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Minimals Transversals in hypergraphic Wireless Sensor Network

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Abstract: *Such as background,* **Wireless sensors networks have become a very widespread domain in the research field along with their representation. The latter has become an important matter to reduce the energy's consumption as well as to expand the longevity. Thereby, the appearance of a new concept of "Hyper-graphs" in the last decades has paved the path for studies in all topical areas. That is why we are interested in this new axis.**
The objective **is precisely to determine the connectivity nodes in a network (the nodes play a very important role and ensure the continuity of the information and thus the longevity of the network). Mainly that the major present studies, are the surveillance of wireless sensors networks after deployment.**
The principle of our method, **is that the network of wireless sensors is presented in the form of a "Voronoi diagram" in order to reduce the complexity of the network through an optimal placement of the nodes.**
These nodes will be transformed to a hyper-graph through the generation of vertices and hyper-edges where each hyper-edge represents a collection metric. Basing on the properties of hyper-graphs.

The second point of our method is that, **we shed light on the minimal transversals in order to manage the vertex-cover and on the extraction of connectivity nodes in order to maintain the communication within the wireless sensor network (WSN).**
In addition, as the WSN could be a dense network, we will disclose the notion of hypergraphic decomposition in order to obtain the best results for the extraction problems that are caused by this kind of graphs (which are known for being difficult and NP-complete problem).
As results, **We have studied this contribution with some examples and we have proved in this article with an example the importance of the nodes extracted in our network where we have transformed a sensor network to voronoid diagram,then to hypergraph network, decomposition and finaly, the extraction of transversals nodes.**
Therefore, as conclusion **the determination of this nodes have a considerable place in the network, there play the role of "the connectivity nodes" and its allowed to lengthen network longivity and to guarantee expend the communication.**

Keywords: Network of wireless sensors, surveillance, Voronoi, Connectivity, Hyper-graphs, Minimal Transversals, Decomposition

1. INTRODUCTION

The need to control physical phenomena, such as: luminosity, pressure, and temperature, is crucial for several industrial, scientific, and public purposes. This type of control is managed by minidevices called "sensors" which serve to acquire data about the observed phenomena and, if possible, to execute treatments.

Networks of wireless sensors are nowadays used in several applications that differ in the aim as well as in individual constraints: military surveillance, industrial monitoring, medical telemetry, and environmental monitoring.

However, the common denominator of all applications of wireless sensor networks remains the vulnerability of microsensors due to their limited material resources, the energy being the most restrictive one. The consumption of energy is a crucial matter, especially during the deployment in large, unattainable or even dangerous areas. This inconvenient alters the lifespan of the network in case one or more than one node is depleted.

Similarly, [1], among the important parameters of the WSN's performance is the node connectivity. After the deployment, the nodes should compose a connected network to ensure the continuity of information transfer and exchange.

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Mostly, the connectivity depends on the establishment of a path to guarantee the forwarding of data. Contrariwise, if one of the nodes is flawed by breakdown, movability or even by a change in its topology, this leads to the loss of communication links. This loss induces a partitioned network.

The connectivity is crucial in WSN. Most of the articles focus on the connectivity as a performance measure. However, in this study, we focus on the detection or selection of these nodes that are called "connectivity nodes".

Recently, we could note an infatuation to represent the objects in graphs because graphs offer multiuse alternatives for the characteristic vectors in different domains. To illustrate our network, our site is represented in the form of Voronoi Diagram for different reasons:

- Reducing the complexity (mainly),
- Significantly exploiting the space,
- A sensor attributes the nearest points to it rather than to other sensors.
- The most known property of metrics is their connectivity.

Nonetheless, in particular cases of complex systems, simple graphs do not provide a complete description of these systems [2]. Most studies focus on the surveillance based on generalized graph theory. Recently, the representations of the new concept of hypergraphs become more popular, knowing that they existed a long time ago, but they were rarely used to represent Wsns.

We have chosen to use the hypergraphic representation for several reasons, including the possibility to investigate several phenomena in the same network and thus to investigate the variety of these properties that we can utilize in different fields of study such as the present one about the use of the minimal transversals properties [3].

Hypergraphs have the concept of non-graph oriented, the particularity of the hypergraphical representation is characterized by the identification of hyper-edges, where the hyper-edge no longer connects one or two vertices, but any number of vertices [4] (from one to the number of vertices of the hypergraph).

For this reason, some researchers have recently expressed their interest in the application of the hypergraph model in wireless sensor networks, where: In [5], the authors proposed the use of hypergraphical formalism to model the interconnection between different component networks (CN). Due to its structure, an online WSN supervision strategy for surfing on the graph can be set. Nevertheless, from the bottom up, it is possible to detect faults in WSN, when some missions are not fulfilled. This can be done by relying on the concept of degree of filling. The suggested algorithm provides a list of faults that can be detected and isolated.

In [6], a model based on hypergraph structure is suggested to group a set of graphs. First, a clustering technique based on the selection of prototypes is proposed to group the graphs into k independent clusters (k is automatically defined using a type of threshold). Secondary, these clusters are organized so they overlap partially to define the final hypergraph structure.

And in [7], The authors suggested a theory based on hypergraphs to solve the problems caused by graphs such as high computational complexity and the large solution space when managing WSN at large scale. They proposed an algorithm covering hyper-edges to calculate the paths with the minimum transmission power distribution defined for WSN networks and demonstrate the effectiveness of the method by maintaining the quality of transmission and exhibiting high fault tolerance.

Other authors were attracted by another concept, as they suggested an algorithm for grouping and mapping networks of sensors in hypergraphs [8], to apply a hierarchical iterative clustering algorithm to the hypergraph, thus dividing the hypergraph into several parts. And a cluster head is selected in each part. To improve the partitioning process, a new modularity function has been proposed, the simulation shows that the algorithm works better in terms of energy consumption and network life.

We have noticed that hypergraphic representation is becoming center of interest in the research field. That is why our contribution is to realize a mappage of Voronoi hypergraphs where the vertices constitute the graph of our network and the hyper-edges represent the physical parameter that is collected about the environment to utilize the hypergraph property:"minimal transversals" as well as to apply the extraction of sensors to identify our graphs of connectivity.

Calculating or recognizing the minimal transversals of a hypergraph is a frequent problem in the practical field [9]. One usually faces this problem in the important application of theoretical data, boolean switching theory, logic, and artificial intelligence, and particularly in the model-based diagnosis.

The minimal transverses correspond to a set of vertices which intersects all hyper-edges of a hypergraph, and is minimal in the way of insertions [10]. Ties will be seen as valuable nodes that maintain network coverage and connectivity, one of the important properties in WSN.

Our article shows the interest of extracting transversals in a network and more precisely the impact of this extraction in wireless sensor networks, where we have demonstrated the main role played by these extracted nodes which represent essential points of identification to maintain connectivity and exchange of messages in the networks. The loss of one of these nodes can lead to the loss of communication and the loss of information and thus the breakdown of networks.

Due to the density of the sensor networks and the difficulties of the transversals extraction method, we thought of the experiment with a solution of decomposing our hypergraph network in subhypergraph (This decomposition ensures the saving of memory, complexity and the speed of response) in subhypergraph.

Therefore, Our article is organized as follows : first, an overview of Voronoi representation of wireless sensor networks, then we will proceed by explaining the hypergraph and properties, minimals transversals and decomposition. Finally, we will end with some examples to explain our research principle.

2. Background: Preliminaries and State of Art

Sensors have become inevitable elements in all systems where exterior information are necessary for both evaluating and proceeding. Therefore, acquiring a complete knowledge about this field requires the deployment of several sensors and, if possible, the study of all information that are resent to optimize the act of the undertaking process [6].

A sensor's graph is mainly composed of four basic units; a probe or sensor unit, the treatment unit, the transmission unit, and energy control unit. Depending on the application domain, it can also contain additional modules such as the localization system (GPS) or an energy production system (solar or photovoltaic cells).

A wireless sensor network consists of a set of graphs that are deployed in a given environment, regarding a particular application (temperature's statement, precision's statement, control, surveillance, intruder detection, air humidity, agriculture, home automation, medical domain etc). The sensors intercommunicate in order to pass information to a "SINK' graph that communicates with the user's interface. A wireless sensor network WSN is thus composed of several autonomous microsensors that are able to collect and collaborate to transfer information about the environment.

There are several researchers who are interested in studying as well as in proposing a representation of WSNs that usually consist of simplified models of sensors and networks, according to the cover, deployment, distance etc. We can cite: the graphical representation, Delaunay triangulation, VORONOI diagram. The latter is our model of interest..

A. Diagram of Voronoi

Due to the usually limited resources and for covering the zone with the minimum of sensors, researchers are interested in the VORONOI diagram in various applications.

Some methods propose to deploy the sensors' graphs using tools of algorithmic geometry such as the "Voronoi diagram" to maximize the coverage of the surveillance zone. Each sensor's graph that we desire to relocate must form a Voronoi polygon regarding to the location of the other near graphs.

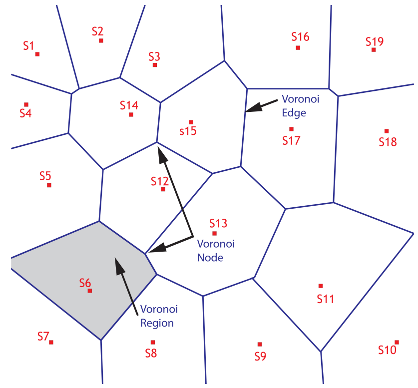

Figure 1. Voronoi diagram

More precisely, the Voronoi diagram in two dimensions is a set of discrete points (called sites). Thus, the plan is divided into a set of convex polygons, in a way that all points inside the polygon are more close to a unique point as shown in the following figure ***(Figure 1)***:

Definition: *In a plane R^2, **S** represents an finished set of points; Voronoi cell or Voronoi region associated with a point **p** in **S** is the set of points that are neares to **p** than any other point in **S***

$$VORs(p) = x\epsilon R^2/\forall q\epsilon S\,||x-p|| <= ||x-q||$$

We can generalize the notion to a Euclidean space **E** that has an Euclidean distance **d**, the notion becomes:

$$VORs(p) = x\epsilon E/\forall q\epsilon S\,d(x-p) <= d(x-q)$$

Therefore, it is necessary to establish an accurate and effective algorithm for each sensor to autonomously calculate its own Voronoi cell according to the graphs' location in its detection range or according to limited communication so that the complete Voronoi partition of the WSN can be generated in a distributed mode. Nevertheless, there are several articles that propose various methods about cover optimisation by the Voronoi diagram, such as: [11], [12] and [13].....

According to the researches in this context, the Voronoi diagram, also called the Dirichlet tessellation, is among the most important geometric structures in the practical field [12]. We are interested in this diagram because we want to reduce the complexity of our network as we already cited.

After the preview and the brief study about the Voronoi diagram, we pass now to our contribution point which is the principal of hypergraphs through exploring its properties and characteristics such as "Minimal Transversals".

B. The Hypergraphs

Hypergraphs are systems of finite sets and form, probably the most general concept in discrete mathematics. This branch of mathematics has developed very rapidly during the latter part of the twentieth century. Hypergraph theory is a recent theory mostly developed in Hungary and France [4]. The theory of hypergraphs aims at generalizing the graph theory by introducing the concept of hyper-edge which contains families of vertices, unlike the classical edges that join only two vertices. From a theoretical point of view, hypergraphs allow to generalize certain theorems of graphs and to factor several of them into only one. From a practical point of view, hypergraphs are sometimes preferred to graphs since they provide better models to certain some types of constraints.

These mathematical tools can be used to model networks, biology networks, data structures, process scheduling, computers, and a variety of other systems [4].

C. Hypergraph Definition

A hypergraph **H** is a couple **(V , E)**, where $V = v_1, v_2, ..., v_n$ is a non-empty (usually finite) set and $E = E_1, E_2, ..., E_m$ is a family of nonempty parts of V.

Like graphs, we say that:

- The elements of V are the vertices of H.
- The number of vertices n is the order of the hypergraph.
- The elements of E are the edges of H.

Figure 2 shows an example of a hypergraph where:

$V = x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

$E = e_1, e_2, e_3, e_4, e_5, e_6 = \{x_7, x_9\}, \{x_4\}, \{x_1, x_2, x_3, x_9\}$ $, \{x_4\}, \{x_{10}\}, \{x_4, x_6, x_8\}$

1) Simple hypergraph

H = (X, ξ) is said to be a simple hypergraph if for all $e_i \epsilon \xi$ and $e_j \epsilon \xi$, so $e_i \subseteq e_j \Rightarrow i = j$,i.e., no hyper-edge of H contains another hyper-edge, otherwise, H is said to be a multiple hypergraph.

Thus, the definition of hypergraphs includes that of graphs. Indeed, a simple graph is a simple hypergraph of which all hyper-edges are of cardinality 2,

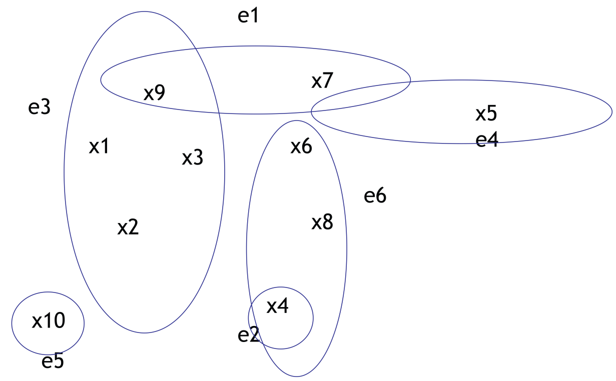

Figure 2. Hypergraph example

$$\text{ie.,}|e_i| = 2, \forall e_i \epsilon \xi$$

The **rank r(H)** of a hypergraph H is the maximum number of vertices of a hyper-edge and is defined by

$$\text{r(H)} = \max|e_i|, \forall e_i \epsilon \xi$$

Hypergraphs have several properties, such as: similarity, Helly property, stable hypergrah, transversals ...... etc. In our article, we are interested in the property of transversals and more precisely in the extraction of minimal transversals to detect the connectivity nodes in our network. For this, an overview of our research in the field of transversals is described below:

D. Cross Members and Minimum Transversals

The number of minimal traversals is polynomial and thus the extraction remains in shadow because of their difficulty of determination of minimal transversals.

1) Transversals:

Definition: In hypergraph theory, a transversal is a part which meets all edges of the starting hypergraph. The set of transversals is the gate.

2) Minimal Transversals

Extracting the minimal traversal of a hypergraph is a problem known to be quite difficult [14], which has attracted various researchers to explore this type of quest.

A minimal transversal corresponds to a set of vertices which intersects all hyper-edges of a hypergraph that is minimal in the sense of inclusion ***as show in Figure 3***. The extraction of these minimal traversals has been the subject of several works in the literature due to the diversity of its applications in various fields such as artificial intelligence, data mining, cryptography, semantic web ... etc.

3) Definition:

Let a be a hypergraph H = (X, ξ). All traversals of H, denoted γ_H, are equal to [5]:

$$\gamma_H = T \subset X | T \cap e_i \neq \emptyset, \forall i = 1, .., |\xi|$$

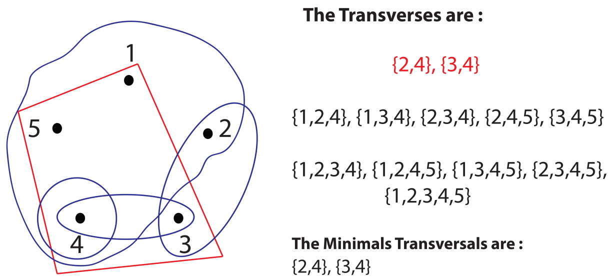

Figure 3. Graphic representation and Minimal transversals extraction.

A traversal T of γ_H is said to be minimal if it does not exist another transversal S of γ_H included in T:

$$\nexists S \epsilon \gamma_H \text{ s.t } S \subset T$$

Note : The set of minimal transversals form a hypergraph called a "transversal hypergraph".
Some research works are below :

The 1st author who proposed a minimal transversals algorithm is called Berge's algorithm as cited in [6].

The principle is simple; we start by calculating the whole minimal transversals of the first hyper-edge. Then, we update this set of minimal transversals by adding other hyper-edges, one by one.

The downside is that the algorithm still needs to store the intermediate minimal transversals before going to the next step of adding a new hyperedge. Therefore, Berge's algorithm becomes impracticable on large hypergraphs.

Moreover, there have been improvements proposed by Kavvadias and Stavropoulos [6] considering only the transversals which intersect the new processed hyperedge and also taking only the vertices of ξ_i which do not already belong to the already identified minimal transversals.

In [15], the authors propose an approach based on the principle of a Boolean matrix where the vertices correspond to patterns and hyper-edges to objects. The MtMiner algorithm adopts two pruning strategies when browsing at a candidate levels in the generated lattice. The first is based on the anti-monotonicity property of minimality of equivalence classes, according to which if a set of vertices does not constitute a minimal generator, then the search space generated from it is pruned. The second pruning strategy is to eliminate oversets of a set of vertices which is a minimal traversal, since they do not satisfy the condition of minimality in the sense of inclusion.

The MtMiner algorithm performs a width traversal by starting the scan of the search space by the vertices.

In [14], the authors established a link between the concepts of data mining and those of the theory of hypergraphs, thus proposing a methodological framework for the calculation of minimal traversals. They introduced the notion of irredundant minimal traversals, from which they could find the global set of all the minimal traversals, using the proposition of a new algorithm supposing that the vertices of the hypergraph are sorted by lexicographic order and by carrying out a traversal in width.

The authors of [16] examine two decision problems on hypergraphs; the saturation of the hypergraph and the recognition of the transversal hypergraph, and they discuss their importance for several research problems in applied computer sciences, that is to say, given two hyper-graphs H1; H2, deciding whether the sets of H2are all minimal transversals of H1.
Calculating or recognizing minimal transversals of a hypergraph is a common problem in practice, which is underscored by identifying important applications in database theory, Boolean switching theory, logic, and AI, and particularly in model-based diagnostics.

3. Analysis And Discussion

In previous section, we have mentioned the definition and explication of the essential points of our research. We have observed that the problem of energy and network longevity was been treated on the algorithm optimization side or position node side.
Moreover, our study is interested about the fact of node and its role in Wsn. In fact, the determinations of the connectivity node in the network have an important pact to keep a communication and so, extend network life time.

Wireless sensor networks have become very interesting for researchers given the number of methods, approaches and algorithms proposed in the literature. Therefore, the monitoring of wireless sensor networks is an important problem in this type of research because of the high energy consumption and the density of the networks. For this, many types of research have been opened to deal with this problem.

Among the important performance measures in WSN, we distinguish the connectivity which is a fundamental condition for the network to be operational.The connectivity is a measure of the network portion in the WSNs. Knowing that, the routing and maintenance of the network must be well studied to resist the failure of the graph. Our proposal is to determine these connectivity graphs to be able to manage, monitor, and maintain our network.

Once the extraction and selection of these connection graphs are defined, we can, for example, monitor their energy and replace them before deficiency to guarantee communication between the graphs, and also to ensure the exchange of information and the longevity of the network. The failure of these graphs has the following consequences:

1) Isolation of graphs;
2) Loss of communication links;
3) Network partitioning.

Knowing that among the essential questions that network analysis strives to deal with is the identification of individuals occupying a determining role in the network. Therefore, based on the research and the links between WSN and hypergraphs, we became more interested in the former traction of minimal transverslas and this is the principle of our contribution: to find the graph occupying a fundamental place in our network which will be called ”the connectivity graph”.

4. Contribution : Methods

A. WSN Modeling

We consider that our network is represented in the form of a Voronoi diagram. The main question we investigate is where does the interest in a representation in the form of a Voronoi diagram come from?

As an answer, the initial goal is the gain in complexity and the second most important point is the cover that is one of the important aspects of wireless sensor networks and thus the approach of the Voronoi diagram maximizes the coverage knowing that the minimal deployment of graphs to achieve full coverage is essential for the research.

Our study basis is to map this diagram to a hypergraph as follows:

B. Hypergraph representation

In this article, we establish a link between the concepts of ” Wireless Sensor Networks ” and those of the theory of hypergraphs, by proposing a methodological approach to construct a hypergraph that covers all networks.

As we have already mentioned, the hypergraphic representation means determining the vertices and hyper-edges. In the case of wireless sensor networks, our hypergraphic model is based on the physical parameters collected by the sensors' nodes (a sensor transmits its data totally or part by part to the wells). The assembling of nodes according to the physical phenomenon, has not been studied yet in the literature.
Concretely, these collected phenomenon are:

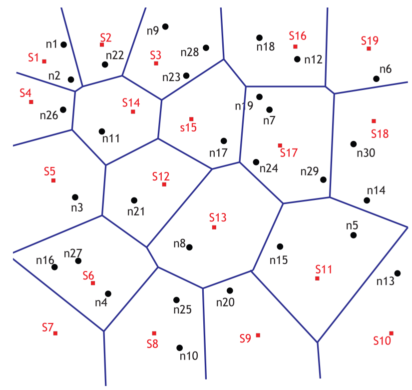

Figure 4. Voronoi Example-Basis.

1) Temperature reading(RT),
2) Pressure reading (RP),
3) Control reading (RC),
4) Intrusion Detection (DI),
5) Air Humidity (HA),
6) ....etc

These measures are not limited to those mentioned above, but rather depend on the study domain: agricultural, demotic, medical, ... etc. For example, in [17] the hyperedge are organised as the mission of WSN like Redundancy, Data collection..etc.

Our wireless sensor network is represented as a Voronoi hypergraph $H = V, E$, where the graphs V are the normal nodes and base nodes.

$$V = BN \cup SN,$$

And our hyperedges E are the set of physical phenomena (PP)

$$E = PP = RT \cup RP \cup RC \cup HA.$$

In conclusion:

$$H = \{V, PP\}$$

1) Example

Underneath is an example of a WSN in the form of a Voronoi Diagram, where the writer proposes an algorithm that consists of devising the region into several parts using a Voronoi Diagram and connecting nodes in the region's localization.

We take this example ***(Figure 4)*** as the basis of our study to concretize our contribution in it.

The example sheds light on the different WSN nodes

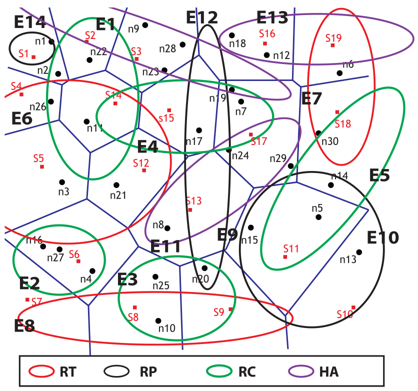

Figure 5. Hypergraphe example Basis.

that are divided in the form of Voronoi Diagrams. In the underneath image, the mapping to the hypergraphic representation is in the form of hyperedges of the different collected physical phenomena.

$$H = \{\{s_i \cup n_i\}, \{RT \cup RP \cup RC \cup HA\}\}$$

In the previous image ***Figure 5***, our network consists of hyperedges collected from a physical phenomena (the latter are attributed in an arbitrary way), where:

1) E1,E2,E3,E4,E5 : represent the temperature collection
2) E6,E7,E8 : the pressure's reading,
3) E9,E10 : control reading,
4) E11,E12 : Air Humidity.

Once the transformation of our network into a hypergraph is achieved, our second point of contribution is extracting the minimal transversals to manipulate the connectivity of our network by detecting the nodes that occupy an important function and a decisive function in the network. Moreover, the failure of this type of nodes would impact the communication and the cover of our sensor network.

5. Contribution : Experimental

For our experimental, we have exploit some example of WSN to see the impact of the "connectivity nodes" on the network after there determination. For the purpose of a comprehensive modelling, we have used one of this example **"Pedagogic example"** in the article to demonstrate the method of extraction, select Minimals Transversals and prove the importance of this nodes.

A. Transversals Extraction

The reason behind our interest in the extraction of minimal transversals is to highlight the nodes that ensure the connectivity, which is a sensitive point in the WSN. A disconnection can end the mission of a deployed network by isolating certain nodes of the network. Nevertheless, in case these nodes are determined previously by our method, we will be able to intervene before losing these nodes and thus before losing the network.

The major problem in extracting the minimal transversals M_H is the density of the network and because our hypergraph is defined as an extension of a traditional graph, the calculation of TM is a polynomial problem [18] [19]. Therefore, we have chosen to reduce our hypergraph without any loss of information, following the idea described as follows:

Proposition:*By Kavvadias et Stavropoulos en 2005*
Let a be H and G two simple hypergraphs. The minimal transversals of the hypergraphs H and G are minimal couples, in the sense of inclusion, generated by the Cartesian product of the minimal transversals sets of H and G.

$$TM_{H \cup G} = MINTM_H \times TM_G$$

1) Divide our network into k-hypergraphs (K = a range of an input hypergraph) to reduce the research space;
   The rank r(H) of a hypergraph H is the maximum number of vertices of a hyperedge and is defined by:
   $$r(H) = max|e_i|, \forall e_i \in \xi$$
   - In the previous example, K=7 is the equivalence of the maximal number of nodes which is in the hyperedge **E6** $E6 = \{n_{26}, n_{11}, s_{14}, s_{12}, n_{21}, n_3, s_5\}$ and according to this we conclude that we could divide the global hypergraph to maximum 7 sub-hypergraphs in an arbitrary way to facilitate the search for minimal transversals.
2) Study each newly obtained partial hypergraph and extract minimal transversals TM_H;
3) Assemble all of the obtained transversals by combining all subsolutions of the subproblems by the Cartesian product (the latter allows finding the set of minimal transversals TM_H)

The idea of decomposition is inspired from the study

[20] The difference is that instead of using the transversality number, we have used the **"The rank"** property of a hypergraph (which we already presented).

6. Contribution : Results and Discussion

On the basis of our example, after the decomposition of our hypergraph to 3 sub-hypergraphs, the study and extraction of the minimal transversals TM is the Cartesian product

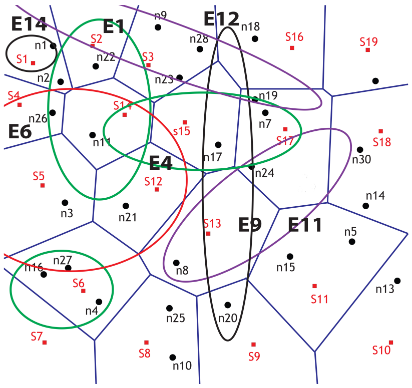

Figure 6. sub-hypergraph 1.

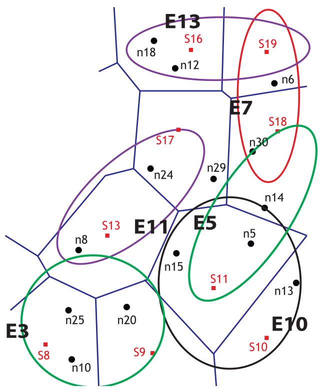

Figure 7. sub-hypergraph 2.

of the minimal transversal TM1 of the sub-hypergraph 1 ***(Figure 6)*** , TM2 of the sub-hypergraph 2 ***(Figure 7)***, and TM3 of the sub-hypergraph 3 ***(Figure 8)***.
$TM = TM1 \times TM2 \times TM3$

Our sub-hypergraph is equal to the set of our hyper-edges as follows:

- $SH1 = E1, E2, E4, E6, E9, E11, E12, E14$

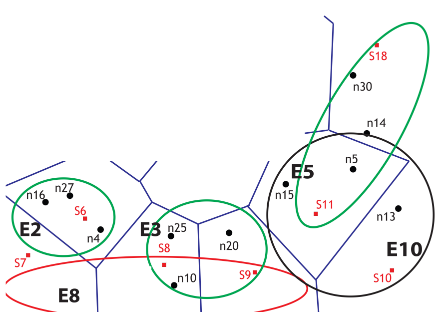

Figure 8. sub-hypergraph 3.

- $SH2 = E5, E3, E7, E10, E11, E13$
- $SH3 = E2, E3, E5, E9, E10$

The extraction of transversals is :

- $TM1 = s2, n22, n26, n11, s14, n19, n24, s13, n20$
- $TM2 = s19, s18, n30, n14, s11, n5$
- $TM3 = n10, s8, s9, n5, n14, n5, s11$

As we mentioned previously, our minimal transversals are the Cartesian products of the three minimal transversals extracted above. Therefore, the set of TM graphs are: TM=; according to this, we can conclude that the graphs of these minimal transversals represent our identified (connectivity graphs) in our network.

The loss of one of these graphs, either by energy exhausting or by another incident, causes the loss of communication between the hyper-edges (message exchange) in the network as we can show that as follows: The elimination or absence of certain nodes that belong to the CN (connected nodes) has an impact on our network; for instance: s2, s14, s19, n20.

The loss of nodes: s2, s14 reduces the hyper-edge E1 of 6 nodes in countenance to 4 nodes in countenance and makes us loose the communication links between the hyper-edges E1, E12 and E4, E6 and E1. Thus, the same remark to the node s19, the absence of this node influences our network by the cessation of connectivity between the hyper-edges: E13 and E5.

The ablation of message exchange and communication in the network is basically caused by our nodes $EN \epsilon CN$ (our minimal transversals already extracted), which plays a crucial role in the connectivity and longevity of our network.

In our principal contribution, by mapping in hyper-graphs and extracting minimal transversals, sit helps us to detect the decisive and crucial nodes in the contact link of the wireless sensor networks.

7. Conclusions and Future Work

The wireless sensor networks are becoming among the most studied fields in the last decades. Thus, the originality of proposing to study this network with the principal of this hyper-graphic transforming and the study of the extraction property of minimal transversals to detect the connectivity nodes and the impact of the failure of these nodes on our network. .

Due to the vulnerability of the wireless sensors networks as the energy limitation, the node breaks the weak bandwidth and calculation space. We are interested in the property of minimal transversals extraction for the selection of nodes that play a essential role in the network. As shown in the article, the latter represents the linking sensors that maintain the communication; also the loss of one of these nodes can cause the interruption of sending and receiving messages and the loss of connectivity between the different network structures.

The hypergraphic representation allows us to explore the property of ”minimal transversals” to give a sense to the nodes extracted by the terminology of ”connectivity nodes” because the latter plays a determining role in our network.

We first reduced our network to the Voronoi configuration to minimize the load and manage the partitioning, which mainly helped for the extraction of transversals which is determined by a polynomial problem (which remains a important point of study).

Our proposition, either the hypergraphic representation or the extraction of minimal transversals, aims at granting links of communication between the nodes and to maintain the network working as long as possible.The determination of these points or nodes of connectivity gives us the possibility of carrying out a monitoring on these nodes to avoid the distribution of the network with the attribution of another node which will play the role of a node of connectivity before the loss or disappearance of the already selected node.

Our contribution ensures the prediction of communication loss and gives a major advantage to solve this problem by the choice or the mobility of other nodes to ensure the connectivity of the network.

As a perspective, the application of our proposal in a simulator such as NS3 or Omnet++.

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