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Ideal Economic Dispatch of Thermal Power Units

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Abstract: One of the main concerns posed to power generation operations is the problem of economic dispatch. This problem can be resolved by minimizing fuel costs and satisfying system constraints to generate optimal amounts of power from generating units in the system. The accuracy of this solution depends on fuel cost parameters within the generating units. This paper explores a solution method for this economic dispatch problem and thermal generator units in power systems. To examine this solution, the input/output curves of the generators are considered as a quadratic function. In addition, a least-squares method is utilized to calculate the coefficient of these quadratic functions. To consolidate the suggested approach, a case study is analyzed by considering the generators of the IEEE Reliability Test System (IEEE – RTS). The results garnered from this application are then presented. Some of the results are verified with corresponding literature, thus proving the performance of computational algorithms successful.

Keywords: Economic Dispatch, Thermal Power, Fuel Cost, Quadratic Function, Least-Square, Power System

1. Introduction

The optimal operation of electric power systems has gained greater importance in recent years due to ever-increasing fuel costs. Electric power utilities are required to operate their systems more efficiently and economically. This requirement means that a series of optimizations must be solved to operate and design a power system for maximum efficiency [13]. Usually, electric power systems contain both thermal and hydro stations. Due to the complexity and size of hydrothermal systems, different problems have been identified in optimizing their operation over different time scales: long term, medium-term, and short term [14, 15].

The coordination of thermal generation and hydro generation is an attempt to minimize the total fuel bill of the system. This idea comprises the principal avoidable operating cost for a given system configuration. The costs of hydro station operations are virtually independent of the generation. Thermal station fuel costs are also virtually independent of the generation, where thermal station fuel costs vary considerably from one station to another depending on the type of fuel used and the efficiency of the machine installed. Hence, the more expensive thermal station must be used to the least possible extent, and wasting water resources spilling must be minimized [16].

Electric power systems have a fundamental operating future: electrical energy production and consumption are virtually simultaneous. To ensure that customers receive energy whenever desired, the system providing that energy must be substantially reliable, i.e., high reliability is an essential characteristic. Maintenance of power system equipment is also a significant component in providing this reliability in two ways. Firstly, preventative maintenance of the equipment is crucial to ensure that the equipment continues to operate at peak performance, thereby being reliable. Secondly, while the equipment is being maintained, it is unavailable for service, and the system is consequently weekend. These include generating units that are implicitly related to power system reliability and have a tremendous impact on the operation of the power system. Therefore, there is a need to consider the effect of maintenance on the system's reliability and ensure that the maintenance schedule itself does not affect the reliability too adversely.

Generating unit maintenance touches upon many problems. Inappropriate scheduling can lead to many undesirable situations from an economic point of view, especially the huge direct cost of generating unit maintenance. In addition, there are some latent costs,

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including that of electric energy generated during generating unit maintenance time, power generation when a less efficient generating unit replaces a highly efficient generating unit, and investment in maintenance reserve capacity that has to be added to ensure the electricity supply reliability of the system. As for the system reliability, the risk of the system supply's inefficiency may increase during the scheduled maintenance outage. The problem is especially prominent for systems that have a shortage of reserve capacity [17].

In the last two decades, there has been a growing interest in research into both theoretical and methodological approbation to the shedding of generating unit maintenance. This interest relies on power system planning design and operation management; both research and practice have shown that the power system maintenance schedule is a long-term constraint optimized program. The maintenance schedule that satisfies all the constraints is called a feasible solution [18]. Certain constraints should be set up according to the real conditions of the power system in order to make the maintenance plan feasible, which can be divided into three main categorize: time constraints, maintenance crew constraints, and resource constraints. The time constraint means some generating units must or cannot be scheduled for maintenance during a certain time interval, while the maintenance crew constraint means that there are limitations due to maintenance manpower availability. It should be noted that two generating unit maintenances cannot be scheduled to be carried out by one group of the crew in the same time interval. The cost resource constraint relates to the maximum resource provided for certain maintenance [19]. Reference [18] surveys some maintenance scheduling, shedding light on such constraints [18].

This paper explores relevant problem formulation and solution methods. Significant research in the field of maintenance scheduling can be summarized as follows. Reference [20] comprises a starting point for many authors, who try either to eliminate the shortcoming existing in this model gradually or to develop radically new models. It is worth saying that despite its numerous shortcomings, this reference positively influenced the development of new automatic maintenance scheduling techniques. Reference [30, 31] introduce the concept of equivalent load capability, which models the impact of thermal units' unavailability on power system reliability, demonstrating how, with simple corrections in existing maintenance scheduling algorithms, the risk can be introduced as an optical criterion instead of reserving the proposed method [19, 33, 34].

It is recognized that maintenance scheduling units found a significant part of the overall operational management of an electric power utility. Optimizing the outage decisions of the units is therefore essential for ensuring an effete operation of an electric power system. Real electric power systems usually contain thermal generation and hydro generation. Some solutions suggest considering only thermal generation except for reference [34], where only the hydro maintenance scheduling problem is analyzed. However, other solutions proposing considering both types of units are not apparent in the current literature review due to some factors. One of the reasons is because it is recognized that the coordination of the operation of the system of hydrothermal electric generation plans is usually more complex than the scheduling of an all thermal generation system. Still, the improvements and availability of reliable computers have created the opportunity to develop solution methods for this complex combinatorial problem.

This paper develops an efficient methodology to solve the maintenance scheduling problem considering electric power systems containing both hydro and thermal generating units. The advantages of this methodology include the following:

a) It provides a better representation of electric power systems.
b) It produces a more realistic solution for the hydrothermal maintenance scheduling problem.
c) It provides space and opportunity to analyze the effects of unit maintenance outages on the operation and reliability of hydrothermal electric power systems.

On the other hand, this paper also suggests feasible solutions for the economic dispatch problem. The input-output curves of generating are considered as quadratic functions. A least-squares method is used to calculate the coefficients of these quadratic functions, which is further supported by a case study considering the generator of the IEEE-RTS.

In the case study, a power system has thermal and hydro generation units working together. The maintenance is analyzed in sections 1, 8 & 9. The Hydrothermal Maintenance Method uses a coordination methodology; this coordination requires the optimal solution of the economic dispatch problem. This paper presents an alternative approach for solving the economic dispatch problem. Section 2 describes the economic dispatch problem, which requires treating the generation cost of each unit as input data. The research then discusses the modeling of fuel cost in Section 3, followed by two sections exploring solutions for the economic dispatch problem through illustrated algorithms. A case study is then presented in section 6.

2. Description of the Economic Dispatch Problem

The economic dispatch problem is articulated as the principle problem when minimizing fuel costs in supplying a given load. This problem may be concisely stated in mathematical terms. I.E., an objective function total cost of thermal generation (C_T), is equal to the total cost for supplying the indicated load – system load demand (D) is supplied by the Nt_g thermal generating units. The problem lies in minimizing C_T subject to the constraint that the sum of power generated must equal the assigned load. Note that any transmission losses are neglected, and any operating limits are not explicitly stated when formulating this problem: that is [2, 12],

$$C_T = C_1 + C_2 + C_3 + \cdots + C_{Ntg} = \sum_{i=1}^{Ntg} C_i\,(P_{gi}) \quad (1)$$

Where: $C_i\left(P_{gi}\right)$ Generation cost of unit i when it is generating power of (P_{gi}) ($) and P_{gi} Generation of the unit (MW).

$$\emptyset = 0 = D - \sum_{i=1}^{Ntg} P_{gi} \quad (2)$$

By using an advanced calculus method involving the $LaGrange$ function, this problem of constraint optimization can be attacked formally.

The $LaGrange$ function, as shown in equation (3), establishes the necessary conditions for an extreme value of the objective function by adding the constraint function to the objective function after the constraint function has been multiplied by an undetermined multiplier.

$$\varsigma = C_T + \lambda\, \phi \quad (3)$$

The conditions for an extreme value of the objective function result are known when one takes the first derivative of the $LaGrange$ function concerning each independent variable and sets the derivatives equal to zero. In this case, there are $Ntg + 1$ variables, the Ntg values of power output, P_{gi} plus the undetermined $LaGrange$ multiplier λ. The derivative of the $LaGrange$ function, for the undetermined multiplier, merely gives back the constraint equation. On the other hand, the Ntg equations result when one takes the partial derivative of the $LaGrange$ function with respect to the power output values one at a time. The set of equations are shown as equation (4).

$$\frac{\partial \varsigma}{\partial P_{gi}} = \frac{dC_i(P_{gi})}{dP_{gi}} - \lambda = 0 \Rightarrow 0 = \frac{dC_i(P_{gi})}{dP_{gi}} - \lambda \quad (4)$$

The incremental cost rates of all the units must be equal to some undetermined value λ to meet the necessary condition for the existence of a minimum cost-operating condition for the thermal power system. In addition, the constraint equation – that the sum of the power outputs must equal the power demanded by the load must be included. Furthermore, two inequalities must be satisfied for each of the units. That is, the power output of each unit must be greater than or equal to the minimum power permitted and must also be less than or equal to the maximum power permitted on that particular unit.

These conditions and inequalities may be summarized as shown in the set of equations making up equation (5).

$$\frac{dC_i(P_{gi})}{dP_{gi}} = \lambda$$

$$\sum_{i=1}^{Ntg} P_{gi} = D$$

$$P\ P_{gi.min} \leq P_{gi} \leq P_{gi.\max} \quad (5)$$

When one recognizes the inequality constraints, the necessary conditions may be expanded slightly, as shown in the set of equations making up the equation (6).

$$\frac{dC_i(P_{gi})}{dP_{gi}} = \lambda \; for \; P_{gi.min} < P_{gi} < P_{gi.max}$$

$$\frac{dC_i(P_{gi})}{dP_{gi}} =\leq \lambda \; for \; P_{gi} = P_{gi.max}$$

$$\frac{dC_i(P_{gi})}{dP_{gi}} =\geq \lambda \; for \; P_{gi} < P_{gi.min} \quad (6)$$

Where: $P_{gi.min}$. $P_{gi.max}$ Minimum and maximum generating unit (MW).

3. MODELLING FUEL COSTS FOR THERMAL GENERATION

As far as economic studies are concerned, a thermal generating unit is considered an input-output type model. In this case, the input is the fuel cost, with the unit's active power generation being the output. By and large, the thermal plant's input is normally expressed in $/h. The function relating fuel cost and generation is a nonlinear function.

An additional curve is used to represent the characteristic of this model: heat rate-power is the relation in this nonlinear function. The units of heat rate are in M.J./kWh. The typical heat rate of data for sample unit sizes of steam units with coal, oil, and gas as energy sources are illustrated in Table 1 [3, 4, 11].

Loading (output) levels whereby a new steam admission valve is opened are called valve points. At these levels, discontinuities in the cost curves and the incremental heat rate curves occur due to the sharp increases in throttle losses. With a gradual lifting of the valve, losses are decreased until the valve is completely open. The area of the valve points is difficult to determine through actual testing, which influences how to determine the shape of the input-output curve.

TABLE 1. TYPICAL FOSSIL GENERATION AND NET HEAT RATES

Fossil fuel	Unit rating	Output (M.J./kWh) 100%	80%	60%	40%	25%
Coal	50	11.59	11.69	12.05	12.82	14.13
Oil	50	12.12	12.22	12.59	13.41	14.78
Gas	50	12.33	12.43	12.81	13.64	15.03
Coal	200	10.01	10.09	10.41	11.07	12.21
Oil	200	10.43	10.52	10.84	11.54	12.72
Gas	200	10.59	10.68	11.01	11.72	12.91
Coal	400	9.49	9.53	9.75	10.31	11.25
Oil	400	9.91	9.96	10.18	10.77	11.75
Gas	400	10.01	10.06	10.29	10.88	11.88
Coal	600	9.38	9.47	9.77	10.37	11.40
Oil	600	9.80	9.90	10.20	10.84	11.91
Gas	600	9.91	10.01	10.31	10.96	12.04
Coal	800/1200	9.22	9.28	9.54	10.14	
Oil	800/1200	9.59	9.65	9.92	10.55	
Gas	800/1200	9.70	9.75	10.03	10.67	

Most utility systems are satisfied with the input-output characteristic, often represented as a smooth curve defined by a polynomial [3]. In this study, the input-output curves are modeled as quadratic functions, expressed for unit i as equation (7)

$$Y_i = C_i\left(P_{gi}\right) = \propto_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \quad (7)$$

Where: $\alpha_i, \beta_i, \gamma_i$ Cost function coefficients of unit i

The determination of the parameters $\propto_i$, β_i and γ_i requires the availability of data relating to the cost $C_i\left(P_{gi}\right)$ to the generation level P_{gi} One may then use a simple least-square estimation algorithm to do so [5]. Given Np points where the cost $C_i\left(P_{gi}\right)$ and the power P_{gi} are known, the parameters are determined such that a least square error is involved. The problem is then to minimize $E_i(\propto_i, \beta_i, \gamma_i)$ with respect to $\propto_i, \beta_i$ and γ_i, where $E_i(\propto_i, \beta_i, \gamma_i)$ is expressed in equation (8)

$$E_i(\propto_i, \beta_i, \gamma_i) = \sum_{K=1}^{Np}(\propto_i P_{gik}^2 + \beta_i P_{gik} + \gamma_i - Y_{ik})^2 \quad (8)$$

The solution is obtained by setting the derivatives of $E_i(\propto_i, \beta_i, \gamma_i)$ with respect to $\propto_i, \beta_i$ and γ_i) to zero. The resulting relations are shown in the set of equations making up equation (9).

$$\frac{\partial E_i(\propto_i,\beta_i,\gamma_i)}{\partial \propto_i} = 2 = \sum_{K=1}^{Np}(\propto_i P_{gik}^2 + \beta_i P_{gik} + \gamma_i - Y_{ik})^1.\left(P_{gik}^2\right) = 0$$
$$\frac{\partial E_i(\propto_i,\beta_i,\gamma_i)}{\partial \beta_i} = 2 = \sum_{K=1}^{Np}(\propto_i P_{gik}^2 + \beta_i P_{gik} + \gamma_i - Y_{ik})^1.\left(P_{gik}\right) = 0 \quad (9)$$
$$\frac{\partial E_i(\propto_i,\beta_i,\gamma_i)}{\partial \gamma_i} = 2 = \sum_{K=1}^{Np}(\propto_i P_{gik}^2 + \beta_i P_{gik} + \gamma_i - Y_{ik})^1.(1.0) = 0$$

Rearranging, one can obtain the following set of equations:

$$\left(\sum_{k=1}^{Np} P_{gik}^4\right).\propto_i + \left(\sum_{k=1}^{Np} P_{gik}^3\right).\beta_i + \left(\sum_{k=1}^{Np} P_{gik}^2\right).\gamma_i = \sum_{k=1}^{Np} Y_{ik} P_{gik}^2$$
$$\left(\sum_{k=1}^{Np} P_{gik}^3\right).\propto_i + \left(\sum_{k=1}^{Np} P_{gik}^2\right).\beta_i + \left(\sum_{k=1}^{Np} P_{gik}\right).\gamma_i = \sum_{k=1}^{Np} Y_{ik} P_{gik} \quad (10)$$
$$\left(\sum_{k=1}^{Np} P_{gik}^2\right).\propto_i + \left(\sum_{k=1}^{Np} P_{gik}\right).\beta_i + Np.\gamma_i = \sum_{k=1}^{Np} Y_{ik}$$

Solving the above linear set of equations in $\propto_i, \beta_i$ and γ_i yields the desired estimates. An algorithm was developed to solve the above linear set of equations, tested using the data given in Table 1.

TABLE 2. TYPICAL COST COEFFICIENTS (M.J./MWh)

Unit Size (M.W.)	Coal $\propto$ (10^{-3})	Coal β	Coal γ	Oil $\propto$ (10^{-3})	Oil β	Oil γ	Gas $\propto$ (10^{-3})	Gas β	Gas γ
50	10.307	10.064	49.915	11.570	10.471	52.856	11.647	10.662	53.607
200	2.2921	8.6718	173.63	2.3829	9.0391	180.69	2.3509	9.1949	182.65
400	1.4629	8.1441	300.82	1.4966	8.5222	312.67	1.4940	8.6135	316.36
600	0.5332	8.2800	462.11	0.5636	8.6473	483.31	0.5882	8.7316	489.87
800	1.0002	7.4758	752.70	1.0781	7.7316	794.14	1.1722	7.7261	825.47
1200	0.6667	7.4759	1129.0	0.7185	7.7319	1191.1	0.7810	7.7269	1237.9

The results are illustrated in Table 2. These results can be compared with the results obtained in reference [4]. This finding determines that the results obtained with the developed algorithm and those from the reference are almost equal. This test is a verification that the algorithm is working well.

4. Equal incremental cost loading considering quadratic functions of the fuel costs

The fuel cost function for each thermal generating unit is considered a quadratic function, as shown in equation (11).

$$C_i(P_{gi}) = \propto_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \quad (11)$$

In section 2, it was stated that all the thermal generating units must have equal incremental cost ($\curlywedge$) to have an optimal operation. Mathematically this is

$$\frac{dC_i(P_{gi})}{dP_{gi}} = \curlywedge$$
(12)

If the units of C_i are in \$/hr, the units of the $\curlywedge$ are in \$/MWh. The $\curlywedge$ figure represents the increase in cost rate per increase in M.W. output, or equivalently the increase in cost per increase in MWh [1].

If a quadratic function of the fuel cost is considered, the generation of a thermal unit can be expressed as a function of the incremental cost $\curlywedge$ as follows;

$$P_{gi}(\curlywedge) = \frac{\curlywedge - \beta_i}{2\propto_i}$$
(13)

The value of $\curlywedge$ can be determined in the following way. First, we need to consider the constraint of the sum of the thermal generation that must be equal to the given demand and including equation (13), which produces:

$$\sum_{i=1}^{Ntg} \frac{\curlywedge - \beta_i}{2\propto_i} = D$$
(14)

From equation (14), the value of is calculated as follows:

$$\curlywedge = \frac{\sum_{i=1}^{Ntg} \frac{\curlywedge - \beta_i}{2\propto_i}}{\sum_{i=1}^{Ntg} \frac{1}{2\propto_i}}$$
(15)

In the following section, the solution method to include the output limits of the generating units is discussed. This method for a solution $P_{gi}(\curlywedge)$ from equation (13) yields equal incremental costs for each unit, unless a lower or an upper limit prevents this from occurring – as indicated in the set of equations making up (6).

5. AN ALGORITHM TO OBTAIN AN OPTIMAL ECONOMIC DISPATCH WITH QUADRATIC FUNCTION OF THE FUEL COST

Several solution methods to obtain the incremental costs consider the generator output limits, according to [2, 6, 12]. Two methods are presented as a reference to this [6]. The first one searches for a value of $\curlywedge$ that satisfies

$$\emptyset(\curlywedge) = \sum_{i=1}^{Ntg} P_{gi}(\curlywedge) - D = 0$$
(16)

With

$$P_{gi}(\curlywedge) = \min\left[\max\left(\frac{\curlywedge - \beta_i}{2\propto_i}, P_{gi\,.min}\right). P_{gi.max}\right]$$
(17)

The value of $\curlywedge$ is limited between a lower value $\curlywedge_{max}$ and an upper value , with

$$\curlywedge_{min} = \min_i [2 \propto_i P_{gi.max} + \beta_i]$$
(18)

And

$$\curlywedge_{max} = \max_i [2 \propto_i P_{gi.max} + \beta_i]$$
(19)

Function $\emptyset(\curlywedge)$ is a piecewise-linear, non-decreasing function of $\curlywedge$ between the values $\curlywedge_{min}$ and $\curlywedge_{max}$ with $\emptyset(\curlywedge_{min}) \leq 0$ and $\emptyset(\curlywedge_{max}) \geq 0$. Solving $\emptyset(\curlywedge) = 0$ can be determined, such as with the bisection method, by each time the interval in which the solution is present. This idea can be tested because $\emptyset(\curlywedge)$ is positive if $\curlywedge$ is too large, and it is negative when $\curlywedge$ is too small. The start interval can be $(\curlywedge_{min}, \curlywedge_{max})$. This iterative process is stopped if $|\emptyset(\curlywedge)| \geq \in$, with $\in > 0$ as a predefined accuracy measure.

The second method (which is stated to be more accurate and faster because it avoids the many iterations of the previous method) is based on the following,

$$PN = \sum_{i=1}^{Ntg} \max[0, P_{gi}(\curlywedge) - P_{gi.max}]$$
(20)

$$PX = \sum_{i=1}^{Ntg} \max[0, P_{gi}(\curlywedge) - P_{gi.max}]$$
(21)

When PN and PX are the sums of violations to lower limits and upper limits, respectively, in reference [6] the following theorem is given,

$$If\ PN = PX = P_{gi} = P_{gi}\ (\lambda), i = 1 \ldots\ldots Ntg.$$

It is the optimal solution to the economic dispatch problem

$If\ PN \leq PX$ all the units with $P_{gi}\ (\lambda), > P_{gi\,max}$ will have $P_{gi} = P_{gi\,max}$ in the optimal solution

$If\ PN > PX$ all the units with $P_{gi}\ (\lambda), < P_{gi\,max}$ will have $P_{gi} = P_{gi\,max}$ in the optimal solution

The implementation of a computational algorithm took place using this solution method. In the next section, results obtained using this algorithm are presented.

6. CASE STUDY USING THE IEEE-RTS

An economic dispatch study for the IEEE-RTS, as described in [7], has been undertaken in this section. The maximum thermal generation of the IEEE-RTS is 3105 MW. This system comprises 26 thermal generating units accompanied by seven types of thermal generating units. Table 3 illustrates the relevant information for these units, such as size and type. Additionally, these units' heat rates for several capacity levels are also shown in Table 3. In order to estimate coefficients of fuel-generation curves of these thermal generating units, the capacity levels' heat rates are used. The estimated values are given in Table 4. All fuel-generation curves are shown in Figures 1 & 2. These figures are drawn on as verification for the estimated values produced to fit the given points. These curves appear to be acceptable.

Considering the fuel costs given in reference [7] and shown in Table 5, one can obtain the cost coefficients of the cost-generation function. It is achieved by multiplying the previously estimated coefficients by the fuel costs. The results are shown in Table 6. To obtain the cumulative cost curve of each unit, these coefficient costs, especially $\propto$ and, are used. The use of equation (13) allows these incremental cost curves to be obtained. This equation can be rewritten as follows:

$$\lambda = 2 \propto_i P_{gi} + \beta_i \quad (22)$$

Figure 3 presents the obtained curves, which are straight lines considering two points for each unit: minimum and maximum output capacity. It also shows that the cheapest one is the 400 MW (nuclear) unit, with the 20 M.W. and the 12 M.W. units being the most expensive. The maximum capacity obtained from the IEEE-RTS hydro units is 300 MW (6 units of 50 M.W. each). The maximum load of the system in the year is 2850 MW. If one considers hydro to generate their maximum capacity, the maximum load for the thermal generating units will be 2550 MW. Therefore, economic dispatch will be carried out for a load of 2550 MW using all the available thermal units. The iterative process to obtain the incremental cost is shown in Tables 7 & 8. Rather than using multiple settings in Table 7 for lower or upper limits, a single setting was used in their place. This process verifies the multiple setting – as shown in Table 8. Only four iterations are required to obtain the value of λ (28.60868 \$/MWh), the two tests give the same value.

TABLE 3. DATA OF THE THERMAL GENERATING UNITS OF THE IEEE-RTS

Size (MW)	Type	Fuel	Output %	Heat rate KBtu/kWh
12	Fossil steam	6 oil	20.0 50.0 80.0 100.0	15.600 12.900 11.900 12.000
20	Combus. turbine	2 oil	40.0 80.0 100.0	18.000 15.000 14.500
76	Fossil steam	coal	20.0 50.0 80.0 100.0	15.600 12.900 11.900 12.000
100	Fossil steam	6 oil	25.0 55.0 80.0 100.0	13.000 10.600 10.100 10.000
155	Fossil steam	coal	35.0 60.0 80.0 100.0	11.200 10.100 9.800 9.700
197	Fossil steam	6 oil	35.0 60.0 80.0 100.0	10.750 9.850 9.840 9.600
350	Fossil steam	coal	40.0 65.0 80.0 100.0	10.200 9.600 9.500 9.500
400	Nuclear steam	LWR	25.0 50.0 80.0 100.0	12.550 10.825 10.170 10.000

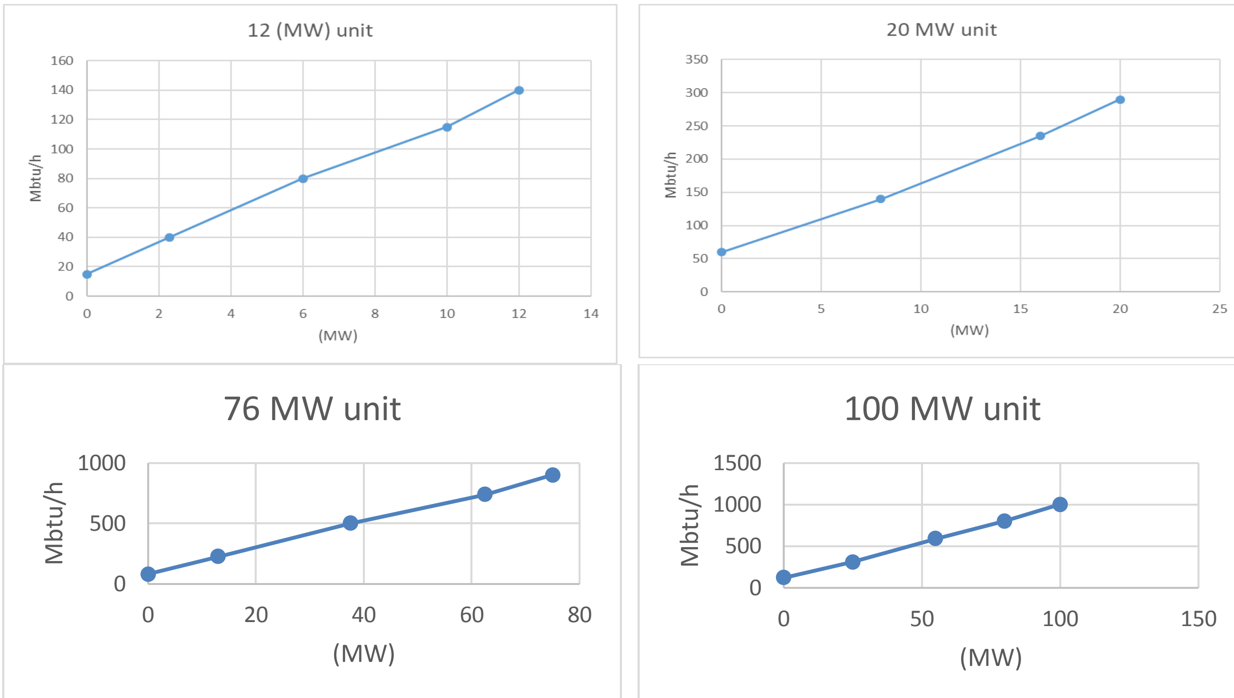

Figure 1. Fuel-generation curves of the thermal generating units of the IEEE-RTS

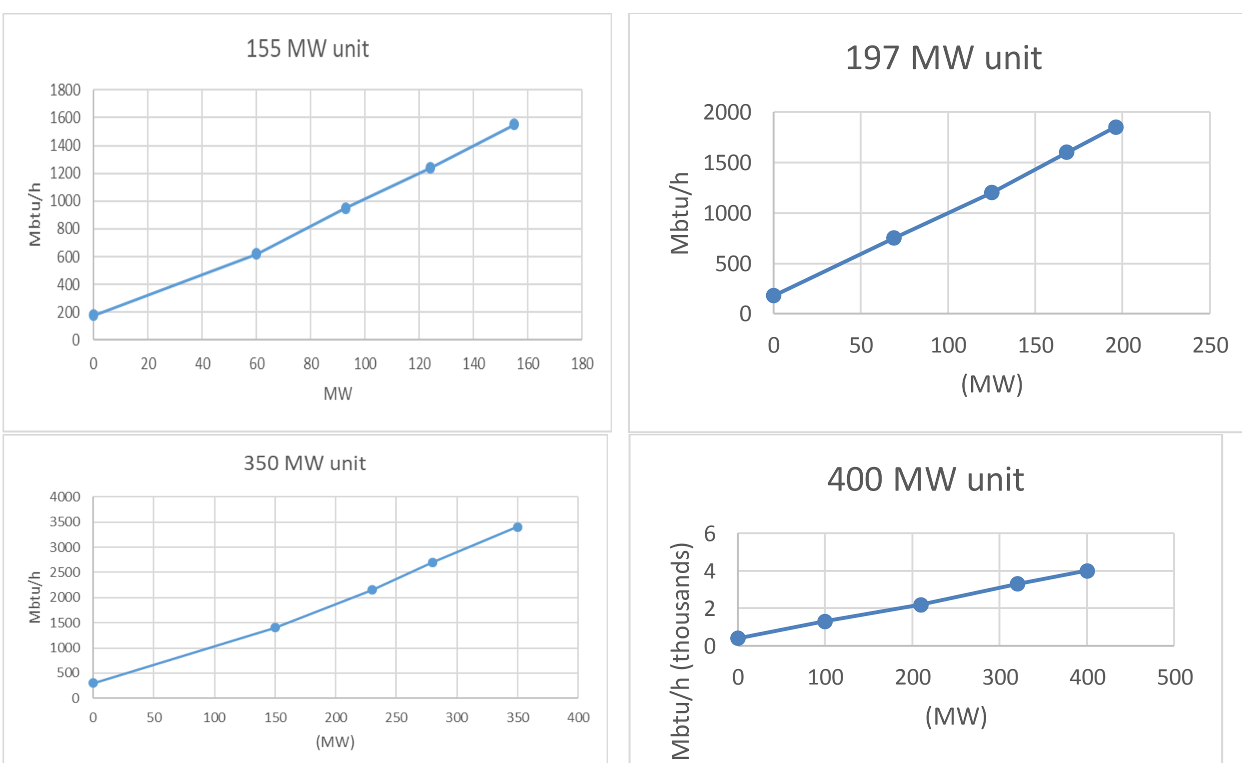

Figure 2. Fuel-generation curves of the thermal generating units of the IEEE-RTS

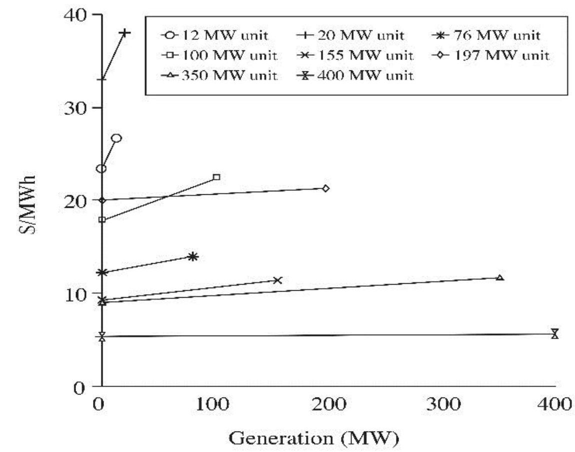

Figure 3. Incremental costs of the thermal generating units of the IEEE-RTS

TABLE 4. COEFFICIENTS OF FUEL-GENERATION CURVES OF THE THERMAL GENERATING UNITS OF THE IEEE-RTS

Unit Size (M.W.)	$\propto$ (10^{-3})	β	γ
12	59.7267	10.1205	13.21639
20	41.6816	10.9996	53.33580
76	9.42690	10.12085	83.69745
100	9.57697	7.79295	124.4536
155	5.56228	7.7253	172.2633
197	1.30370	8.7055	130.968
350	3.26315	7.43297	323.5509
400	0.460694	8.90839	360.9801

TABLE 5. FUEL COSTS

Fuel	Cost
#6 oil	$ 2.30 MBtu
#2 oil	$ 3.0 MBtu
Coal	$ 1.20 MBtu
nuclear	$ 0.60 MBtu

TABLE 6. COST COEFFICIENTS OF THE THERMAL GENERATING UNITS OF THE IEEE-RTS

Number Of Units	**Unit Size (M.W.)**	$\propto$ (10^{-3})	β	γ
5	12	137.3714	23.27716	30.39769
4	20	125.0449	32.99875	160.0074
4	76	11.3123	12.14502	100.4369
3	100	22.0270	17.92378	286.2434
4	155	6.67474	9.27036	206.716
3	197	2.99851	20.02265	301.2264
1	350	3.91578	8.91956	388.2611
2	400	0.276416	5.34503	216.588

One should keep in mind the condition needed to obtain the optimal solution, which is explained in the set of equations making up equation (6) and Figure 3, to obtain a logical explanation of the iterative process shown in Table 8. In the first iteration, a value of 8.32002 is obtained when λ . P.X. is greater than P.N. With this, some units, units 25 and 26, are set to their maximum output limit (400 MW). If a horizontal straight line is drawn at 8.32002 $/MWh, the generation values can be obtained graphically in Figure 3. When the incremental cost curve reaches this straight line, all generation values are given.

TABLE 7. ITERATIVE PROCESS TO OBTAIN THE INCREMENT COST λ

Iter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
PX	9962.7	7969.4	2676.3	2325.8	2064.2	1760.4	1403.4	977.84	777.47	551.47	294.58	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
PN	8212.7	6564.0	2200.3	1849.8	1588.2	1296.2	979.58	602.05	424.32	264.02	249.44	232.73	187.24	141.14	94.418	47.056	37.880	28.588	19.179	9.6508	.0
	8.32	9.971	16.576	17.158	17.592	18.096	18.688	19.395	19.727	20.102	20.529	21.018	20.938	20.857	20.774	20.691	20.675	20.658	20.642	20.625	20.608
1	-54.4	-48.4	-24.4	-22.3	-20.7	-18.9	-16.7	-14.1	-12.9	-11.5	-10.0	-8.2	-8.5	-8.8	-9.1	-9.4	.0	.0	.0	.0	.0
2	-54.4	-48.4	-24.4	-22.3	-20.7	-18.9	-16.7	-14.1	-12.9	-11.5	-10.0	-8.2	-8.5	-8.8	-9.1	-9.4	-9.5	.0	.0	.0	.0
3	-54.4	-48.4	-24.4	-22.3	-20.7	-18.9	-16.7	-14.1	-12.9	-11.5	-10.0	-8.2	-8.5	-8.8	-9.1	-9.4	-9.5	-9.5	.0	.0	.0
4	-54.4	-48.4	-24.4	-22.3	-20.7	-18.9	-16.7	-14.1	-12.9	-11.5	-10.0	-8.2	-8.5	-8.8	-9.1	-9.4	-9.5	-9.5	-9.6	.0	.0
5	-54.4	-48.4	-24.4	-22.3	-20.7	-18.9	-16.7	-14.1	-12.9	-11.5	-10.0	-8.2	-8.5	-8.8	-9.1	-9.4	-9.5	-9.5	-9.6	-9.6	.0
6	-98.7	-92.1	-65.7	-63.3	-61.6	-59.6	-57.2	-54.4	-53.1	-51.6	-49.9	-47.9	.0	.0	.0	.0	.0	.0	.0	.0	.0
7	-98.7	-92.0	-65.7	-63.3	-61.6	-59.6	-57.2	-54.4	-53.1	-51.6	-49.9	-47.9	-48.2	.0	.0	.0	.0	.0	.0	.0	.0
8	-98.7	-92.0	-65.7	-63.3	-61.6	-59.6	-57.2	-54.4	-53.0	-51.6	-49.9	-47.9	-48.2	-48.5	.0	.0	.0	.0	.0	.0	.0
9	-98.7	-92.1	-65.7	-63.3	-61.6	-59.6	-57.2	-54.4	-53.0	-51.5	-49.9	-47.9	-48.2	-48.5	-48.9	.0	.0	.0	.0	.0	.0
10	-169.1	-96.0	195.9	221.6	240.8	263.0	289.2	320.5	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0
1	-	-	195.	221.	240.	263.	289.	320.	335.	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0

	1	169.1	96.0	9	6	8	0	2	5	2												
	12	-169.1	-96.0	195.9	221.6	240.8	263.0	289.2	320.5	335.2	351.7	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0
	13	-169.0	-96.0	195.9	221.6	240.8	263.0	289.2	320.5	335.2	351.7	370.6	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0
	14	-218.0	-180.5	-30.6	-17.4	-7.5	3.9	17.4	33.4	40.9	49.5	59.1	70.2	68.4	66.6	64.7	62.8	62.5	62.1	61.7	61.3	60.9
	15	-218.0	-180.5	-30.6	-17.4	-7.5	3.9	17.4	33.4	40.9	49.5	59.1	70.2	68.4	66.6	64.7	62.8	62.5	62.1	61.7	61.3	60.9
	16	-218.0	-180.5	-30.6	-17.9	-7.5	3.9	17.4	33.4	40.9	49.5	59.1	70.2	68.4	66.6	64.7	62.8	62.5	62.1	61.7	61.3	60.9
	17	-71.2	52.5	547.3	590.9	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
	18	-71.2	52.5	547.3	590.9	623.4	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
	19	-71.2	52.5	547.3	590.9	623.4	661.1	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
	20	-71.2	52.5	547.3	590.9	623.4	661.1	705.5	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0	155.0
	21	-1951.4	-1675.9	-574.7	-477.6	-405.3	-321.2	-222.4	-104.6	-49.2	13.4	84.5	166.0	152.7	139.1	125.4	111.5	108.8	106.1	103.3	100.5	97.7
	22	-1951.4	-1675.9	-574.7	-477.6	-405.3	-321.2	-222.4	-104.6	-49.2	13.4	84.5	166.0	152.7	139.1	125.4	111.5	108.8	106.1	103.3	100.5	97.7
	23	-1951.4	-1675.9	-574.7	-477.6	-405.3	-321.2	-222.4	-104.6	-49.2	13.4	84.4	166.0	152.7	139.1	125.4	111.5	108.8	106.1	103.3	100.5	97.7
	24	-76.5	134.4	977.7	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0	350.0
	25	5381.4	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0
	26	5381.3	8369.4	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0	400.0

TABLE 8. ITERATIVE PROCESS TO OBTAIN THE INCREMENTAL COST (MULTIPLE SETTING)

Iter		1	2	3	4
PX		9962.713	2676.328	.0	.0
PN		8212.713	2200.329	232.7313	.0
		8.32002	16.57642	21.01817	20.60868
	1	-54.44	-24.39	-8.22⇑	.00
	2	-54.44	-24.39	-8.22⇑	.00
	3	-54.44	-24.39	-8.22⇑	.00
	4	-54.44	-24.39	-8.22⇑	.00
	5	-54.44	-24.39	-8.22⇑	.00
	6	-98.68	-65.67	-47.91⇑	.00
	7	-98.68	-65.67	-47.91⇑	.00
	8	-98.68	-65.67	-47.91⇑	.00
	9	-98.68	-65.67	-47.91⇑	.00
	10	-169.06	195.87⇓	76.00	76.00
	11	-169.06	195.87⇓	76.00	76.00
	12	-169.06	195.87⇓	76.00	76.00
Generating unit	13	-169.06	195.87⇓	76.00	76.00
	14	-218.00	-30.58	70.24	60.95
	15	-218.00	-30.58	70.24	60.95
	16	-218.00	-30.58	70.24	60.95
	17	-71.19	547.29	155.00	155.00
	18	-71.19	547.29	155.00	155.00
	19	-71.19	547.29	155.00	155.00
	20	-71.19	547.29	155.00	155.00
	21	-1951.41	-574.66	166.00	97.72
	22	-1951.41	-574.66	166.00	97.72
	23	-1951.41	-574.66	166.00	97.72
	24	-76.55	977.69⇓	350.00	350.00
	25	5381.36⇓	400.00	400.00	400.00
	26	5381.36⇓	400.00	400.00	400.00

⇓Setting at upper limit, ⇑Setting at lower limit

When the 400 MW units are set to their maximum output, the next iteration is realized, giving a value of 16.5764. The second equation of the set of equations shown in (6)is verified by this increment in the value of ⋋ . This equation states that this unit must be at its upper limit if the incremental cost of the unit is lower than the incremental cost of the system. When some units are set at their lower limits, the ⋋ of the system decreases. An example is between iterations 3 and 4, where the ⋋ decreases from 21.01817 to 20.60868. The 12 and 20 M.W. units are set to their lower limits (0.0 MW). This reduction in ⋋ verifies the third equation of the set of equations making up equation (6). As seen from Figure 3, if a horizontal line is drawn at 20.60868 $/MWh, the incremental cost of these two units is greater than the λ of the system. Only units with a maximum capacity of 100 MW (units14-16) and 197 MW (units 21-23) can generate the optimal solution between their maximum and minimum limits.

The incremental cost curve of the IEEE-RTS is shown in Figure 4. The load variation is from zero to the maximum load supplied by the thermal generating system. It can be seen from this Figure that λ has a minimum that is equal to the incremental cost of its cheapest unit (400 MW-nuclear unit), as well as a maximum that is equal to the incremental cost of the most expensive unit (20 M.W. unit).

The following test is undertaken to measure the effect of taking out a unit for maintenance on the incremental cost. The result of this test is shown in Table 9, which shows that the λvalues are not affected when one of the two most expensive units (12 and 20 M.W.) is under maintenance – this is explained as follows. At the load levels considered, these units are always at their lower limits (0 M.W.). This condition is the same whether they are connected to the system or are under maintenance. The greatest effect is when one of the 400 MW units is under maintenance. The value of λ for a load of 2850 MW changes from 20.6087 to 22.1089 $/MWh.

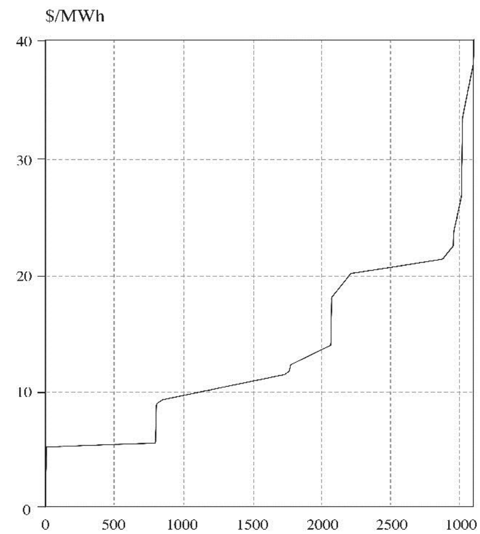

Figure 4. Incremental cost curve of the IEEE-RTS

TABLE 9. INCREMENTAL COSTS OF THE IEEE-RTS WITH UNITS UNDER MAINTENANCE

Load (M.W.)	Unit under maintenance (capacity in M.W.)							
	12	20	76	100	155	197	350	400
1650	11.1547	11.1547	11.1547	11.1547	12.3430	11.1547	13.4459	13.7287
1675	11.2132	11.2132	11.2132	11.2132	12.4844	11.2132	13.5873	17.9385
1700	11.2717	11.2717	11.2717	11.2717	12.6258	11.2717	13.7287	18.3056
1725	11.3302	11.3302	11.3302	11.3302	12.7672	11.3302	17.9385	18.6727
1750	11.5040	11.5040	11.5040	11.5040	12.9086	11.5040	18.3056	19.0398
1775	12.1733	12.1733	12.1827	12.1733	13.0500	12.1733	18.6727	19.4069
1800	12.3147	12.3147	12.3713	12.3147	13.1914	12.3147	19.0398	19.7740
1825	12.4561	12.4561	12.5598	12.4561	13.3328	12.4561	19.4069	20.0368
1850	12.5975	12.5975	12.7483	12.5975	13.4742	12.5975	19.7740	20.0808
1875	12.7389	12.7389	12.9369	12.7389	13.6156	12.7389	20.0368	20.1248
1900	12.8803	12.8803	13.1254	12.8803	13.7570	12.8803	20.0808	20.1688
1925	13.0217	13.0217	13.3140	13.0217	18.0119	13.0217	20.1248	20.2128
1950	13.1631	13.1631	13.5025	13.1631	18.3790	13.1631	20.1688	20.2568
1975	13.3045	13.3045	13.6910	13.3045	18.7461	13.3045	20.2128	20.3008
2000	13.4459	13.4459	17.9531	13.4459	19.1132	13.4459	20.2568	20.3448
2025	13.5873	13.5873	18.3203	13.5873	19.4804	13.5873	20.3008	20.3887
2050	13.7287	13.7287	18.6874	13.7287	19.8475	13.7287	20.3448	20.4327
2075	17.9385	17.9385	19.0545	17.9458	20.0456	17.9385	20.3887	20.4767
2100	18.3056	18.3056	19.4216	18.4965	20.0896	18.3056	20.4327	20.5207
2125	18.6727	18.6727	19.7887	19.0472	20.1336	18.6727	20.4767	20.5647
2150	19.0398	19.0398	20.0386	19.5978	20.1776	19.0398	20.5207	20.6087
2175	19.4069	19.4069	20.0826	20.0331	20.2216	19.4069	20.5647	20.6527
2200	19.7740	19.7740	20.1266	20.0789	20.2656	19.7740	20.6087	20.6967
2225	20.0368	20.0368	20.1706	20.1248	20.3096	20.0427	20.6527	20.7406
2250	20.0808	20.0808	20.2146	20.1706	20.3536	20.1050	20.6967	20.7846
2275	20.1248	20.1248	20.2585	20.2164	20.3975	20.1672	20.7406	20.8286
2300	20.1688	20.1688	20.3025	20.2622	20.4415	20.2295	20.7846	20.8726
2325	20.2128	20.2128	20.3465	20.3080	20.4855	20.2918	20.8286	20.9166
2350	20.2568	20.2568	20.3905	20.3538	20.5295	20.3540	20.8726	20.9606

2375	20.3008	20.3008	20.4345	20.3997	20.5735	20.4163	20.9166	21.0046
2400	20.3448	20.3448	20.4785	20.4455	20.6175	20.4785	20.9606	21.0486
2425	20.3887	20.3887	20.5225	20.4913	20.6615	20.5408	21.0046	21.0925
2450	20.4327	20.4327	20.5665	20.5371	20.7055	20.6030	21.0486	21.1365
2475	20.4767	20.4767	20.6104	20.5829	20.7494	20.6653	21.0925	21.1805
2500	20.5207	20.5207	20.6544	20.6287	20.7934	20.7275	21.1365	21.3747
2525	20.5647	20.5647	20.6984	20.6746	20.8374	20.7898	21.1805	21.7418
2550	20.6087	20.6087	20.7424	20.7204	20.8814	20.8520	21.3747	22.1089

7. CONCLUSION

This paper has explored the solution to the economic dispatch problem. This problem is described as minimizing fuel costs in supplying a given load, comprising a constraint optimization problem solved using the *LaGrange* function. The fuel cost function for each thermal generating unit was considered as a quadratics function. It was stated that the conditions for an extreme value of the objective function, considered the total thermal generating cost, result when one takes the first derivative of the function with respect to each of the independent variables and sets the derivatives equal to zero.

A solution methodology considering the minimum and maximum output capacity for each generating unit was described and tested. In addition, a solution method for the economic dispatch problem has been described in this paper. The input/output curves of the generators are considered as quadratic functions. A least-square estimation method is used to calculate the coefficients of these quadratic functions. These two methods have been implemented in a computational algorithm. A case study considering the IEEE-RTS was carried out. Results of this application were presented, with some of them being verified against those of the literature review, proving the computational algorithms' successful performance.

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