International Journal of Computational and Theoretical Statistics
ISSN (2210-1519)
Int. J. Comp. Theo. Stat. 7, No. 1 (May-2020)

http://dx.doi.org/10.12785/ijcts/070104

Exponential Type Estimator for Estimating Finite Population Mean

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Received December 22, 2019, Revised March 18, 2020, Accepted April 15, 2020, Published May 1, 2020

Abstract: In this paper, we suggest an exponential type estimator for estimating population mean under simple random sampling without replacement and also utilize this estimator for missing data under varied imputation techniques. Expressions for Bias and MSE's are acquired in the form of population parameters up to the first order of approximation. The empirical study in support of theoretical results is also included.

Keywords: Bias, Exponent type Estimator, Mean Square Error, SRSWOR.

1. INTRODUCTION

In the field of survey sampling, the auxiliary information plays an important role for improvement in the precision of the estimators. The use of auxiliary information at the estimation stage comes in sight with the work of Watson (1937), Cochran (1940), Robson (1957) and Murthy (1964). Behl and Tuteja (1991) proposed exponential ratio and product type estimator. Singh et al. (2007), Yadav and Kadilar (2013),Singh and Solanki (2013), Vishwakarma et al. (2014),Singh and Kumar (2011), Singh et al. (2016), Etuk et al. (2016), Singh et al. (2017), Khare et al. (2019) proposed improved estimators using auxiliary information.

Consider Ω (= Ω_1, Ω_2,…..,Ω_N) be the finite population of size N, whereby sample is drawn of size n under simple random sampling without replacement (SRSWOR) technique. Let $(\bar{y},\bar{x})$be the sample mean estimators of the population means $(\bar{Y},\bar{X})$ respectively.S_Y^2 and S_X^2 are the population variance of study and auxiliary variables separately.
In order to find the bias and mean square error (MSE) of proposed estimator, we consider the large sample approximation. Let,
$\bar{y}=\bar{Y}(1+e_0),\ \bar{x}=\bar{X}(1+e_1)$ where, $|e_i|<1 \quad (i=0,1)$

such that, $E(e_0)=E(e_1)=0$ and $E(e_0^2)=\theta C_Y^2, E(e_1^2)=\theta C_X^2, E(e_0e_1)=\theta\rho C_Y C_X$
where, $\theta=\left(\frac{1}{n}-\frac{1}{N}\right), C_Y^2=\frac{S_Y^2}{\bar{Y}^2}, C_X^2=\frac{S_X^2}{\bar{X}^2}$ and ρ is correlation coefficient between study variable y and auxiliary variable x.

2. ESTIMATORS IN LITERATURE:

In this section we discuss some of the estimators present in sampling literature.

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TABLE 1.0. EXISTING ESTIMATORS IN LITERATURE

S.no.	Estimators	Min. MSE
1	$T_u = \bar{y}$	$V(\bar{y}) = \theta\bar{Y}^2C_Y^2$
2	$T_r = \bar{y}\frac{\bar{X}}{\bar{x}}$,Cochran (1940)	$MSE(T_r) = \theta\bar{Y}^2(C_Y^2 + C_X^2 - 2\rho C_Y C_X)$
3	$T_r = \bar{y}\frac{\bar{x}}{\bar{X}}$, Robson (1957) and Murthy (1964)	$MSE(T_p) = \theta\bar{Y}^2(C_Y^2 + C_X^2 + 2\rho C_Y C_X)$
4	$T_{reg} = \bar{y} + b(\bar{X} - \bar{x})$ Watson (1937)	$MSE(T_{reg}) = \theta\bar{Y}^2C_Y^2(1 - \rho^2)$
5	$T_{er} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ Behl and Tuteja (1991)	$MSE(T_{er}) = \theta\bar{Y}^2\left(C_Y^2 + C_X^2\left(\frac{1}{4} - \frac{\rho C_Y}{C_X}\right)\right)$
6	$T_{rpn} = \bar{y}\left[\alpha\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + (1-\alpha)\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)\right]$,Singh et. al (2008)	$MSE(T_{rpn}) = \theta\bar{Y}^2C_Y^2(1 - \rho^2)$

3.PROPOSED ESTIMATOR

We propose the following estimator,

$$\hat{\theta}_{PExpi} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{\alpha(\bar{X} - \bar{x})}{\alpha(\bar{X} + \bar{x}) + 2\beta}\right) \qquad (i = 1,2,3,4) \qquad (1)$$

Under the large sample approximation, (1) is defined as,

$$\hat{\theta}_{PExpi} = \bar{Y}\left((w_1 + w_2)\left(1 + e_0 - \lambda e_1 + \frac{3}{2}\lambda^2 e_1^2 - \lambda e_0 e_1\right) + w_2\left((1+\lambda)e_1^2 - e_1 - e_0 e_1\right)\right)$$

$$\text{Where, } \lambda = \frac{\alpha\bar{X}}{2(\alpha\bar{X} + \beta)}$$

$$\hat{\theta}_{PExpi} - \bar{Y} = \bar{Y}\left((w_1 + w_2 - 1) + (w_1 + w_2)\left(e_0 - \lambda e_1 + \frac{3}{2}\lambda^2 e_1^2 - \lambda e_0 e_1\right) + w_2\left((1+\lambda)e_1^2 - e_1 - e_0 e_1\right)\right) \qquad (2)$$

Now, taking expectation both the sides of (2), we get the bias of proposed estimator $\hat{\theta}_{PExp}$ as,

$$Bias(\hat{\theta}_{PExpi}) = \bar{Y}\left((w_1 + w_2 - 1) + (w_1 + w_2)\theta\left(\frac{3}{2}\lambda^2 C_X^2 - \lambda C_{YX}\right) + w_2\theta\left((1+\lambda)C_X^2 - C_{YX}\right)\right) \qquad (3)$$

Square and take expectation both the sides of (2) ,we get the mean square error (MSE) of proposed estimator $\hat{\theta}_{PExp}$ as,

$$MSE(\hat{\theta}_{PExpi}) = \bar{Y}^2(1 + w_1^2 A + w_2^2 B - 2w_1 C - 2w_2 D + 2w_1 w_2 E) \qquad (4)$$

Where,

$$A = 1 + \theta(C_Y^2 + 4\lambda^2 C_X^2 - 4\lambda C_{YX}), \qquad B = 1 + \theta(C_Y^2 + (4\lambda^2 + 4\lambda + 3)C_X^2 - 4(\lambda + 1)C_{YX})$$

$$C = 1 + \theta\left(\frac{3}{2}\lambda^2 C_X^2 - \lambda C_{YX}\right), \quad D = 1 + \theta\left(\left(\frac{3}{2}\lambda^2 + \lambda + 1\right)C_X^2 - (\lambda + 1)C_{YX}\right)$$

$$E = 1 + \theta(C_Y^2 + (4\lambda^2 + 2\lambda + 1)C_X^2 - 2(2\lambda + 1)C_{YX})$$

To obtain the expression for the optimum value of w_1 and w_2 we partially differentiate $MSE(\hat{\theta}_{PExp})$ with respect to w_1 and w_2 and then equate the results to zero as;

$$w_1 = \frac{C - w_2 E}{A} \quad \text{and } w_2 = \frac{D - w_1 E}{B}$$

Further, the expressions for optimum values of $w_i, (i = 1,2)$denoted by w_i^{opt}(i=1,2) are obtained as,

$$w_1^{opt} = \frac{BC - DE}{AB - E^2} \text{ and } w_2^{opt} = \frac{AD - CE}{AB - E^2}$$

Substituting optimum values of w_1^{opt} and w_2^{opt} in (4), we get the minimum MSE of proposed estimator $\hat{\theta}_{PExp}$ denoted by $MSE(\hat{\theta}_{PExp})_{min}$ as,

$$\mathrm{MSE}\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{\min} = \bar{Y}^2\left(1-\frac{(BC^2+AD^2-2CDE)}{\mathrm{AB}-\mathrm{E}^2}\right) \quad (i=1,2,3,4) \tag{5}$$

TABLE 2.0 FAMILY OF $\hat{\theta}_{\mathrm{Pr\,Expi}}$ FOR DISTINCT CHOICE OF α AND β

α	β	Estimators
1	-1	$\hat{\theta}_{\mathrm{PExp1}} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})-2}\right)$
1	0	$\hat{\theta}_{\mathrm{PExp1}} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})-2}\right)$
1	1	$\hat{\theta}_{\mathrm{PExp3}} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2}\right)$
0	1	$\hat{\theta}_{\mathrm{PExp4}} = \bar{y}\left(w_1 + w_2\frac{\bar{X}}{\bar{x}}\right)$

4. EFFICIENCY COMPARISONS

In this section we compare the efficiency of proposed estimator with respect to other estimators.

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_u), (i=1,2,3,4)$$
$$\theta C_Y^2 - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{6}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_r), (i=1,2,3,4)$$
$$\theta(C_Y^2 + C_X^2 - 2\rho C_Y C_X) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{7}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_p), (i=1,2,3,4)$$
$$\theta(C_Y^2 + C_X^2 + 2\rho C_Y C_X) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{8}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_{reg}), (i=1,2,3,4)$$
$$\theta C_Y^2(1-\rho^2) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{9}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_{er}), (i=1,2,3,4)$$
$$\theta\left(C_Y^2 + C_X^2\left(\frac{1}{4}-\frac{\rho C_Y}{C_X}\right)\right) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{10}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_{ep}), (i=1,2,3,4)$$
$$\theta\left(C_Y^2 + C_X^2\left(\frac{1}{4}+\frac{\rho C_Y}{C_X}\right)\right) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{11}$$

$$MSE\left(\hat{\theta}_{\mathrm{PExpi}}\right)_{min} < Var(T_{rpn}), (i=1,2,3,4)$$
$$\theta C_Y^2(1-\rho^2) - \left[\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \geq 0 \tag{12}$$

From (6) to (12), we observe that the conditions cannot be obtained in explicit form. So, the conditions can be verified with help of numerical illustrations.

5. EMPIRICAL STUDY

For numerical study, we use three population data sets.

Population1: [Source: Murthy (1967, p. 228)]
Y: Output and X: Number of workers.
$\bar{Y} = 5182.64, \bar{X} = 283.875, C_Y = 0.352, C_X = 0.943, \rho_{yx} = 0.9136, N = 923, n = 180$

Population2:[Source: Koyuncu and Kadilar (2009)]
Y: Number of teachers in both primary and secondary schools,
X: Number of students in both primary and secondary schools.
$\bar{Y} = 436.4345, \bar{X} = 11440.4984, C_Y = 1.7183, C_X = 1.8645, \rho_{yx} = -0.9199, N = 923, n = 180$

Population 3:[Source: Khoshnevisan et al. (2007)]
$\bar{Y} = 19.55, \bar{X} = 18.8, C_Y = 0.3552, C_X = 0.3943, \rho_{yx} = -0.91990, N = 20, n = 8$

Percent relative efficiency (PRE) of the estimators is calculated by using the formula,

$$PRE(\blacksquare, \bar{y}) = \frac{V(\bar{y})}{MSE(\blacksquare)} * 100$$

TABLE 1.2. PREs OF VARIOUS ESTIMATORS UNDER SRSWOR:

Estimators	Population 1	Population 2	Population 3
T_u	100.00	100.00	100.00
T_r	30.472	939.708	23.395
T_p	7.650	23.538	526.498
$T_{reg.}$	604.832	1119.676	650.263
T_{er}	288.421	386.3150	42.9340
T_{ep}	19.0776	42.9213	348.534
T_{rpn}	604.832	1119.676	650.263
P1	606.137	1133.802	656.375
P2	606.154	1133.800	656.003
P3	606.171	1133.798	655.704
P4	649.690	1127.285	654.596

6. RESULTS

From Table 1.2, it is evident that families of estimators generated from distinct values of α and β are more efficient than $T_u, T_r, T_p, T_{reg}, T_{er}, T_{ep}$ and T_{rpn}. among the proposed families of $\hat{\theta}_{PrExpi}$ $(i = 1,2,3,4)$ it is seen that $\hat{\theta}_{PrExp1}, \hat{\theta}_{PrExp2}, \hat{\theta}_{PrExp3}$ are uniformly efficient than other estimators. Whereas, $\hat{\theta}_{PrExp4}$ is most efficient among all. Hence, the family of proposed estimators is suggested for use in obtaining greater efficiency.

ACKNOWLEDGMENT

The authors are thankful to learned referee(s) for their valuable comments and suggestions that helped in the improvement and refinement of the present manuscript.

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