International Journal of Computational and Theoretical Statistics
ISSN (2210-1519)
Int. J. Comp. Theo. Stat. 6, No. 2 (Nov-2019)

http://dx.doi.org/10.12785/ijcts/060203

GMM Estimation of AR(1) Time Series Model with One Additional Regressor

B F Chakalabbi[1], Sanmati Neregal[1] and Sagar Matur[1]

[1]*Department of Statistics, Karnatak University's Karnatak Arts College, Dharwad – 580001, India*

Received July 7, 2018, Revised March 10, 2019, Accepted May 3, 2019, Published November 1, 2019

Abstract: GMM estimators properties for panel data have been very well known in the econometric literature and it has been observed that for small sample cases, they perform well. The OLS (Ordinary Least Squares) is not applicable when lagged endogenous and exogenous variables are correlated with the error term. Hence, here an attempt is made to estimate AR(1) time series model with one additional regressor by considering First-difference GMM and Level GMM estimation methods proposed by Arellano and Bond (1991) and Arellano and Bover (1995) respectively. In order study the performances of the above mentioned estimators in comparison with the OLS estimator Monte Carlo simulation study is carried out. Further, a comparison among these estimators has been done in terms of bias and RMSE. Study disclose that for an autoregressive parameter, Level GMM estimator performs better than First-difference GMM and OLS estimators when T, the sample size is small and ρ, the autoregressive parameter is close to unity. Whereas for the parameter of additional regressor β, Level GMM estimator performs better than the other two mentioned estimators for all the values of ρ and T.

Keywords: AR(1) with additional regressor, First-difference GMM, Level GMM, Bias, RMSE, OLS, Monte-Carlo simulation.

1. INTRODUCTION

An enormous amount of research has been done on estimation of the first order autoregressive (AR(1)) time series model (see [3], [6], [2], [9], [11], [5] and [12]). In our previous work [1], we have applied two GMM estimation methods for AR(1) time series model to examine the performance of the mentioned estimators in comparison with the OLS estimator. In this study, the first order autoregressive time series model with additional regressor is considered. To estimate the interested model, three different estimators are considered namely, Ordinary Least Squares (OLS) used for the estimation of AR(1) model [4], one-step first-difference GMM estimator is proposed by Arellano and Bond [7] and level GMM estimator proposed by Arellano and Bover [8].

For the above two GMM estimators, two distinct cases are taken into account. In the first case, the additional regressor is correlated with the white noise error term. This paper analyzes through Monte-Carlo simulation results, where the simulation design is analogous to that of Soto [10] and the results are appropriate only for the estimators are considered in the first case, where the additional regressor is correlated with the white noise error term. The bias and RMSE of the aforementioned two estimators are compared together with OLS estimator.

The article is arranged as follows. Section 2 contains an Autoregressive model with additional regressor, assumptions and two GMM estimators. Section 3 encompasses Monte-Carlo simulation to investigate the performances of the stated estimators. Section 4 presents discussion and results. The last section concludes the paper.

2. THE MODEL AND ESTIMATORS

The first-order autoregressive model with additional regressor is given by,

$$y_t = \alpha + \rho\, y_{t-1} + \beta x_t + u_t, \qquad t = 2, 3, \ldots, T. \tag{1}$$

$$x_t = \alpha + \delta x_{t-1} + \theta u_t + e_t, \qquad t = 2, 3, \ldots, T. \tag{2}$$

E-mail address: bfckcd@gmail.com , naregalsanmati@gmail.com , matursagar@gmail.com

where α is a constant, ρ and δ are autoregressive parameters with $|\rho| < 1$ and $|\delta| < 1$, β is the coefficient of additional regressor x_t, T is the time period, u_t and e_t are the disturbances with the following assumptions.

Assumption 1: $\{u_t\}: iid(0, \sigma_u^2)$
Assumption 1: $\{e_t\}: iid(0, \sigma_e^2)$
Assumption 1: $\{u_t\}$ and $\{e_t\}$ are independent of each other.

On the basis of the above three assumptions, we take into consideration two types of estimation methods first one, First-difference GMM estimation method and the second one, Level GMM estimation method.

A. First-Difference GMM Estimation

In the model (1), the constant α leads to a correlation between the lagged endogenous variable y_{t-1} and error term u_t with the additional assumption of no correlation between x_t and u_t. The first differences of the model (1) is performed to eliminate the constant to meet out the orthogonality condition.

By, first-differencing model (1), we obtain

$$\Delta y_t = \rho\, \Delta y_{t-1} + \beta\, \Delta x_t + \Delta u_t, \qquad t = 3, \dots, T. \tag{3}$$

Case 1: If $E(x_t u_t) \neq 0$, the case when the additional regressor x_t is correlated with the white noise error term u_t, the one-step first-difference GMM estimator is based on the below $2(T-2)$ moment conditions,

$$E(Z'_{d1}\, \Delta u_t) = 0 \tag{4}$$

where, Z_{d1} is a $(T-2) \times 2(T-2)$ instrumental matrix and Δu_t is a $(T-2) \times 1$ vector.

$$Z_{d1} = \begin{bmatrix} y_1 & x_2 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & y_2 & x_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & y_{T-2} & x_{T-1} \end{bmatrix}, \quad \Delta u_t = \begin{bmatrix} \Delta u_3 \\ \Delta u_4 \\ \vdots \\ \Delta u_T \end{bmatrix}$$

With reference to the moment condition (4), the criterion function for the one-step first-difference GMM estimator is given by,

$$Q_{dif1} = \Delta u_t' Z_{d1} W_{d1}^{-1} Z_{d1}' \Delta u_t \tag{5}$$

By minimizing the criterion function (5) w.r.t $[\rho_{dif1}\ \beta_{dif1}]'$, the one-step first-difference GMM estimator is obtained and is as follows,

$$\begin{pmatrix} \hat{\rho} \\ \hat{\beta} \end{pmatrix}_{dif1} = \left\{ \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix} Z_{d1} W_{d1}^{-1} Z_{d1}' \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix}' \right\}^{-1} \left\{ \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix} Z_{d1} W_{d1}^{-1} Z_{d1}' \Delta y_t \right\} \tag{6}$$

where, $\begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix}$ is $2(T-2)$ matrix $(\Delta y_2, \Delta x_3, \Delta y_3, \Delta x_4, \dots, \Delta y_{T-1}, \Delta x_T)$, $W_{d1} = Z_{d1}' G_d Z_{d1}$ is a $2(T-2) \times 2(T-2)$ weight matrix with G_d is same as H in the estimator proposed by Arellano and Bond [7].

Case 2: If $E(x_t u_t) = 0$, the additional regressor x_t is not correlated with the white noise error term u_t, the one-step first-difference GMM estimator is based on the below $(T-1)$ moment conditions,

$$E(Z'_{d2}\, \Delta u_t) = 0 \tag{7}$$

where, Z_{d2} is a $(T-2) \times (T-1)$ instrumental matrix and Δu_t is a $(T-2) \times 1$ vector.

$$Z_{d2} = \begin{bmatrix} y_1 & 0 & \dots & 0 & \Delta x_3 \\ 0 & y_2 & \dots & 0 & \Delta x_4 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & y_{T-2} & \Delta x_T \end{bmatrix}, \quad \Delta u_t = \begin{bmatrix} \Delta u_3 \\ \Delta u_4 \\ \vdots \\ \Delta u_T \end{bmatrix}$$

In accordance with the moment condition (7), the criterion function for the one-step first-difference GMM estimator is given by,

$$Q_{dif2} = \Delta u_t' Z_{d2} W_{d2}^{-1} Z_{d2}' \Delta u_t \quad (8)$$

By minimizing the criterion function (8) w.r.t $[\rho_{dif2} \;\; \beta_{dif2}]'$, the obtained one-step first-difference GMM estimator is presented below,

$$\begin{pmatrix} \hat{\rho} \\ \hat{\beta} \end{pmatrix}_{dif2} = \left\{ \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix} Z_{d2} W_{d2}^{-1} Z_{d2}' \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix}' \right\}^{-1} \left\{ \begin{pmatrix} \Delta y_{t-1} \\ \Delta x_t \end{pmatrix} Z_{d2} W_{d2}^{-1} Z_{d2}' \Delta y_t \right\} \quad (9)$$

where, $W_{d2} = Z_{d2}' G_d Z_{d2}$ is a $(T-1) \times (T-1)$ weight matrix.

B. Level GMM Estimation

In the manner of Arellano and Bover [8], to comply with the orthogonality condition, the constant α is wiped out from the instrumental variable.

Case 1: If $E(x_t u_t) \neq 0$, the additional regressor x_t is correlated with the white noise error term u_t, based on the following $2(T-2)$ moment conditions the one-step level GMM estimator is written as,

$$E(Z_{l1}' u_t) = 0 \quad (10)$$

where, Z_{l1} is a $(T-2) \times 2(T-2)$ instrumental matrix and u_t is a $(T-2) \times 1$ vector.

$$Z_{l1} = \begin{bmatrix} \Delta y_2 & \Delta x_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \Delta y_3 & \Delta x_4 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \Delta y_{T-1} & \Delta x_T \end{bmatrix}, \quad u_t = \begin{bmatrix} u_3 \\ u_4 \\ \vdots \\ u_T \end{bmatrix}$$

The criterion function for the one-step level GMM estimator is based on the moment conditions (10) becomes,

$$Q_{lev1} = u_t' Z_{l1} W_{l1}^{-1} Z_{l1}' u_t \quad (11)$$

By minimizing the criterion function (11) w.r.t $[\rho_{lev1} \;\; \beta_{lev1}]'$, the one-step level GMM estimator is obtained and is as follows,

$$\begin{pmatrix} \hat{\rho} \\ \hat{\beta} \end{pmatrix}_{lev1} = \left\{ \begin{pmatrix} y_{t-1} \\ x_t \end{pmatrix} Z_{l1} W_{l1}^{-1} Z_{l1}' \begin{pmatrix} y_{t-1} \\ x_t \end{pmatrix}' \right\}^{-1} \left\{ \begin{pmatrix} y_{t-1} \\ x_t \end{pmatrix} Z_{l1} W_{l1}^{-1} Z_{l1}' y_t \right\} \quad (12)$$

where, $\begin{pmatrix} y_{t-1} \\ x_t \end{pmatrix}$ is $2(T-2)$ matrix $(y_2, x_3, y_3, x_4, \dots, y_{T-1}, x_T)$, $W_{l1} = Z_{l1}' G_l Z_{l1}$ is a $2(T-2) \times 2(T-2)$ weight matrix with G_l is a $(T-2) \times (T-2)$ identity matrix.

Case 2: If $E(x_t u_t) = 0$, the case when the additional regressor x_t is not correlated with the white noise error term u_t, the one-step level GMM estimator is based on the below $(T-1)$ moment conditions,

$$E(Z_{l2}' u_t) = 0 \quad (13)$$

where, Z_{l2} is a $(T-2) \times (T-1)$ instrumental matrix and u_t is a $(T-2) \times 1$ vector.

$$Z_{l2} = \begin{bmatrix} y_2 & 0 & \dots & 0 & x_3 \\ 0 & y_3 & \dots & 0 & x_4 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & y_{T-1} & x_T \end{bmatrix}, \quad u_t = \begin{bmatrix} u_3 \\ u_4 \\ \vdots \\ u_T \end{bmatrix}$$

Based on the moment conditions (13), the criterion function for the one-step level GMM estimator is given by,

$$Q_{dif2} = u_t' Z_{l2} W_{l2}^{-1} Z_{l2}' u_t \quad (14)$$

By minimizing the criterion function (14) w.r.t $[\rho_{lev2} \;\; \beta_{lev2}]'$, the obtained one-step level GMM estimator is given by,

$$\begin{pmatrix}\hat{\rho}\\ \hat{\beta}\end{pmatrix}_{lev2} = \left\{\begin{pmatrix}y_{t-1}\\ x_t\end{pmatrix} Z_{l2} W_{l2}^{-1} Z_{l2}' \begin{pmatrix}y_{t-1}\\ x_t\end{pmatrix}'\right\}^{-1} \left\{\begin{pmatrix}y_{t-1}\\ x_t\end{pmatrix} Z_{l2} W_{l2}^{-1} Z_{l2}' y_t\right\} \quad (15)$$

where, $W_{l2} = Z_{l2}' G_l Z_{l2}$ is a $(T-1)\times(T-1)$ weight matrix.

3. MONTE CARLO SIMULATION DESIGN

In this simulation design, the data generating process has been done from the following AR(1) model with one additional regressor to explore the finite sample performance of the above stated two estimators.

$$y_t = \alpha + \rho\, y_{t-1} + \beta x_t + u_t, \qquad t = 2, 3, \dots, T.$$

$$x_t = \alpha + \delta x_{t-1} + \theta u_t + e_t, \qquad t = 2, 3, \dots, T.$$

The initial observations of y_t and x_t are given by,

$$y_1 = \frac{(1-\delta+\beta)}{(1-\rho)(1-\delta)} + \beta\theta r_1 + \beta s_1 + v_1$$

and

$$x_1 = \frac{\alpha}{1-\delta} + \theta p_1 + q_1$$

where,

$u_t \sim N(0, \sigma_u^2);\ e_t \sim N(0, \sigma_e^2);\ q_1 \sim N(0, \sigma_p^2);\ q_1 \sim N(0, \sigma_q^2)$ and $v_1 \sim N(0, \sigma_v^2)$.

The variance of white noise error term $\sigma_u^2 = 1$, we set $\rho = \delta \in (0,1), \alpha = \beta = 1, \theta = -0.1$, $\sigma_e^2 = 0.16$, $\sigma_v^2 = \frac{\sigma_u^2}{1-\rho^2}, \sigma_p^2 = \frac{\sigma_u^2}{1-\delta^2}, \sigma_q^2 = \frac{\sigma_e^2}{1-\delta^2}, \sigma_r^2 = \sigma_u^2(1-\phi_2)/[(1+\phi_2)((1-\phi_2)^2-\phi_1^2)]$ and $\sigma_s^2 = \sigma_e^2(1-\phi_2)/[(1+\phi_2)((1-\phi_2)^2-\phi_1^2)]$ with $\phi_1 = \rho+\delta$, $\phi_2 = -\rho\delta$ and for the sample size, we choose $T = 5, 10, 20, 50$ and 75. The number of replication is 10000 for all cases.

4. DISCUSSION OF THE RESULTS

The results obtained from the simulation study are explained through the tables and graphs. Table 1 gives the values of bias, RMSE and estimated values of an autoregressive parameter (ρ) through first-difference GMM (dif), level GMM (lev) and OLS estimators (ols). When $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) are compared, it is observed that, $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) have an identical bias for all the values of ρ over the range $T = 5$ to 75. In the case of $T = 5$, the bias of $\hat{\rho}$ (lev) is the smallest among three estimators. When $T = 10$ and 20, $\hat{\rho}$ (lev) has less bias than $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) for the values of $\rho > 0.3$and $\rho > 0.6$ respectively. When the sample sizes are 50 and 75, $\hat{\rho}$ (lev) has more bias than $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) for all the values of ρ. In other words, when the sample size is too small and ρ is very close to unity, $\hat{\rho}$ (lev) is more preferable to the other two with respect to the bias.

Pertaining to the RMSE, when $T = 5, \hat{\rho}$ (lev) has the smallest RMSE among the above stated estimators excluding at the values of $\rho = 0.3$ and 0.5. For $T = 10$, $\hat{\rho}$ (lev) has less RMSE except at the values of $\rho = 0.1$. For the cases $T = 20, 50$ and 75, the RMSE of $\hat{\rho}$ (lev) is smaller than the RMSE of $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) except at the values of $\rho < 0.5, \rho < 0.8$ and $\rho < 0.9$ respectively.

Table 2 shows the values of bias, RMSE and the estimated values of parameter of an additional refressor (β) through first-difference GMM, level GMM and OLS estimators. Upon comparison of three estimated values namely $\hat{\beta}$ (dif), $\hat{\beta}$ (lev) and $\hat{\beta}$ (ols), it is found that $\hat{\beta}$ (dif) and $\hat{\beta}$ (ols) have equal bias for all the considered ρ values over the entire range of T. When $T = 5$, bias of $\hat{\beta}$ (lev) is the smallest among mentioned three estimators except at the values of $\rho = 0.1$ and 0.3. when the sample sizes are $20, 50$ and 75, bias of $\hat{\beta}$ (lev) is smallest for all the values of ρ. More understandably, for all the values of T and $\rho, \hat{\beta}$ (lev) has the least bias compared to the remaining estimators.

The simulation results are summarized in figure $1-4$. From all the graphs it is noticed that, in the matter of bias and RMSE, $\hat{\rho}$ (dif), $\hat{\rho}$ (ols) and $\hat{\beta}$ (dif), $\hat{\beta}$ (ols) perform equally for all the considered range of T and ρ. Figure 1

TABLE1. SIMULATION RESULTS OF COMPARISON OF THE BIAS AND RMSE OF $\hat{\rho}$ (DIF), $\hat{\rho}$ (LEV) AND $\hat{\rho}$ (OLS) (T= 5, 10, 20 50 AND 75)

T		ρ								
		0.1	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
5	$\hat{\rho}$ (dif)	-0.248	-0.045	-0.104	-0.232	-0.03	0.099	0.126	0.184	0.391
	$\hat{\rho}$ (lev)	0.168	0.28	0.324	0.383	0.455	0.622	0.691	0.783	0.914
	$\hat{\rho}$ (ols)	-0.248	-0.045	-0.104	-0.232	-0.03	0.099	0.126	0.184	0.391
	Bias(dif)	-0.348	-0.245	-0.404	-0.632	-0.53	-0.501	-0.574	-0.616	-0.509
	BIas(lev)	0.068	0.08	0.024	-0.017	-0.045	0.022	-0.009	-0.017	0.014
	Bias(ols)	-0.348	-0.245	-0.404	-0.632	-0.53	-0.501	-0.574	-0.616	-0.509
	RMSE(dif)	1.73	1.526	1.903	3.688	1.267	2.42	1.464	2.787	1.368
	RMSE(lev)	1.401	1.227	2.07	1.374	1.31	1.036	0.929	0.948	0.453
	RMSE(ols)	1.73	1.526	1.903	3.688	1.267	2.42	1.464	2.787	1.368
10	$\hat{\rho}$ (dif)	-0.052	0.016	0.103	0.155	0.233	0.334	0.436	0.522	0.679
	$\hat{\rho}$ (lev)	0.326	0.385	0.451	0.539	0.609	0.678	0.757	0.839	0.925
	$\hat{\rho}$ (ols)	-0.052	0.016	0.103	0.155	0.233	0.334	0.436	0.522	0.679
	Bias(dif)	-0.152	-0.184	-0.197	-0.245	-0.267	-0.266	-0.264	-0.278	-0.221
	BIas(lev)	0.226	0.185	0.151	0.139	0.109	0.078	0.057	0.039	0.025
	Bias(ols)	-0.152	0.184	-0.197	-0.245	-0.267	-0.266	-0.264	-0.278	-0.221
	RMSE(dif)	0.38	0.395	0.408	0.43	0.457	0.458	0.444	0.469	0.392
	RMSE(lev)	0.426	0.378	0.363	0.323	0.303	0.255	0.195	0.16	0.096
	RMSE(ols)	0.38	0.395	0.408	0.43	0.457	0.458	0.444	0.469	0.392
20	$\hat{\rho}$ (dif)	0.032	0.125	0.218	0.3	0.38	0.476	0.579	0.687	0.821
	$\hat{\rho}$ (lev)	0.359	0.426	0.515	0.579	0.649	0.717	0.784	0.852	0.917
	$\hat{\rho}$ (ols)	0.032	0.125	0.218	0.3	0.38	0.476	0.579	0.687	0.821
	Bias(dif)	-0.068	-0.075	-0.082	-0.1	-0.12	-0.124	-0.121	-0.113	-0.079
	BIas(lev)	0.259	0.226	0.215	0.179	0.149	0.117	0.084	0.052	0.017
	Bias(ols)	-0.068	-0.075	-0.082	-0.1	-0.12	-0.124	-0.121	-0.113	-0.079
	RMSE(dif)	0.238	0.232	0.236	0.238	0.247	0.246	0.227	0.209	0.161
	RMSE(lev)	0.332	0.294	0.283	0.241	0.206	0.171	0.132	0.092	0.042
	RMSE(ols)	0.238	0.232	0.236	0.238	0.247	0.246	0.227	0.209	0.161
50	$\hat{\rho}$ (dif)	0.079	0.173	0.268	0.371	0.474	0.569	0.679	0.787	0.892
	$\hat{\rho}$ (lev)	0.383	0.455	0.533	0.607	0.68	0.744	0.805	0.861	0.921
	$\hat{\rho}$ (ols)	0.079	0.173	0.268	0.371	0.474	0.569	0.679	0.787	0.892
	Bias(dif)	-0.021	-0.027	-0.032	-0.029	-0.026	-0.031	-0.021	-0.013	-0.008
	BIas(lev)	0.283	0.255	0.233	0.207	0.18	0.144	0.105	0.061	0.021
	Bias(ols)	-0.021	-0.027	-0.032	-0.029	-0.026	-0.031	-0.021	-0.013	-0.008
	RMSE(dif)	0.144	0.143	0.135	0.133	0.127	0.117	0.1	0.077	0.046
	RMSE(lev)	0.308	0.282	0.255	0.227	0.199	0.16	0.117	0.071	0.028
	RMSE(ols)	0.144	0.143	0.135	0.133	0.127	0.117	0.1	0.077	0.046

T		ρ								
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
75	$\hat{\rho}$ (dif)	0.084	0.185	0.279	0.384	0.489	0.592	0.698	0.801	0.903
	$\hat{\rho}$ (lev)	0.381	0.461	0.537	0.612	0.685	0.746	0.807	0.865	0.922
	$\hat{\rho}$ (ols)	0.084	0.185	0.279	0.384	0.489	0.592	0.698	0.801	0.903
	Bias(dif)	-0.016	-0.015	-0.021	-0.016	-0.011	-0.008	-0.002	0.001	0.003
	BIas(lev)	0.281	0.261	0.237	0.212	0.185	0.146	0.107	0.065	0.022
	Bias(ols)	-0.016	-0.015	-0.021	-0.016	-0.011	-0.008	-0.002	0.001	0.003
	RMSE(dif)	0.115	0.108	0.109	0.106	0.097	0.086	0.069	0.053	0.028
	RMSE(lev)	0.299	0.277	0.252	0.225	0.195	0.155	0.115	0.071	0.026
	RMSE(ols)	0.115	0.108	0.109	0.106	0.097	0.086	0.069	0.053	0.028

$\hat{\rho}$ (dif) = First-difference GMM estimator, $\hat{\rho}$ (lev) = Level GMM estimator, and $\hat{\rho}$ (ols) = Ordinary Least Square estimator, Bias(dif) = Bias of First-difference GMM estimator, Bias(lev) = Bias of Level GMM estimator, Bias(ols) = Bias of OLS estimator, RMSE = Root Mean Square Error, RMSE(dif) = RMSE of First-difference GMM estimator, RMSE(lev) = RMSE of Level GMM estimator, RMSE(ols) = RMSE of OLS estimator

TABLE2. SIMULATION RESULTS OF COMPARISON OF THE BIAS AND RMSE OF $\hat{\beta}$ (DIF), $\hat{\beta}$ (LEV) AND $\hat{\beta}$ (OLS) (T= 5, 10, 20 50 AND 75)

T		$\beta = 1$								
		ρ								
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
5	$\hat{\rho}$ (dif)	0.649	0.349	0.488	0.374	0.345	0.596	0.506	0.275	0.393
	$\hat{\rho}$ (lev)	1.651	1.546	1.581	1.618	1.538	1.283	1.334	1.268	0.955
	$\hat{\rho}$ (ols)	0.649	0.349	0.488	0.374	0.345	0.596	0.506	0.275	0.393
	Bias(dif)	-0.351	-0.651	-0.512	-0.626	-0.655	-0.404	-0.494	-0.725	-0.607
	BIas(lev)	0.651	0.546	0.581	0.618	0.538	0.283	0.334	0.268	-0.045
	Bias(ols)	-0.351	-0.651	-0.512	-0.626	-0.655	-0.404	-0.494	-0.725	-0.607
	RMSE(dif)	5.929	5.546	5.906	6.861	4.596	4.299	6.67	6.735	4.72
	RMSE(lev)	3.157	2.801	5.281	3.705	3.714	3.515	4.113	5.347	5.104
	RMSE(ols)	5.929	5.546	5.906	6.861	4.596	4.299	6.67	6.735	4.72
10	$\hat{\rho}$ (dif)	0.424	0.434	0.426	0.423	0.497	0.581	0.555	0.605	0.504
	$\hat{\rho}$ (lev)	1.321	1.297	1.273	1.174	1.145	1.118	1.047	0.96	0.824
	$\hat{\rho}$ (ols)	0.424	0.434	0.426	0.423	0.497	0.581	0.555	0.605	0.504
	Bias(dif)	-0.576	-0.566	-0.574	-0.577	-0.503	-0.419	-0.445	-0.395	-0.496
	BIas(lev)	0.321	0.297	0.273	0.174	0.145	0.118	0.047	-0.04	-0.176
	Bias(ols)	-0.576	-0.566	-0.574	-0.577	-0.503	-0.857	-0.445	-0.395	-0.496
	RMSE(dif)	1.251	1.188	1.281	1.23	1.224	1.19	1.2	1.231	1.305
	RMSE(lev)	0.817	0.8	0.838	0.78	0.853	0.857	0.807	0.922	1.021
	RMSE(ols)	1.251	1.188	1.281	1.23	1.224	1.19	1.2	1.231	1.305
20	$\hat{\rho}$ (dif)	0.417	0.448	0.479	0.508	0.572	0.609	0.653	0.737	0.791
	$\hat{\rho}$ (lev)	1.243	1.2	1.121	1.083	1.027	0.977	0.929	0.887	0.914
	$\hat{\rho}$ (ols)	0.417	0.448	0.479	0.508	0.572	0.609	0.653	0.737	0.791
	Bias(dif)	-0.583	-0.552	-0.521	-0.492	-0.428	-0.391	-0.347	-0.263	-0.209
	BIas(lev)	0.243	0.2	0.121	0.083	0.027	-0.023	-0.071	-0.113	-0.086
	Bias(ols)	-0.583	-0.552	-0.521	-0.492	-0.428	-0.391	-0.347	-0.263	-0.209

	RMSE(dif)	0.833	0.836	0.779	0.791	0.745	0.719	0.687	0.627	0.576
	RMSE(lev)	0.498	0.461	0.457	0.439	0.425	0.433	0.443	0.472	0.429
	RMSE(ols)	0.833	0.836	0.779	0.791	0.745	0.719	0.687	0.627	0.576
50	$\hat{\rho}$ (dif)	0.396	0.433	0.458	0.507	0.549	0.602	0.673	0.772	0.853
	$\hat{\rho}$ (lev)	1.196	1.143	1.075	1.008	0.937	0.881	0.837	0.83	0.867
	$\hat{\rho}$ (ols)	0.396	0.433	0.458	0.507	0.549	0.602	0.673	0.772	0.853
	Bias(dif)	-0.604	-0.567	-0.542	-0.493	-0.451	-0.398	-0.327	-0.228	-0.147
	BIas(lev)	0.196	0.143	0.075	0.008	-0.063	-0.119	-0.163	-0.17	-0.133
	Bias(ols)	-0.604	-0.567	-0.542	-0.493	-0.451	-0.398	-0.327	-0.228	-0.147
	RMSE(dif)	0.698	0.667	0.639	0.592	0.555	0.5	0.421	0.339	0.273
	RMSE(lev)	0.322	0.302	0.262	0.247	0.26	0.267	0.275	0.275	0.245
	RMSE(ols)	0.698	0.667	0.639	0.592	0.555	0.5	0.421	0.339	0.273
75	$\hat{\rho}$ (dif)	0.402	0.438	0.461	0.509	0.543	0.611	0.685	0.757	0.852
	$\hat{\rho}$ (lev)	0.402	0.438	0.461	0.509	0.543	0.611	0.685	0.757	0.852
	$\hat{\rho}$ (ols)	0.402	0.438	0.461	0.509	0.543	0.611	0.685	0.757	0.852
	Bias(dif)	-0.598	-0.562	-0.539	-0.491	-0.457	-0.389	-0.315	-0.243	-0.148
	BIas(lev)	0.196	0.134	0.066	-0.003	-0.078	-0.126	-0.167	-0.189	-0.145
	Bias(ols)	-0.598	-0.562	-0.539	-0.491	-0.457	-0.389	-0.315	-0.243	-0.148
	RMSE(dif)	0.658	0.625	0.603	0.56	0.52	0.452	0.384	0.314	0.219
	RMSE(lev)	0.289	0.242	0.216	0.201	0.203	0.223	0.248	0.258	0.212
	RMSE(ols)	0.658	0.625	0.603	0.56	0.52	0.452	0.384	0.314	0.219

$\hat{\beta}$(dif) = First-difference GMM estimator, $\hat{\beta}$ (dif) (lev) = Level GMM estimator, and $\hat{\beta}$ (dif) (ols) = Ordinary Least Square estimator, Bias(dif) = Bias of First-difference GMM estimator, Bias(lev) = Bias of Level GMM estimator, Bias(ols) = Bias of OLS estimator, RMSE = Root Mean Square Error, RMSE(dif) = RMSE of First-difference GMM estimator, RMSE(lev) = RMSE of Level GMM estimator, RMSE(ols) = RMSE of OLS estimator

illustrates the comparison of the performance of $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) with reference to the true line over the positive range of ρ. From figure 1, it is evident that. When T is not considerably large, $\hat{\rho}$ (lev) has lesser bias than the other two estimators, but as T increases bias of $\hat{\rho}$ (lev) also increases. Referring to figure 2, it is apparent that $\hat{\beta}$ (lev) has a smaller bias than other two in all the cases of T and ρ. Figure 3 depicts the distinction of RMSE of $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) for the positive range of ρ. From figure 3, it is noticed that. $\hat{\rho}$ (lev) has less RMSE, especially when T is too small. As T setsup $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) perform better than $\hat{\rho}$ (lev) but as ρ approaches unity, $\hat{\rho}$ (lev) performs extremely better than other two estimators. The RMSE of $\hat{\beta}$ for above considered estimators is plotted in figure 4 and it is observed that, in all the cases of T and ρ, $\hat{\beta}$ (lev) has least RMSE as compared to the remaining two estimators.

5. Conclusion

In this paper, an estimation of AR(1) time series model with one additional regressor is done by using First-difference GMM and Level GMM estimation methods. Monte-Carlo simulation is conducted to examine the performances of the considered estimators. Based on the simulation results, it is noticed that, in the case of an autoregressive parameter (ρ), when the sample size is too small the bias and RMSE of level GMM estimator is better compared to remaining estimators. As sample size increases first-difference GMM and OLS estimators perform better than level GMM estimator. But when ρ is very close to unity the level GMM estimator is more efficient than remaining two estimators. The additional regressor parameter (β) has the smallest bias and is more efficient than first-difference GMM and OLS estimators.

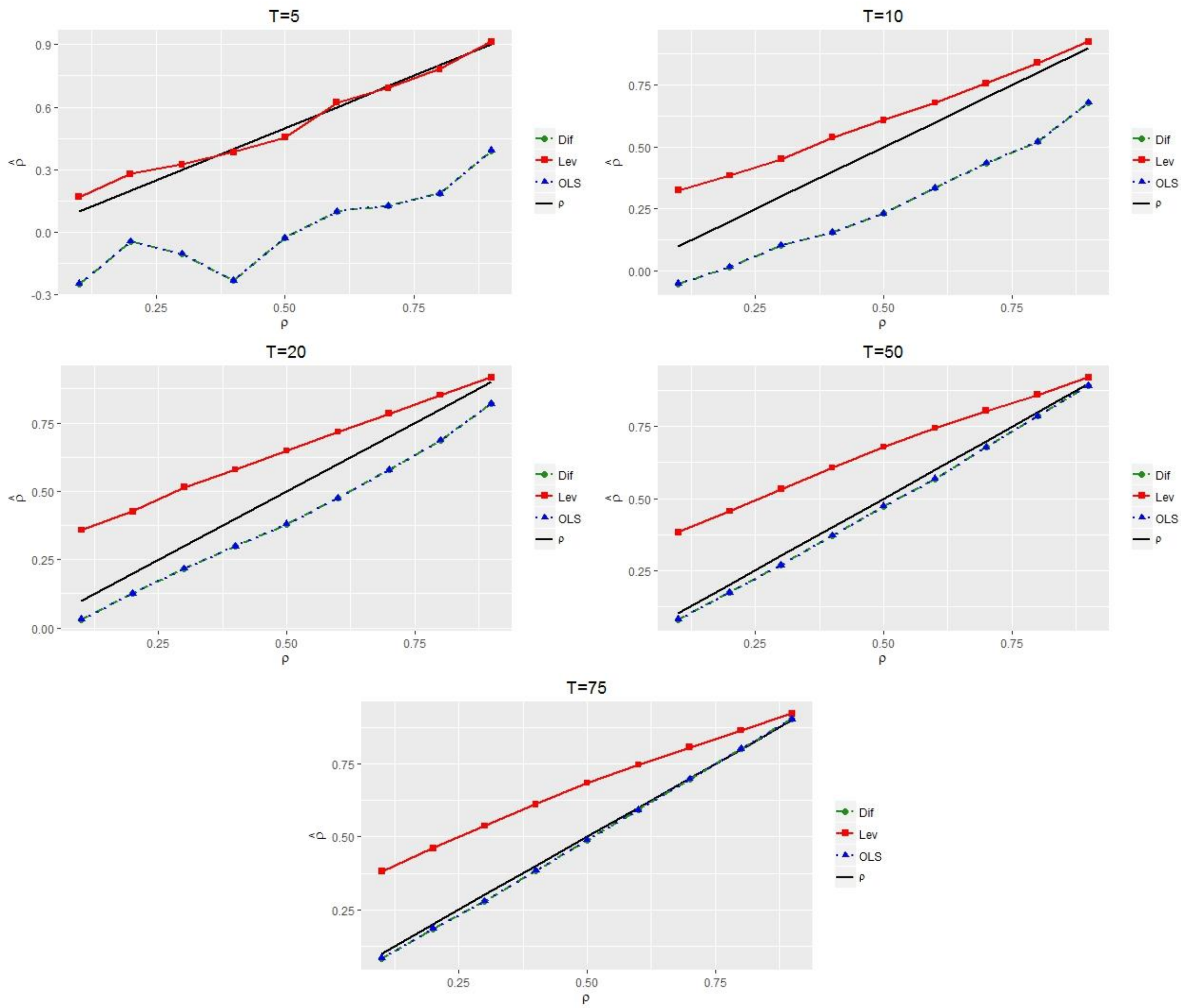

Figure 1. Means of First-difference GMM, Level GMM and OLS estimators for autoregressive parameter.

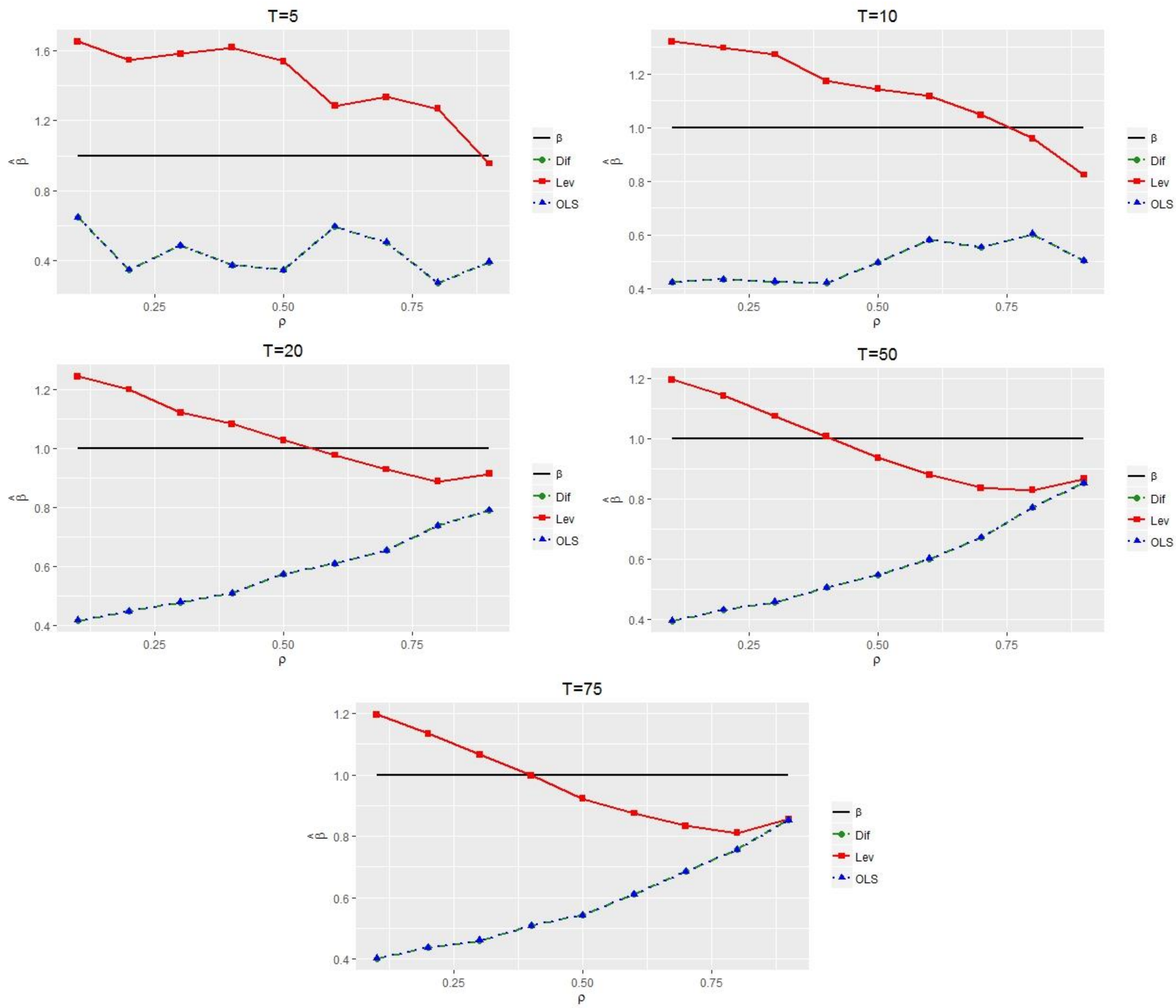

Figure 2. Means of First-difference GMM, Level GMM and OLS estimators for additinal regressor parameter.

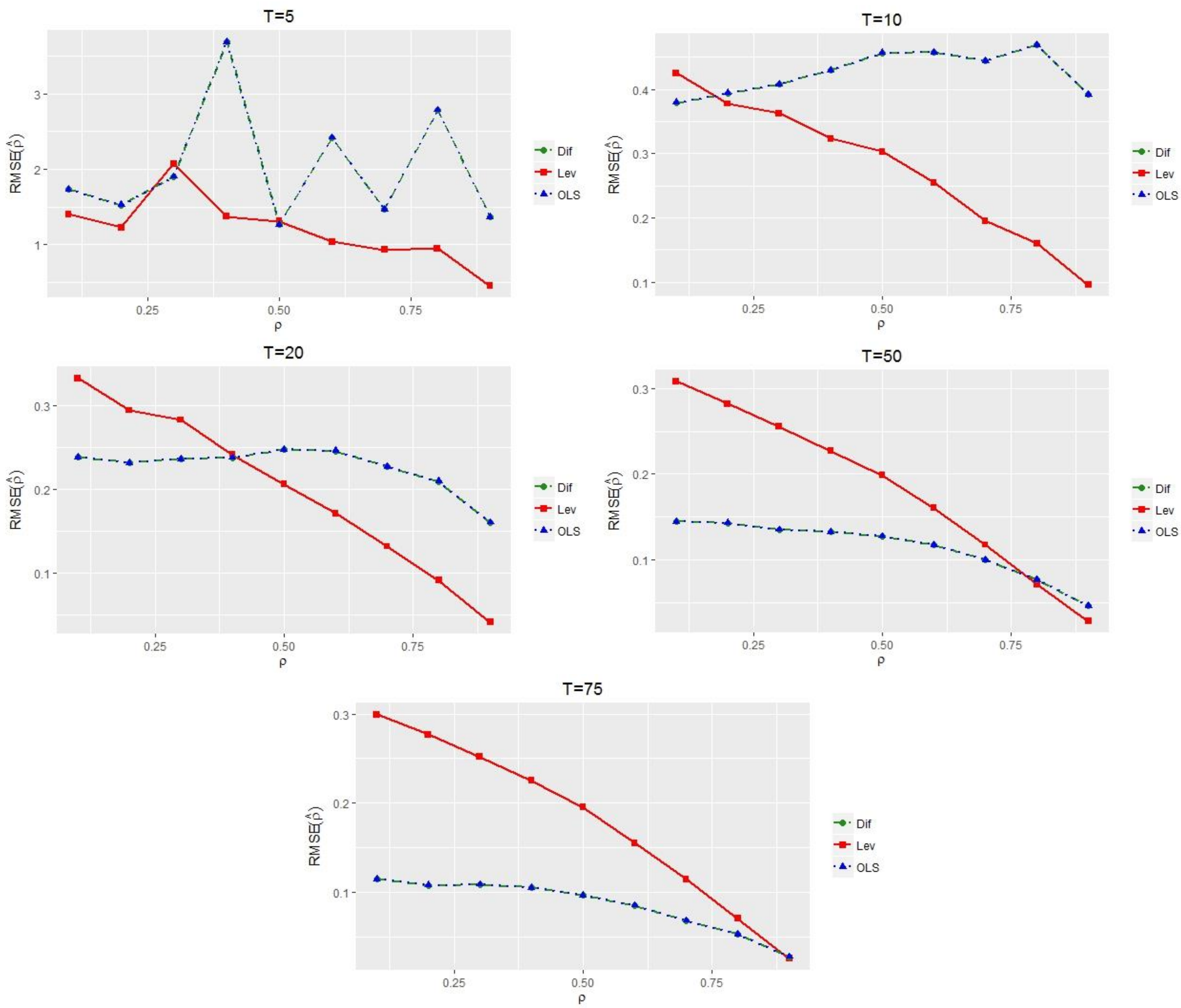

Figure 3. RMSEs of Difference GMM, Level GMM and OLS estimators for autoregressive parameter.

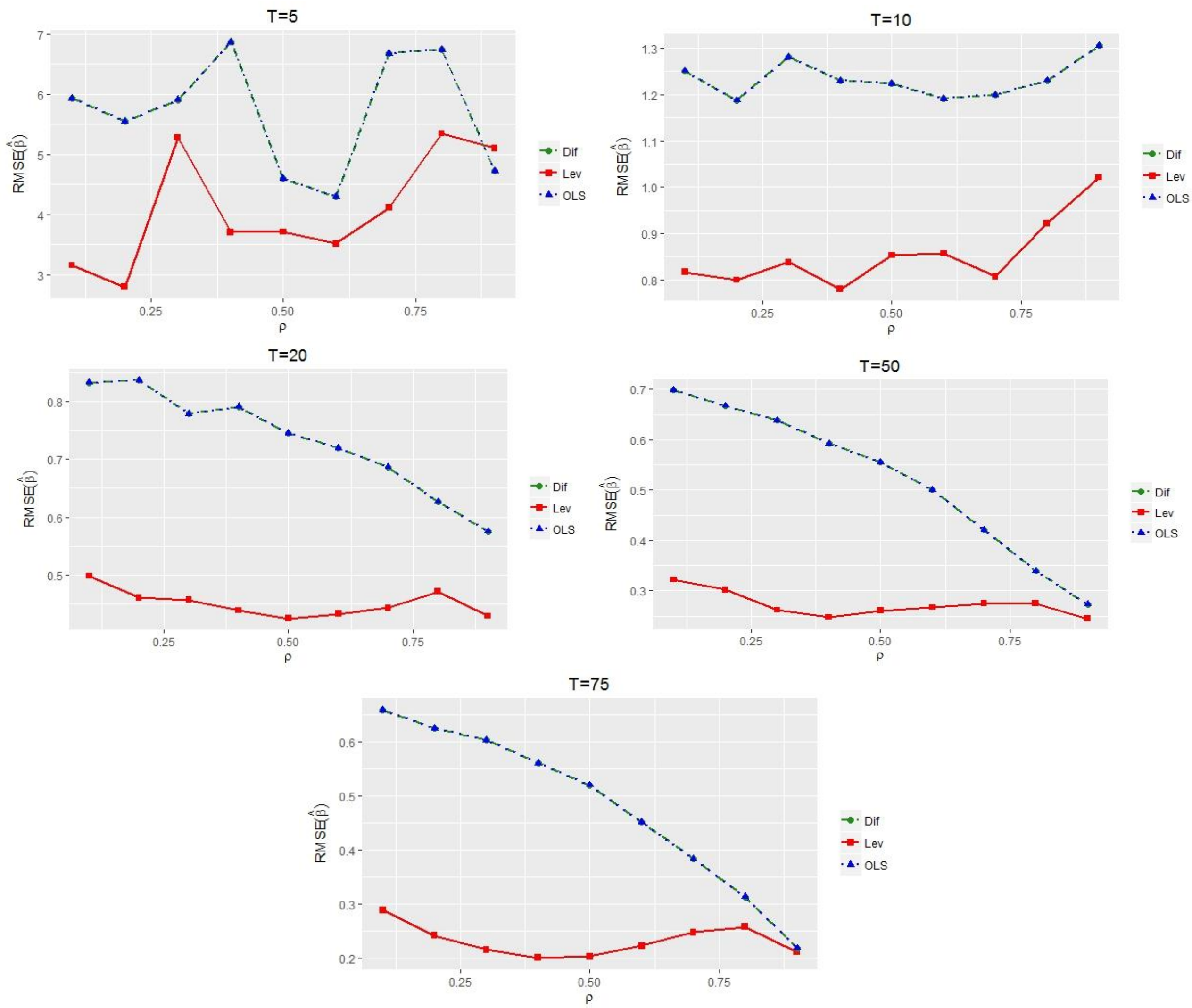

Figure 4. RMSEs of Difference GMM, Level GMM and OLS estimators for additinal regressor parameter.

REFERENCES

[1] B. F. Chakalabbi, S. Neregal and S. Matur, “Improved GMM estimation of AR(1) Time series with a Root near 1”, International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol. 5, pp. 1-10, 2018.

[2] F. H. C. Marriott and J. A. Pope, “Bias in the Estimation of Autocorrelations”, Biometrika, Vol. 41, pp. 390-402.

[3] G. H. Orcutt, “A Study of the Autoregressive Nature of the Times Series Used for Tinbergenfs Model of the Economic System of the United States”, Journal of the Royal Statistical Society, Series B, pp. 1-45, 1948.

[4] H. B. Mann and A. Wald, “On the statistical treatment of linear stochastic difference equations”, Econometrica: Journal of the Econometric Society, pp.173-220, 1943.

[5] K. Tanaka, “Asymptotic exxpansions associated with the AR(1) model with unknown mean”, Econometrica: Journal of the Econometric Society, pp.1221-1231, 1983.

[6] L. Hurvicz, “Least Squares Bias in Time Series. In T. Koopmans (ed.), Statistical Inference in Dynamic Economic Models”, New York: Wiley, pp. 365-383, 1950.

[7] M. Arellano and S. Bond, "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations", Review of Economic Studies, Vol. 58, pp. 277-298, 1991.

[8] M. Arellano and O. Bover, "Another Look at the Instrumental Variable Estimation of Error-Components Models", Journal of Econometrics, Vol. 68, pp. 29-51, 1995.

[9] M. G. Kendall, "Note on the Bias in the Estimation of Autocorrelation", Biometrika, Vol. 41, pp. 403-404, 1954.

[10] M. Soto, "System GMM estimation with small sample", (working paper No. 395). Retrived from Barcelona Graduate School of Economic, website: https://econpapers.repec.org/paper/bgewpaper/395, 2009.

[11] P. C. B. Phillips, "Approximations to Some Finite Sample Distributions Associated with a First Order Stochastic Difference Equation", Econometrica, Vol. 45, pp. 463-485, 1977.

[12] P. Shaman and R. A. Stine, "The bias of autoregressive coefficient estimators", Journal of the Americal Statistical Association, Vol. 83(403), pp. 842-848, 1988.