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Time Control Chart – Log Logistic Distribution

B. SriRam [1], A.Suhasini [2] and R.R.L.Kantam [3]

[1] *Department of Science and Humanities, Acharya Nagarjuna University College of Engineering and Technology, Acharya Nagarjuna University, Guntur - 522 510*
[2] *Department of Statistics, S. D. M. Siddhartha Mahila Kalasala ,Vijayawada -520013.*
[3] *Department of Statistics, Acharya Nagarjuna University, Guntur-522 510.*

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Abstract: The time to failure of a product is considered as a quality characteristic of following Log-Logistic distribution ($\beta = 3$). Control limits are evaluated for the time to failure. Life time data are compared with the control limits to judge the quality performance of the product.

Keywords: Log-Logistic distribution ($\beta = 3$), Time Control Chart, Reliability, Hazard function, $\bar{x}$ & R Chart.

1. INTRODUCTION

Life time data generally contain the failure times of sample products or inter failure times or number of failures experienced in given time. Assuming a suitable probability model, the reliability of the product is computed using life time data and the quality with respect to reliability would be generally assessed. From a different point of view if the specific life time data contain larger gap between successive failure so that the obtain product is large. Hence the product is preferable, that is detection of out of control above the upper control limit (UCL) is desirable times between successive failures times and then probability limits for such data can be parametric approach. Taking central line at the median of distribution of the data, the (probability limits can be though as usual control limits of a control chart for the data). Points above the upper control limits (UCL) of such a data would be an encouraging characteristic of the product because they lead to a large gap between successive failure so that the obtain product is large hence the product is preferable. Thus detection of out of control above the upper control limit (UCL) is desirable and its causes are to be preserved or encouraged. Similarly detection of out control below lower control limit (LCL) results is shorter gaps between successive failures.

The assignable causes for this detection are to be minimized, or eliminated. Points within the control limits indicate a smooth and satisfactory failure phenomenon. Thus such a set of control limits would be helpful in assessing the quality of the product based inter failure time data. The control chart may be accordingly named as Time Control Chart. Xie *et al*.(2002) has suggested control charts for failure data modelled by the well-known exponential distribution. Ravi Kumar and Kantam (2010) adopted the principle of Xie *et al*.(2002) to develop time control chart for gamma distribution and half logistic distribution. In this paper we adopt the same principle to develop time control chart for life time data modelled by well-known log-logistic distribution. The rest of paper is organized as, Section 2 deals with generalized theory Xie *et al*.(2002) and its application to the log-logistic distribution. Section 3 consists of evaluation of control limits for the time control chart with a live data from Aarset (1987). An extension of time control chart for time to every r^{th} failure called t_r control chart is also developed using cumulative distribution function of highest order statistic in the given sub group.

2. TIME CONTROL CHART

Let F(x) be the cumulative distribution function of a continuous positive valued random variable, f(x) be its probability density function. If the random variable represents inter failure time of a device (Time lapse between successive failures), a control chart for such data would be based on 0.9973 probability limits (on par with the

E-mail address: sriram_stat@rediffmail.com, allada_suhasini@yahoo.co.in, kantam_rrl@rediffmail.com

probability, constant chosen by Shewhart for the classical control chart) of the times between failure random variable say X. These limits and central line are the respective solutions of the following equations taking equitailed probabilities.

$$F(x) = 0.99865 \quad (2.1)$$
$$F(x) = 0.5 \quad (2.2)$$
$$F(x) = 0.00135 \quad (2.3)$$

Let $F_U(x)$, $F_C(x)$, $F_L(x)$ be respective solutions of the Equations (2.1), (2.2) and (2.3) with standard form i.e.

$$X_U = F^{-1}(0.99865) \quad (2.4)$$
$$X_C = F^{-1}(0.5) \quad (2.5)$$
$$X_L = F^{-1}(0.00135) \quad (2.6)$$

The graph between the serial number of the failure and corresponding inter failure time together with 3 parallel lines to the horizontal axis at X_U, X_C, X_L is the time control chart. In our study we consider the log-logistic distribution whose probability density function and cumulative distribution function respectively given by

$$f(z) = \begin{cases} \frac{\beta z^{\beta-1}}{(1+z^{\beta})^2} & ; z \geq 0, \beta > 1 \\ 0 \end{cases} \quad (2.7)$$

where β is the shape parameter of the distribution and

$$F(z) = \frac{z^{\beta}}{(1+z^{\beta})} \quad (2.8)$$

Substituting (2.8) in (2.4), (2.5) and (2.6) we get the percentiles useful for the time control chart. Infact, these are the solutions of the following equations in z i.e, when F is replaced successively by (0.99865), (0.5), (0.00135)

$$1 - F(z) = 1 - \frac{z^{\beta}}{(1+z^{\beta})} \quad (2.9)$$

where F(.) is given by (2.8) the control limits and central line based on the parameters of the population and can be estimated only from a given data supposed to have been following population. Sometime the lapse of time up to r^{th} failure becomes a deciding factor about the failure trend in a given sample inter failure times. If we are given a series of n inter failure times, let r be a natural number less than n. The $\sum_{i=1}^{r} X_i$, $\sum_{i=r+1}^{2r} X_i$, $\sum_{i=2r+1}^{3r} X_i$,...... etc. represents the lapse of time consequently between every r^{th} failure. A control chart for time between every r^{th} failure would throw more light on the out of control signals that of inters failure times. Xie *et al*.(2002) named such control chart as t_r control chart and developed control limits using sampling distribution of $\sum_{i=1}^{r} X_i$ and have taken example of exponential distribution and used the theory that the sum of exponential variate is a gamma variate to get the percentiles of t_r control charts with the help of cumulative Poisson summations. If inter failure times are exponentially distributed, the control limits of t_r chart of Xie *et al*.(2002) cannot be used. Kantam and Ravi Kumar (2012a, 2013b) are the authors who have worked in this direction overcoming the drawback, we suggest the following alternative approach to get control limits of t_r chart for any distribution. If $(X_1, X_2, \ldots X_r)$; $(X_{r+1}, X_{r+2}, \ldots X_{2r})$; $(X_{2r+1}, X_{2r+2}, \ldots X_{3r})$ etc., are regarded as independent samples of size r each iid random variable having F(x) as their common model then $Y_1 = X_1$, $Y_2 = \sum_{i=1}^{2} X_i$, $Y_3 = \sum_{i=1}^{3} X_i$, ... $Y_r = \sum_{i=1}^{r} X_i$ become an ordered sample of size r representing the time to 1^{st} failure, time to 2^{nd} failure, time to r^{th} failure respectively. Y_r is the highest ordered statistics in an ordered sample $Y_1 < Y_2 < \ldots < Y_r$. Thus the t_r chart is the control chart Y_r as the points on it representing the time of every r^{th} failure. Therefore, r is fixed, the percentiles of highest order statistic in a sample of size r would serve the purpose of control limits for the t_r chart.

We know that $[F(x)]^r$ cumulative distribution function of r^{th} order statistic in a sample of size r for the percentiles of t_r chart with 0.9973 coverage probability would be the solution of $[F(x)]^r = 0.99865$ and $[F(x)]^r = 0.00135$. The control limits of t_r chart would be the solution of $[F(x)]^r = 0.5$. In this paper we develop control limits for t_2 chart based on analytical expression involving the parameters of log-logistic distribution.

3. ILLUSTRATION

Aarset (1987) gives the data representing the life times of 50 devices as in Table3.1 given below

Table 3.1 Aarset 1987) data

S.No	Devices	S.No	Devices	S.No	Devices	S.No	Devices	S.No	Devices
1	0.1	11	7	21	36	31	67	41	84
2	0.2	12	11	22	40	32	67	42	84
3	1	13	12	23	45	33	67	43	84
4	1	14	18	24	46	34	67	44	85
5	1	15	18	25	47	35	72	45	85
6	1	16	18	26	50	36	75	46	85
7	1	17	18	27	55	37	79	47	85
8	2	18	18	28	60	38	82	48	85
9	3	19	21	29	63	39	82	49	86
10	6	20	32	30	63	40	83	50	86

Here n=50 for the sake of explanation we develop Time Control Chart and t_2 control chart for this data then UCL, CL, LCL are

$$\left.\begin{aligned} UCL &= 166.0819 \\ CL &= 53.79115 \\ LCL &= 2.373845 \end{aligned}\right\} \qquad (3.1)$$

Comparison with the given data of size 50 we see that number of points below LCL = 8, and above UCL = 0 and between LCL and UCL = 42, from Figure 1 this revels that the data has shorter, moderate life but not longer life with reference to survival chance based on the notion of longer lives as better quality characteristics. The data can be branded as of moderate quality. The following is the distribution of points with respect to the above control limit of the chart points.

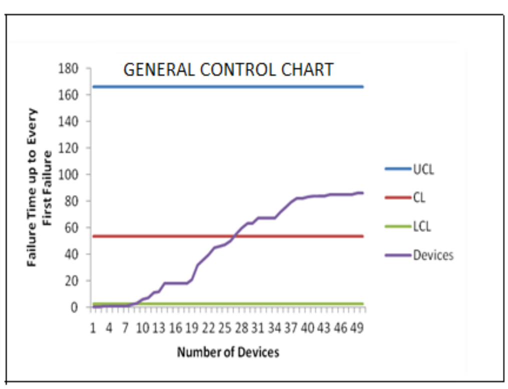

Figure 1. General Control Chart

Adopting the concept t_2 control chart we have grouped the 50 observations into 25 disjoint successive groups of size2 each. Using the percentiles of highest order statistic in a sample of size 2, the control limits for t_2 control chart are

$$\left.\begin{array}{l} UCL = 348.9743 \\ CL = 143.1927 \\ LCL = 25.00029 \end{array}\right\} \quad (3.2)$$

The sum of the 2 observations in each successive sub groups shall become a point on the t_2 chart with the above control limits. The following is the spread of the 25 points on t_2 chart given in Figure 2 and are the distribution of points with respect to the control limits of t_2 chart.

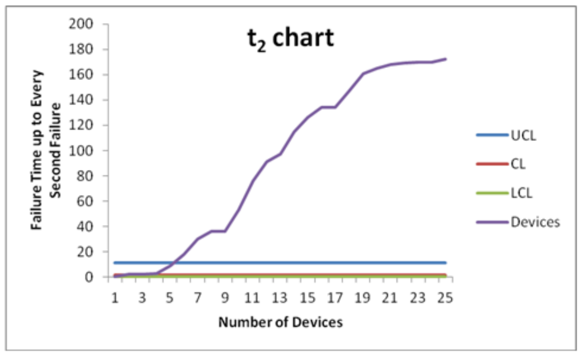

Figure 2. spread of 25 points on t2 Chart

The arithmetic means of the 25 subgroups are compared on a typical average control chart drawn for a skewed distribution, adopting the skewness correction control limits suggested by Chan and Cui (2003). The skewness of the log-logistic distribution estimated from the data is

$$\beta_1 = 2.07 \quad (3.3)$$

$$Sk_{(Bowley)} = 0.18 \quad (3.4)$$

Corresponding to this quantum of skewness the corrected control limits for mean chart borrowed from Chan and Cui (2003) for a subgroup size n = 2(1)5, 7 and 10 limits for $\bar{X}$ – Chart and R-Chart given in Table 3.2 .

Table 3.2 Mean Chart & Range Chart Limits

Sample sizes	Mean Chart limits		Range chart limits	
n	A*U	A*L	D*4	D*3
2	2	1.73	4.16	0
3	1.08	0.98	2.99	0
4	0.77	0.71	2.6	0
5	0.6	0.56	2.35	0.12
7	0.43	0.41	2.11	0.25
10	0.32	0.3	1.93	0.36

The control limits for $\bar{X}$ – Chart are

$$\left.\begin{array}{l} UCL = \bar{\bar{X}} + A_U^* \bar{R} \\ CL = \bar{\bar{X}} \\ LCL = \bar{\bar{X}} - A_L^* \bar{R} \end{array}\right\} \quad (3.5)$$

The control limits for R-Chart are

$$\left.\begin{array}{l} UCL = D_4^* \bar{R} \\ CL = \bar{R} \\ LCL = D_3^* \bar{R} \end{array}\right\} \quad (3.6)$$

where $A_U^*, A_L^*, D_3^*, D_4^*$ are given in Chan and Cui (2003)

Comparison of the ranges and averages of the 25 subgroups with the control limits of the respective control chart, we notice of following spread of the points.

$\bar{X}$ – Chart:-

Bowleys:-

$$\left.\begin{array}{l} UCL = 49.374 \\ CL = 45.686 \\ LCL = 42.3852 \end{array}\right\} \quad (3.7)$$

Number of points between the limits zero and below LCL is 9 and above UCL is 45.686. The following is the distribution of points of $\bar{\bar{X}}$ – Chart given in Figure 3.

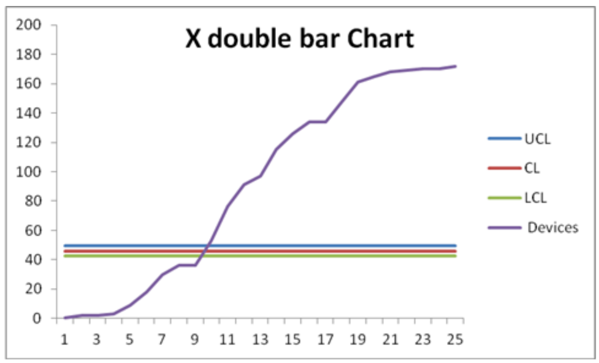

Figure 3. Distribution of points of $\bar{\bar{X}}$ Chart

R –Chart:-

$$\left.\begin{array}{l} UCL = 7.67104 \\ CL = 1.844 \\ LCL = 0 \end{array}\right\} \quad (3.8)$$

Number of points between the limits 24 above UCL is 1.844 and below LCL is zero. The following is the distribution of points of $\bar{R}$-Chart given in Figure 4.

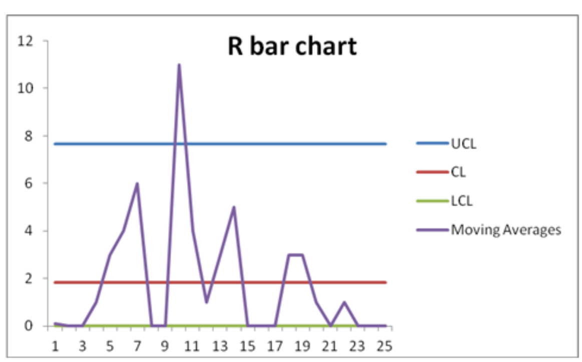

Figure 4. Distribution of points of $\bar{R}$ Chart

The arithmetic means of the 25 subgroups are compared on a typical average control chart drawn for a skewed distribution, adopting the skewness correction control limits suggested by Chan and Cui (2003). The skewness of the log-logistic distribution estimated from the data is

$$\beta_1 = 0.38 \quad (3.9)$$

$$Sk_{(Kelleys)} = 0.59 \quad (3.10)$$

Corresponding to this quantum of skewness the corrected control limits for mean chart borrowed from Chan and Cui (2003) for a subgroup size n = 2(1)5, 7 and 10 limits for $\bar{X}$ – Chart and R-Chart given in Table 3.3.

Table 3.3 Mean Chart & Range Chart Limits

Sample sizes	Mean Chart limits		Range chart limits	
n	A*U	A*L	D*4	D*3
2	2.25	1.58	4.31	0
3	1.19	0.88	3.16	0
4	0.84	0.65	2.77	0.04
5	0.65	0.52	2.5	0.15
7	0.46	0.38	2.26	0.28
10	0.34	0.29	2.07	0.38

$\bar{X}$ – Chart:-
Kelleys

$$\left.\begin{array}{l} UCL = 49.835 \\ CL = 45.686 \\ LCL = 42.77248 \end{array}\right\} \quad (3.11)$$

Number of points between the limits zero and below LCL is 9 and above UCL is 45.686. The following is the distribution of points of $\bar{\bar{X}}$ – Chart given in Figure 5.

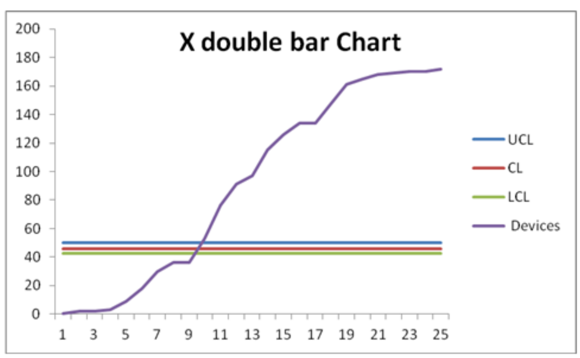

Figure 5. Distribution of points of $\bar{\bar{X}}$ Chart

R –Chart:-

$$\left.\begin{array}{l} UCL = 7.94764 \\ CL = 1.844 \\ LCL = 0 \end{array}\right\} \quad (3.12)$$

Number of points between the limits 24 above UCL is 1.844 and below LCL is zero. The following is the distribution of points of $\bar{R}$-Chart given in Figure 6.

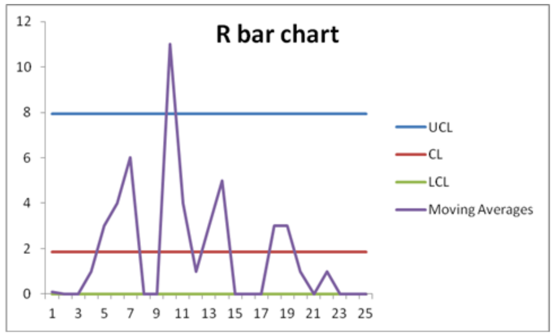

Figure 6. Distribution of points of $\bar{R}$ Chart

CONCLUSION

To sum up, the data treated for the individual observations through the concept of time control chart seems to be of moderate quality. When it is considered as a combination of 25 subgroups of size 2 each, the t_2 chart has also indicated the same quality. The skewness correction chart on the other hand, for sub group mean and range has a contrary conclusion. The effectiveness of these conclusions are subject to the concept of least average run length (ARL) and suitability of the model to the data.

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