found to be as:

$$A = [H_{RD} G H_{SR}] \quad (8)$$

$$B = [H_{RD} G \quad I] \quad (9)$$

And their transpose pairs are:

$$AA^H = [H_{RD} G H_{SR}][H_{SR}^H G^H H_{RD}^H] = H_{RD} G H_{SR} H_{SR}^H G^H H_{RD}^H \quad (10)$$

$$BB^H = [H_{RD} G \quad I] \begin{bmatrix} G^H H_{RD}^H \\ I \end{bmatrix} = (I + H_{RD} G G^H H_{RD}^H) \quad (11)$$

$$R_x = E\{xx^H\} = (P_0 / m_{T_x}) . I_{m_{T_x}} = P_0 / (2 m_{T_x}) \quad (12)$$

$$R_n = E\{nn^H\} = \sigma_n^2 \quad (13)$$

So if we lump up **equations 10, 11, 12 and 13** into **2 equations 14** will emerge as the capacity equation of the two-hop MIMO Relay AF system.

$$\log_2 \det \left\{ \begin{matrix} \mathrm{I} + \frac{\mathrm{P_o}}{2\mathrm{N_S}\sigma_n^2} \mathrm{H_{RD} G H_{SR} H_{SR}^H G^H H_{RD}^H} \\ \times \left(\mathrm{I} + \mathrm{H_{RD} G G^H H_{RD}^H}\right)^{-1} \end{matrix} \right\} \quad (14)$$

The Gain Matrix: As this is a non-regenerative relay channel, the total transmitted power in two time slots is restricted to P0. Thus the amplification factor is selected to satisfy this constrain as below:

$$\mathrm{g} = \sqrt{\frac{\mathrm{P_0}}{\frac{\mathrm{P_0}}{2\mathrm{N_S}} \|\mathrm{H_{SR}}\|_F^2 + 2\mathrm{N_R}\sigma_n^2}} \quad (15)$$

where the Gain matrix is a diagonal matrix with the diagonal elements being g and $\| . \|_F$ is the Frobenius norm or the Hilbert–Schmidt norm. The latter term is often reserved for operators on Hilbert space. This norm can be defined in various ways:

$$\|\mathrm{A}\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2} = \sqrt{trace(A^* A)} \quad (16)$$

Where A* denotes the conjugate transpose of A, σi are the singular values of A, and the trace function is used. Then G matrix is defined as:

$$[\mathrm{G}] = \begin{bmatrix} \mathrm{g} & 0 & 0 & \\ 0 & \mathrm{g} & 0 & \cdots \\ 0 & 0 & \mathrm{g} & \\ & \vdots & & \ddots \end{bmatrix}_{\mathrm{m_{T_x}} \times \mathrm{m_{R_x}}} \quad (17)$$

*C. Case 2: One Relay with direct link between source and destination (P5)*

As shown in **Fig. 2 (b),** introducing the direct link between the source, and the destination, leads to the realization of the second simulated system. The new system communicates over two time slots in the initial transmission but then the Mobile Stations will receive data from the source directly and the relay nodes. Hence the initial transmission delay's effect on the system's spectral efficiency is negligible. The system matrices are driven in a similar manner as done with the first system.
In the first transmission phase; phase I the destination receives the data stream coming from the source:

$$y_1 = H_{SD} x + n_{SD} \quad (18)$$

At the relay, the data received can be modeled as below:

$$r_1 = H_{SR} x + n_{SR} \quad (19)$$

The data stream received at the destination coming from the relay node can be represented as below:

$$y_2 = H_{RD} G r_1 + n_{RD} \quad (20)$$

$$y_2 = H_{RD} G \{H_{SR} x + n_{SR}\} + n_{RD} \quad (21)$$

$$y_2 = H_{RD} G H_{SR} x + H_{RD} G n_{SR} + n_{RD} \quad (22)$$

$$Y = \begin{bmatrix} H_{SD} \\ H_{RD} G H_{SR} \end{bmatrix} x + \begin{bmatrix} I & 0 & 0 \\ 0 & H_{RD} G & I \end{bmatrix} \begin{bmatrix} n_{SD} \\ n_{SR} \\ n_{RD} \end{bmatrix} \quad (23)$$

$$A = \begin{bmatrix} H_{SD} \\ H_{RD} G H_{SR} \end{bmatrix}, A \in \mathbb{C}^{2m_{R_x} \times M m_{rel}} \quad (24)$$

$$B = \begin{bmatrix} I & 0 & 0 \\ 0 & H_{RD} G & I \end{bmatrix}, B \in \mathbb{C}^{2m_{R_x} \times (2m_{R_x} + M m_{rel})} \quad (25)$$

$$P_l = \frac{P}{m_{T_x}} \sum_{i=1}^{m_{rel}} \sum_{k=1}^{m_{T_x}} \left| \left[ H_l^{(1)} \right]_{i,k} \right|^2 + m_{rel} N \quad (26)$$

$$g_l = \sqrt{\frac{P/_M}{P_l}} \tag{27}$$

*D. Case 3: Two Relay with direct link between source and destination (P7)*

Adding another Relay Node to P5 brings us to P7 As shown in **Fig. 2 (c).** The addition of more relay nodes should, intuitively, increase the overall system capacity as the correlation between the MIMO antennas decrease since they are further apart. However, this advantage is being faced with the orthogonal codes hard limits. The problem of finding orthogonal codes for more than two relay nodes is difficult if not impossible to find. The current solution for the two relay nodes is using the distributed Alamouti system. Focusing on just one symbol period, we note that we can again divide transmission in two phases. In the second phase, one of the two relays retransmits a conjugated and negated version of the. Hence our analysis will not include adding more relay nodes, as it is not feasible right now. But looking at the bright side, this hard limit opens the way for future work in looking in to more orthogonal codes.

$$Y = \begin{bmatrix} H_{SD} \\ H_{R1D}G_1H_{SR1} \\ H_{R2D}G_2H_{SR2} \end{bmatrix} x + \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & H_{R1D}G_1 & 0 & I \\ 0 & 0 & H_{R2D}G_2 & I \end{bmatrix} \begin{bmatrix} n_{SD} \\ n_{SR1} \\ n_{SR2} \\ n_{RD} \end{bmatrix} \tag{28}$$

This is under the assumption that.

$$n_{R1D} = n_{R2D} \tag{29}$$

$$g_l = \sqrt{\frac{P/2}{\left|h_r^{(1)}\right|^2 P+N}} \qquad \text{for} \quad r \in \{1,2\} \tag{30}$$

Applying a similar analysis to P6 will lead to its capacity equation matrices listed in the below table. For the other systems having the same configurations, the same capacity equations apply. The more antennas add to larger H matrix dimensions. The grouping of MIMO Relay AF systems is explained in the **Table 2 and 3.**

**TABEL 2. Summary of the capacity equations of systems P4, P5, P6 & P7**

| $p_i$ | A Matrix | B Matrix | Gain Factor |
|---|---|---|---|
| $p_4$ | $[H_{RD}GH_{SR}]$ | $[H_{RD}G \quad I]$ | $\sqrt{\frac{P_0}{\frac{P_0}{2N_s}\lVert H_{SR}\rVert_F^2 + 2N_R\sigma_n^2}}$ |
| $p_5$ | $\begin{bmatrix} H_{SD} \\ H_{RD}GH_{SR} \end{bmatrix}$ | $\begin{bmatrix} I & 0 & 0 \\ 0 & H_{RD}G & I \end{bmatrix}$ | $\sqrt{\frac{P/_M}{P_l}}$ |
| $p_6$ | $\begin{bmatrix} H_{S1D} & H_{S2D} \\ H_{RD}GH_{S1R} & H_{RD}G_2H_{S2R} \end{bmatrix}$ | $\begin{bmatrix} I & 0 & 0 \\ 0 & H_{RD}G & I \end{bmatrix}$ | $\sqrt{\frac{P/_M}{\frac{P}{2}\left(\lvert h_{S1R}\rvert^2 + \lvert h_{S2R}\rvert^2\right) + N}}$ |
| $p_7$ | $\begin{bmatrix} H_{SD} \\ H_{R1D}G_1H_{SR1} \\ H_{R2D}G_2H_{SR2} \end{bmatrix}$ | $\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & H_{R1D}G_1 & 0 & I \\ 0 & 0 & H_{R2D}G_2 & I \end{bmatrix}$ | $\sqrt{\frac{P/2}{\left\lvert h_r^{(1)}\right\rvert^2 P + N}} forr \in \{1,2\}$ |

**TABEL 3. Common features of different *case studies***

| Group | Systems | Common Factor | Difference |
|---|---|---|---|
| Bench Mark | P1, P2 & P3 | None Relay systems | Link Structure |
| Group 1 | P4, P11 & P12 | No SD link | 1, 2 & 4 antennas per node |
| Group 2 | P5, P8 & P10 | Full MIMO Relay AF | 1, 2 & 4 antennas per node |
| Group 3 | P6 | Two antennas at S | One model only |
| Group 4 | P7, P9 & P13 | Two Relay Nodes | 1, 2 & 4 antennas per node |

## 4. Correlated Channel Modeling Approach

Three channel models were implemented in the simulation studies in this paper. The models differ by the amount of complexity included in each model to realize the real world model as close as possible. The purpose of having three levels of channel complexity is to evaluate the effect of different parameters variations on the system's capacity. The first channel model used is the simple channel model. The second channel model used is the Correlation Channel Model. The third level of complexity is achieved via the *outdoor Stochastic Spatial Clustered* (OSSC) channel model [9]. Using this channel model in this simulation is one of the paper contributions.

The following parametrs are considered in the channel model. The element spacing for source is 4λ, and the spacing for both the relay and the source is assumed to be 0.5λ. Intuitively, the relay should lie within the right semi circle with the source at the center, which radius is determined by $d_{SD}$, i.e., $0 < d_{SR} < d_{SD}$, and $-\pi/2 < \theta_S < \pi/2$. $\theta_S$ is the angle between source-relay link and source-destination. The Ricean K factor is assumed as K = 13−0.03d (dB) as in [2]. Furthermore, we assume that the polarizations of all the antennas at the source, relay and destination are perfectly matched, i.e., $\theta p = 0$. The capacity of the relay channel with 2 pairs of cross-polarized patch. The focus was on redefining the channel model to include the line of site as well as the none line of site components. The Rayleigh channel model is widely used in modeling the relay systems as done in [3]. If the Relay Node lies in the LOS between the BS and the MS's then this model wouldn't hold. Thus, the channel model adapted in [1]; combines the LOS and NLOS components in the channel model. **Table 4** lists the used channel models and their differences.

**TABEL 4 Comparison between different channel models**

| Channel Model | Simulated Parameters |
|---|---|
| Simple Channel Model | - The Path loss via Hata Model<br>- The channel matrix elements, random variables |
| Correlation Channel Model | - The Path loss via Hata Model<br>- The channel matrix elements, random variables<br>- The Ricean K factors<br>- LOS and NLOS components<br>- Antenna correlation, Fixed Value |
| OSSC Channel Model | - The Path loss via Hata Model<br>- The channel matrix elements, random variables<br>- The Ricean K factors<br>- LOS and NLOS components<br>- Antenna correlation, Rice Steering Matrix<br>- Antenna Configuration, ULA and UCA<br>- MS movement, speed assumed 1.2 KM/h<br>- Math intensive model when carrying out the simulations. |

Having introduced the above MIMO AF Relay systems and the channel models used in the simulation, next the simulation approach will be explained. The capacity investigations aim at studying the advantages of the MIMO Relay AF systems over the MIMO systems. For this purpose three none relay systems where selected. A SISO, a MIMO and a MISO representing the upper and lower bounds of the investigated systems were used as benchmark systems. The SISO system represents the simplest transmission system having one antenna at the receiver, and one antenna at the transmitter. The MIMO system has two antennas at the receiver and two antennas at the transmitter. The MISO system has three antennas at the receiver and one at the transmitter. To test the upper and lower bounds of the benchmark systems, some of the MIMO Relay AF systems were built. To test for correlation and Antenna structure effect on the system capacity, more systems were introduced. **Table 4** summarizes the different purposes of the MIMO Relay AF systems used.

## 5. NUMERICAL RESULTS

The capacity investigation aims at studying the effect of changing some parameters of the studied systems on the overall system capacity. The standard parameters for any communications systems were included. This included the signal to noise ratio affect on the capacity and probability of outage. With the introduction of the OSSC channel model, more parameters were accessible for investigation. The Angle of Arrival, antenna structure and antenna array spacing were included in the capacity investigations. Having a relay system added more dimension to the communications systems. The Relay Node location and the power allocation between the source to destination and source to relay links were new factors to be considered for the capacity investigations. The capacity investigations are listed in the next paragraphs with the results and conclusions found based on the simulation runs.

**TABEL 5 Capacity Investigations for different MIMO Relay AF systems**

| System | Simulation Role |
|---|---|
| P1, P2 & P3 | Bench Mark Systems<br>Upper and lower bounds for similar MIMO Relay AF systems antenna count |
| P4, P5, P6 & P7 | MIMO Relay AF system capacity vs none Relay systems |
| P6, P8, P9, P10, P11, P12& P13 | The effect of Correlation on the Capacity<br>The effect of adding more antennas on the Capacity<br>AoA variation on the Capacity |
| P10 | Changing the antenna structure ULA vs UCA |

We start first with comparing our work with the published work of [3] to validate our numerical results. Our results are shown in **Fig. 3.** The aforementioned simulation assumptions and parameters were used to generate the curves in figure 3. The order of the curves is maintained, although our channel realizations were limited to 500k realizations only. The center frequency of the simulation was set at 2.53 GHz. The Channel model used here is the simple channel model. The shown results validate our model. All the results after figure 2 were based on the same channel model but with added complexity to investigate different parameters effects on the system's capacity. Design elements and system parameters were studied to verify their effect on the system capacity. The aim of this study is to define the optimal conditions to operate the MIMO Relay AF systems. The comparisons are carried among different MIMO Relay systems.

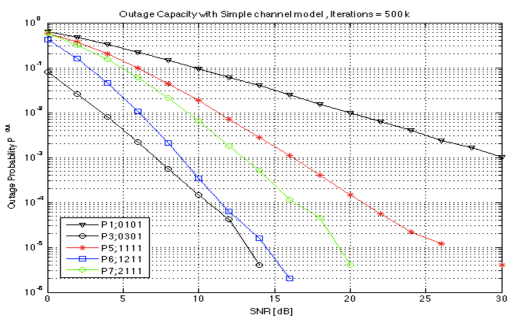



**Figure 3. Outage probability versus SNR for P1,P3,P5,P6 & P7**

*A. Outage probability comparisons Vs. SNR*

**Fig. 4** displays the outage probability curves of systems P1, P2, P3, P6 and P7. The single relay system, P6, outperforms the dual relay system P7 when comparing their resilience. Adding more antennas to the systems does not change the order of performance as shown in **Fig 5**. Adding another antenna at each node converts P6 into P8, the same can be said about P7 and P9. P8 outperforms P9 as clearly shown in **Fig 5**. This favors a single relay system over a two-relay system assuming similar channel conditions for both systems.

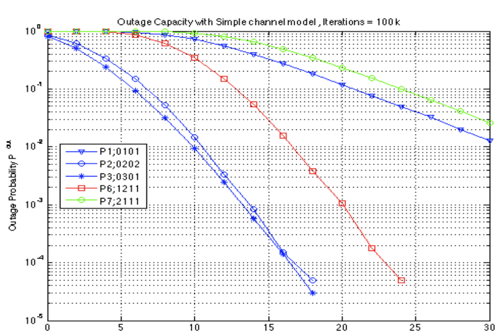



**Figure 4. Outage probability versus SNR**

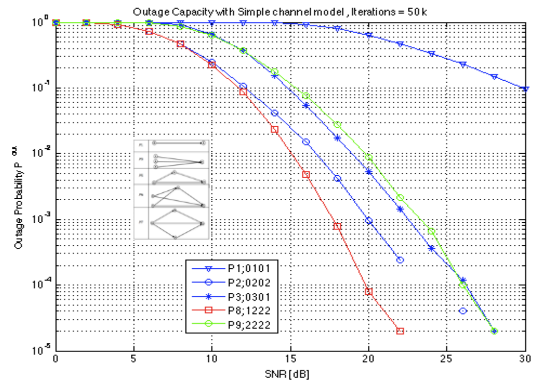



**Figure 5. Outage probability versus SNR**

*B. System capacity Vs. SNR*

Being a two-hope system, P4 will have the lowest capacity among the other systems as shown in **Fig 6**. This is due to the fact that transmission happens over two time slots. The capacity curves of P5 and P1 coincide. But P5 shows a more resilience response in the outage curve as indicated earlier. The capacity curves of P6 and P7 hold the same order as shown in **Fig. 7**. **Fig. 8** indicates the advantage of a single relay over two relays, as P8's capacity is larger than P9's.

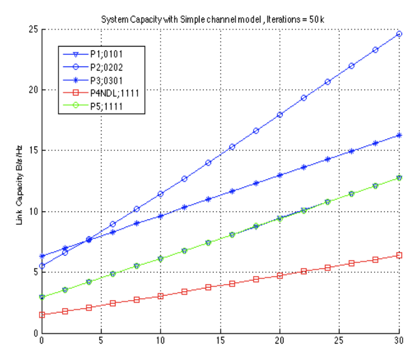



**Figure 6. Capacity versus SNR for P1, P2, P3 P4 & P5 systems**

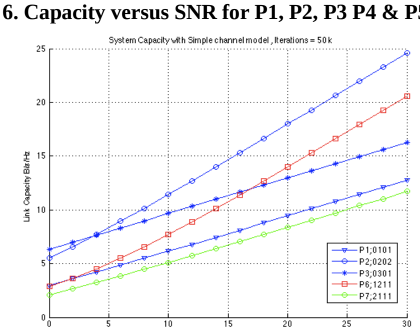



**Figure 7. Capacity versus SNR for P1, P2, P3 P6 & P7 systems**

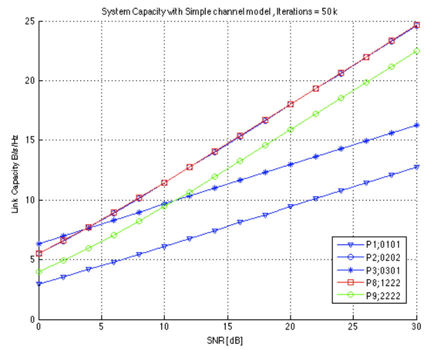



**Figure 8. Capacity versus SNR for P1, P2, P3 P8 & P9 systems**

### *C. Changing antenna parameters effect on system capacity*

Studying the effect of antenna structure and parameters on the link capacity is the focus of this part of the simulations. The antenna correlation effect on the link capacity is evident in **Fig 9**. The curves show how the capacity degrades as the correlation between the antennas increase. This simulation was done via the second channel model, i.e. the correlated channel model shown in **table 2.** The degradation is similar to a normal MIMO system. The curves shown are limited to the direct link between the source and destination.

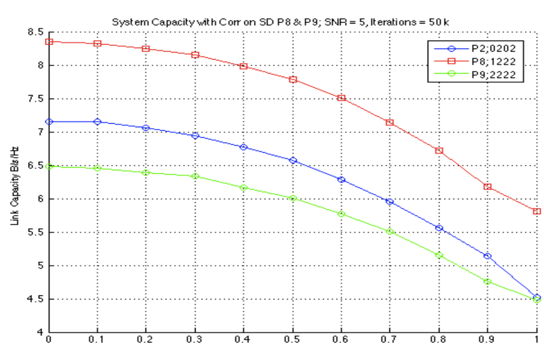



**Figure 9. Capacity versus antenna correlation**

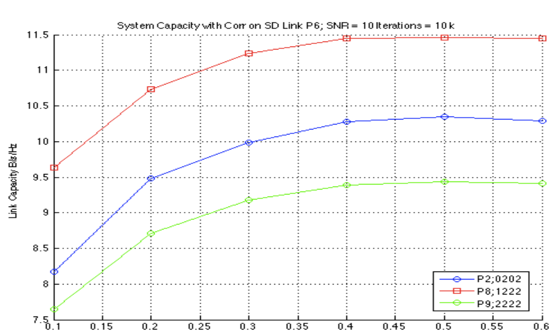



**Figure 10. Capacity versus antenna spacing**

From the curves shown, it is clear that increasing the antenna spacing more than 0.4λ will not affect the capacity positively. Increasing the number of antennas increases the threshold for antenna spacing were 1.5λ is the new threshold as the antenna per node doubled. System P8 and P10 share the same structure but P8 has two antenna per node, whereas P10 has four antennas per node. Uniform Circular Array was simulated in **Fig 11**. The curve shown there indicates that 1.5λ is a good threshold though capacity keeps increasing slowly after that point.

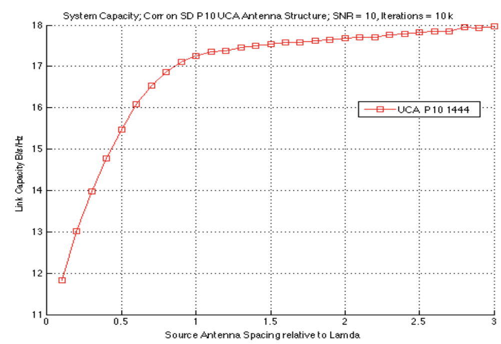



**Figure 11. Capacity versus UCA antenna spacing**

### *D. RN location effect on system capacity*

The location of the Relay Node plays a major role in controlling the capacity of a MIMO Relay AF system. Basically, adding a relay node at the edge of the cell can increase the cell coverage. In this part of the simulations study, the optimum RN location was examined for P12 and P8. As shown in figures 12 and 13 respectively, the RN is best located halfway between the source and destination.

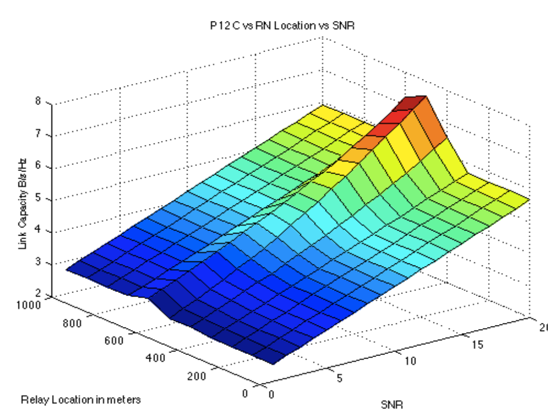



**Figure 12. Capacity versus source antenna spacing for P12 model**

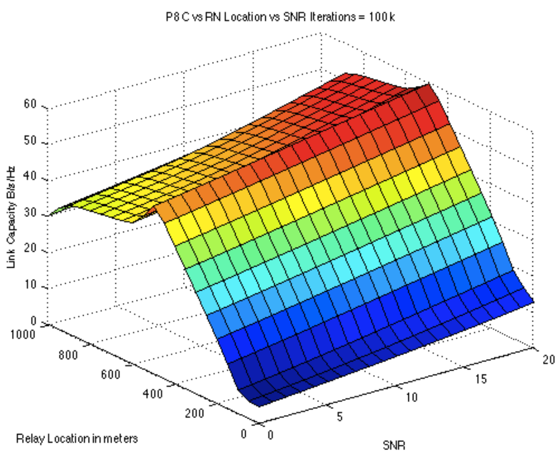



**Figure 13. Capacity versus source antenna spacing for P8 model**

## 6. CONCLUSION

Capacity investigations of different MIMO Relay AF systems modeled under correlated channel conditions have been presented in this paper. It is concluded that the MIMO AF Relay system does not have a capacity advantage over the MIMO systems. However, they are more resilience and outperform the MIMO systems in outage probability. It has been shown that the MIMO AF Relay systems are more immune to system parameters changes on their source to relay and relay to destination links. Their most vulnerable link is the source to destination. Another paper conclusion is that using the source to destination link enhances the system coverage over the two-hop AF system. Also, the antenna geometry parameters and its effects on the system performance have been studied and it is shown that spacing affects the system capacity significantly when the spacing is less than a certain threshold. After this threshold, the antenna spacing does not affect the system capacity. For a ULA antenna structure with two antennas the threshold antenna spacing is 0.4λ. A four antennas array has a higher threshold, 1.5λ. UCA antenna array has a different threshold for the same antennas count. Regarding, the relay node location optimization it is presented that the relay location significantly affects the system capacity for MIMO Relay AF systems. In general, as shown in the numerical results, the antenna should be place halfway through closer to the destination to maintain high system capacity.

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