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Estimation of Multicomponent System Reliability for a Bivariate Generalized Rayleigh Distribution

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Abstract: The study of a multicompnent system with k identical components which are independent to each other is considered in the present work. The components of the system have series structure with two dependent elements that are exposed to a common random stress. Here, strength vectors follow bivariate generalized Rayleigh distribution and a common random stress follow generalized Rayleigh distribution. The s-out-of-k system is said to function if atleast s out of $k(1 \leq s \leq k)$ strength variables exceed the random stress. The estimation of system reliability is studied using maximum likelihood and Bayesian approaches. The maximum likelihood estimates are derived under simple random sampling and ranked set sampling schemes. The approximate Bayes estimates for system reliability are obtained using Lindley's approximation technique. Simulation study is conducted to study the performance of the estimators of reliability using mean squares error criteria.

Keywords: Bivariate Generalized Rayleigh distribution, Stress-strength reliability, Ranked set sampling, Simple random sampling, Maximum Likelihood estimator, Bayes estimator.

1. INTRODUCTION

The study of estimation of "stress-strength" reliability has received a great attention by many authors in the literature, because of its application in different fields like engineering, physics, etc. For example, in a rocket engine, let Y represent the maximum chamber pressure generated by ignition of a solid propellant, and X be the strength of the rocket chamber. Here, the reliability of the rocket engine is the probability of successful firing of the engine. In the simplest term, "stress-strength" reliability can be described as an assessment of reliability of a component in terms of random variable Y representing stress experienced by the component and X representing strength of a component available to overcome the possible stress. If the stress exceeds the strength, then the system will fail. The main idea of stress-strength reliability, $R = P(X > Y)$ was introduced by Birnbaum (1956) and developed by Birnbaum and McCarty (1958). The study of estimation of stress-strength reliability of system when the samples are drawn from various distributions such as exponential, Weibull, normal, gamma etc are considered in the literature. Raqab et. al., (2008) estimated stress-strength reliability for a 3-parameter generalized exponential distribution. Wong (2012) estimated confidence intervals for $P(X > Y)$ when the underlying distribution is from generalized Pareto. Estimation of Reliability for stress-strength model, when X and Y have bivariate exponential distribution was considered by Awed et al,(1981), Jana(1994), Hanagal (1995), while Hanagal (1997) considered the estimation of the reliability when (X, Y) follow bivariate pareto distribution. Angali et al. (2014) considered Bayesian estimation for four parameter bivariate exponential distribution under different loss functions. Hanagal (2003), estimated system reliability, $R = P(X_{k+1} < \min(X_1,...,X_k))$ under the assumption that strengths of the k components $(X_1,...,X_k)$ subjected to a common stress X_{k+1}, when $(X_1,..,X_{k+1})$ follow (k +1) statistically independent gamma or Weibull or Pareto distributions. Also, Pandit and Kantu (2013) considered estimation of multicomponent stress-strength reliability for parallel and series systems when strength and stress variables follow exponential distribution. In recent years, the estimation of multicomponent stress-strength reliability for s-out-of-k systems have been extensively investigated by many authors in the literature.

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An s-out-of-k system functions when the system having k statistically independent and identically distributed components functions if $s(1 \leq s \leq k)$ or more components with stand a common stress, was first developed by Bhattacharyya and Johnson (1974). The applications of these type of systems can be seen in industrial and military (refer Kuo and Zuo (2003)). Estimation of multicomponent stress-strength reliability is considered for the log-logistic (Rao, G. S and Kantum(2010)), generalized exponential(Rao (2012)), Rayleigh (Rao (2012)), Burr Type XII,et. al (Rao(2015)) and generalized Rayleigh distributions(Rao, G. S(2014)) respectively. Recently, Kizilaslan and Nadar (2015, 2016) considered estimation of multicomponent stress-strength reliability using both classical and Bayesian approach when underlying distribution is Weibull and bivariate Kumarswamy distributions.

In the present study the strength variables assumed to follow BGR distribution and stress variable to follow GR distribution. A system having k similar strength component, constitutes a series system of dependent elements experiencing a common stress. The system is said to function if $s(1 \leq s \leq k)$ or more components simulatanously operate. These kinds of situations may occur offenly in real life (refer Nadar, M.and Kizilaslan, F (2016)).

The estimation of multicomponent system reliability are derived using maximum likelihood estimation under simple random sampling (SRS) and ranked set sampling (RSS) scheme. To obtain approximate Bayes estimates of reliability are obtained using Lindley's approximation. RSS method used in this context was introduced by McIntyre (1952). Several authors in different fields have shown interest in RSS for example, reliability (Kvam and Samaniego, 1993,1994), environment (Abu-Dayyeh and Muttlak(1996), Patil et al., 1993; Muttlak, 1997; Yu and Lam, 1997) and quality control (Muttlak and Al- Sabah, 2003). Estimation of stress-strength reliability based on RSS is considered by Sengupta and Mukhati (2008). Muttlak et al. (2010) estimated stress-strength reliability when X and Y follow exponential distribution. Hussian (2014) discussed estimation of stress-strength model for generalized inverted exponential distribution based on RSS and SRS. Maximum likelihood method is used to estimate R using both approaches. Hassan et al. (2015) studied estimation of multicomponent system reliability when X and Y are independently distributed Burr XII random variables based on different sampling schemes.
In section 2, the system reliability is given. In section 3 and 4 Maximum likelihood estimation of $R_{s,k}$ based on SRS and RSS are respectively derived. In section 5, Bayes estimates of $R_{s,k}$ is obtained by Lindley's approximation. In section 6, a simulation study is conducted by obtaining MSEs and conclusions are given in section 7.

2. SYSTEM RELIABILITY

Dina H, Abdul-Hady (2013) proposed a bivariate Generalized Rayleigh(BGR) distribution which can be described as follows:

Let V_1, V_2 and V_3 follow univariate generalized Rayleigh distribution denoted as GR(λ_1, γ), GR(λ_2, γ) and GR(λ_3, γ) and all three distributions are mutually independent. The probability density function (pdf) and the cumulative distribution function (cdf) of a generalized Rayleigh distribution denoted as $X \sim GR(\gamma, \lambda)$ is given by

$$f(x) = \gamma x e^{-\frac{\gamma}{2}x^2}\left(1-e^{-\frac{\gamma}{2}x^2}\right)^{\lambda-1} \quad ; x>0,\ \lambda, \gamma>0$$

and

$$F(x) = \left(1-e^{-\frac{\gamma}{2}x^2}\right)^{\lambda} \quad ; x>0, \lambda, \gamma>0$$

Let $X = \max(V_1, V_3)$ and $Y = \max(V_2, V_3)$. Then the distribution of vector (X, Y) is BGR distribution with the parameters ($\lambda_1, \lambda_2, \lambda_3, \gamma$) and it is denoted as BGR ($\lambda_1, \lambda_2, \lambda_3, \gamma$). If $(X, Y) \sim$ BGR ($\lambda_1, \lambda_2, \lambda_3, \gamma$) is given by

$$f(x,y)=\begin{cases}\lambda_2(\lambda_1+\lambda_3)\gamma^2 xye^{-\frac{\gamma}{2}(x^2+y^2)}\left[1-e^{-\frac{\gamma}{2}x^2}\right]^{\lambda_1+\lambda_3-1}\left[1-e^{-\frac{\gamma}{2}y^2}\right]^{\lambda_2-1} & x>y\\ \lambda_1(\lambda_2+\lambda_3)\gamma^2 xye^{-\frac{\gamma}{2}(x^2+y^2)}\left[1-e^{-\frac{\gamma}{2}x^2}\right]^{\lambda_1-1}\left[1-e^{-\frac{\gamma}{2}y^2}\right]^{\lambda_2+\lambda_3-1} & x<y\\ \lambda_3\gamma xe^{-\frac{\gamma}{2}x^2}\left[1-e^{-\frac{\gamma}{2}x^2}\right]^{\lambda_1+\lambda_2+\lambda_3-1} & x=y\end{cases}$$

where $\lambda_1, \lambda_2, \lambda_3, \gamma > 0$ and $\lambda_1+\lambda_2+\lambda_3=\lambda$. The marginal distribution of X and Y are generalized Rayleigh i.e., X ~ GR($\lambda_1+\lambda_3, \gamma$) and Y ~ GR($\lambda_2+\lambda_3, \gamma$). The distribution of min (X, Y) is generalized Rayleigh with parameters λ and γ. Further, the BGR distribution has both absolute continuous part and a singular part.

Here, the strength vectors $(X_1, Y_1), (X_2, Y_2), \ldots, (X_k, Y_k)$ assumed to follow BGR. Consider $Z_i = \min(X_i, Y_i)$ then $Z_i \sim GR(\lambda, \gamma)$. i = 1, ...,k and a common stress variable T is distributed generalized Rayleigh. The system is working if at least s out of $k (1 \le s \le k)$ of the Z_i strength variables work when common stress variable T is carried out.
Let T, $Z_1,\ldots, Z_k$ be statistically independent, G(t) be the cummulative distribution function (cdf) of T and F(z) be the common cummulative distribution function (cdf) of $Z_1,\ldots,Z_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is given by

$$R_{s,k} = P(\text{at least } s \text{ of the } (Z_1, Z_2, \ldots, Z_k) \text{ exceed } T)$$
$$= \sum_{i=s}^{k}\binom{k}{i}\int_0^\infty (1-F(t))^i (F(t))^{k-i}\, dG(t) \qquad (1)$$

In the operating environment, the system is subjected to stress T which is a random variable with distribution function G(.). The strengths of the components, are independently and identically distributed random variables with distribution function F(.). Then the reliability of the system is given by the equation (1).
Now, we assume that $(X_1, Y_1), (X_2, Y_2), \ldots, (X_k, Y_k)$ be independently and identically distributed random variables having BGR $(\lambda_1, \lambda_2, \lambda_3, \gamma)$ distributed and T is a random variable from GR (δ, γ) distribution. Then $R_{s,k}$ is given by using (1)

$$R_{s,k} = \sum_{i=s}^{k}\binom{k}{i}\int_0^\infty (1-F(t))^i (F(t))^{k-i}\, dG(t)$$
$$= \sum_{i=s}^{k}\binom{k}{i}\int_0^\infty \delta\gamma t e^{-\frac{\gamma}{2}t^2}\left(1-e^{-\frac{\gamma}{2}t^2}\right)^{\delta-1}\left[1-\left(1-e^{-\frac{\gamma}{2}t^2}\right)^{\lambda}\right]^i\left[\left(1-e^{-\frac{\gamma}{2}t^2}\right)^{\lambda}\right]^{k-i} dt$$
$$= \sum_{i=s}^{k}\binom{k}{i}\int_0^1 \delta\left(1-u^{\lambda}\right)^i\left(u^{\lambda}\right)^{k-i} u^{\delta-1} du$$

where, $u = 1-e^{-\frac{\gamma}{2}t^2}$

$$R_{s,k} = \sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^j \frac{\delta}{[\lambda(k+j-i)+\delta]} \qquad (2)$$

3. MLE OF RS,K UNDER SRS

The MLE of $R_{s,k}$ depends on MLE of λ , δ. Hence, the MLE of the $R_{s,k}$ is obtained by deriving that of λ , δ. Let Z_i i = 1,...,n and T_j, j = 1,...,m be two ordered random samples of size n, m respectively.

Then the likelihood function is

$$L_s(\lambda,\delta,\gamma)=\lambda^n\delta^m\gamma^{n+m}\prod_{i=1}^{n} z_i e^{-\frac{\gamma}{2}z_i^2}(1-e^{-\frac{\gamma}{2}z_i^2})^{\lambda-1}\prod_{j=1}^{m} t_j e^{-\frac{\gamma}{2}t_j^2}(1-e^{-\frac{\gamma}{2}t_j^2})^{\delta-1}$$

and the log likelihood function is

$$\log L_s(\lambda,\delta,\gamma)=n\log\lambda+m\log\delta+(n+m)\log\gamma+\sum_{i=1}^{n}\log z_i+\sum_{j=1}^{m}\log t_j-$$

$$\frac{\gamma}{2}\left[\sum_{i=1}^{n} z_i^2+\sum_{j=1}^{m} t_j^2\right]+(\lambda-1)\sum_{i=1}^{n}\log\left(1-e^{-\frac{\gamma}{2}z_i^2}\right)+(\delta-1)\sum_{j=1}^{m}\log\left(1-e^{-\frac{\gamma}{2}t_j^2}\right) \quad (3).$$

Likelihood equations for estimating λ, δ and γ are,

$$\frac{\partial\log L_s}{\partial\lambda}=0\Rightarrow\frac{n}{\lambda}+\sum_{i=1}^{n}\log(1-e^{-\frac{\gamma}{2}z_i^2})=0 \quad (4)$$

$$\frac{\partial\log L_s}{\partial\delta}=0\Rightarrow\frac{m}{\delta}+\sum_{j=1}^{m}\log(1-e^{-\frac{\gamma}{2}t_j^2})=0 \quad (5)$$

$$\frac{\partial\log L_s}{\partial\gamma}=0\Rightarrow\frac{n+m}{\gamma}-\frac{1}{2}\left[\sum_{i=1}^{n} z_i^2+\sum_{j=1}^{m} t_j^2\right]+(\lambda-1)\sum_{i=1}^{n}\frac{e^{-\frac{\gamma}{2}z_i^2}z_i^2}{2\left(1-e^{-\frac{\gamma}{2}z_i^2}\right)}$$

$$+(\delta-1)\sum_{j=1}^{m}\frac{e^{-\frac{\gamma}{2}t_j^2}t_j^2}{2\left(1-e^{-\frac{\gamma}{2}t_j^2}\right)}=0 \quad (6)$$

From (4), (5) and (6), the MLE of λ, δ and γ is

$$\hat{\lambda} = \frac{-n}{\sum_{i=1}^{n} \log(1 - e^{-\frac{\hat{\gamma}}{2} z_i^2})} \qquad (7)$$

$$\hat{\delta} = \frac{-m}{\sum_{j=1}^{m} \log(1 - e^{-\frac{\hat{\gamma}}{2} t_j^2})} \qquad (8)$$

where $\hat{\gamma}$ can be obtained as the solution of non-linear equation:

$$h(\gamma) = \gamma$$

$$= (n+m)\left[\frac{1}{2}\left[\sum_{i=1}^{n} z_i^2 + \sum_{j=1}^{m} t_j^2\right] - (\lambda - 1)\sum_{i=1}^{n} \frac{e^{-\frac{\hat{\gamma}}{2} z_i^2} z_i^2}{2(1 - e^{-\frac{\hat{\gamma}}{2} z_i^2})} - (\delta - 1)\sum_{j=1}^{m} \frac{e^{-\frac{\hat{\gamma}}{2} t_j^2} t_j^2}{2(1 - e^{-\frac{\hat{\gamma}}{2} t_j^2})}\right] \qquad (9)$$

Therefore, $\hat{\gamma}$ can be obtained using "Newton- Raphson" method. Then, the MLE of λ and δ are obtained from (7) and (8). Hence, the MLE of $\hat{R}_{s,k}$, is obtained by applying invariance property.

$$\hat{R}_{s,k}^{Msrs} = \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^j \frac{\hat{\delta}}{[\hat{\lambda}(k+j-i) + \hat{\delta}]} \qquad (10)$$

4. MLE of $R_{s,k}$ under RSS

Let $Z_{(i:m_x)j}$, $i = 1,..., m_x$, $j = 1,..., r_x$ be the set of independent RSS with the sample size $n_x = m_x r_x$ and $T_{(k:m_y)l}$, $k = 1,..., m_y$, $l = 1,..., r_y$ be the set of independent RSS with the sample size $n_y = m_y r_y$, where m_x and m_y are the set sizes and r_x and r_y are the number of cycles . For convenience, we shall denote $Z_{(i:m_x)j}$ and $T_{(k:m_y)l}$ as Z_{ij} and T_{kl} respectively.

The pdf of the random variables Z_{ij} is given by,

$$g_1(Z_{ij}) = \frac{m_x!}{(i-1)!(m_x - i)!} \gamma\lambda z_{ij} e^{-\frac{\gamma}{2} z_{ij}^2} \left[1 - e^{-\frac{\gamma}{2} z_{ij}^2}\right]^{\lambda i - 1} \left[1 - \left(1 - e^{-\frac{\gamma}{2} z_{ij}^2}\right)^{\lambda}\right]^{m_x - i}; \quad z_{ij} > 0, \lambda, \gamma > 0 \qquad (11)$$

and T_{kl} is given by

$$g_2(t_{kl}) = \frac{m_y!}{(k-1)!(m_y - k)!} \gamma\delta t_{kl} e^{-\frac{\gamma}{2} t_{kl}^2} \left[1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right]^{\delta k - 1} \left[1 - \left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)^{\delta}\right]^{m_y - k}; \quad t_{kl} > 0, \delta, \gamma > 0 \qquad (12)$$

The likelihood function of λ, δ and γ are given by:

$$L_r(\lambda,\delta,\gamma)=\xi\prod_{i=1}^{m_x}\prod_{j=1}^{r_x}\gamma\lambda z_{ij}e^{-\frac{\gamma}{2}z_{ij}^2}\left[1-e^{-\frac{\gamma}{2}z_{ij}^2}\right]^{\lambda i-1}\left[1-\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)^{\lambda}\right]^{m_x-i}$$

$$\prod_{k=1}^{m_y}\prod_{l=1}^{r_y}\gamma\delta t_{kl}e^{-\frac{\gamma}{2}t_{kl}^2}\left[1-e^{-\frac{\gamma}{2}t_{kl}^2}\right]^{\delta k-1}\left[1-\left(1-e^{-\frac{\gamma}{2}t_{kl}^2}\right)^{\delta}\right]^{m_y-k}$$

Thus, the log-likelihood function of λ, δ and γ will be

$$\log L_r(\lambda,\delta,\gamma)=\xi^*+m_xr_x\log\lambda+m_yr_y\log\delta+(m_xr_x+m_yr_y)\log\gamma$$

$$+\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}\log z_{ij}+\sum_{k=1}^{m_y}\sum_{l=1}^{r_y}\log t_{kl}-\frac{\gamma}{2}\left[\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}z_{ij}^2+\sum_{k=1}^{m_y}\sum_{l=1}^{r_y}t_{kl}^2\right]$$

$$+\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}(\lambda i-1)\log\left[1-e^{-\frac{\gamma}{2}z_{ij}^2}\right]+\sum_{k=1}^{m_y}\sum_{l=1}^{r_y}(\delta k-1)\log\left[1-e^{-\frac{\gamma}{2}t_{kl}^2}\right]$$

$$+\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}(m_x-i)\log\left[1-\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)^{\lambda}\right]$$

$$+\sum_{k=1}^{m_y}\sum_{l=1}^{r_y}(m_y-k)\left[1-\left(1-e^{-\frac{\gamma}{2}t_{kl}^2}\right)^{\delta}\right] \qquad (13)$$

where ξ* is constant. The likelihood equations for estimating λ, δ and γ are

$$\frac{\partial\log L_r}{\partial\lambda}=0\Rightarrow\frac{m_xr_x}{\lambda}+\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}i\log\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)$$

$$+\sum_{i=1}^{m_x}\sum_{j=1}^{r_x}\frac{(m_x-i)\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)^{\lambda}\log\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)}{\left[1-\left(1-e^{-\frac{\gamma}{2}z_{ij}^2}\right)^{\lambda}\right]}=0 \qquad (14)$$

$$\frac{\partial \log L_r}{\partial \delta} = 0 \Rightarrow \frac{m_y r_y}{\delta} + \sum_{k=1}^{m_y} \sum_{l=1}^{r_y} k \log\left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)$$

$$+ \sum_{k=1}^{m_y} \sum_{l=1}^{r_y} \frac{(m_y - k)\left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)^{\lambda} \log\left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)}{\left[1 - \left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)^{\delta}\right]} = 0 \qquad (15)$$

$$\frac{\partial \log L_r}{\partial \gamma} = 0 \Rightarrow \frac{m_x r_x + m_y r_y}{\gamma} - \frac{1}{2}\left[\sum_{i=1}^{m_x} \sum_{j=1}^{r_x} z_{ij}^2 + \sum_{k=1}^{m_y} \sum_{l=1}^{r_y} t_{kl}^2\right] + \sum_{i=1}^{m_x} \sum_{j=1}^{r_x} \frac{(\lambda i - 1) e^{-\frac{\gamma}{2} z_{ij}^2} z_{ij}^2}{2\left[1 - e^{-\frac{\gamma}{2} z_{ij}^2}\right]}$$

$$+ \sum_{k=1}^{m_y} \sum_{l=1}^{r_y} \frac{(\delta k - 1) e^{-\frac{\gamma}{2} t_{kl}^2} t_{kl}^2}{2\left[1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right]} + \lambda \sum_{i=1}^{m_x} \sum_{j=1}^{r_x} \frac{(m_x - i) e^{-\frac{\gamma}{2} z_{ij}^2} z_{ij}^2 \left(1 - e^{-\frac{\gamma}{2} z_{ij}^2}\right)^{\lambda - 1}}{\left[1 - \left(1 - e^{-\frac{\gamma}{2} z_{ij}^2}\right)^{\lambda}\right]}$$

$$+ \delta \sum_{k=1}^{m_y} \sum_{l=1}^{r_y} \frac{(m_y - k) e^{-\frac{\gamma}{2} t_{kl}^2} t_{kl}^2 \left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)^{\delta - 1}}{\left[1 - \left(1 - e^{-\frac{\gamma}{2} t_{kl}^2}\right)^{\delta}\right]} = 0 \qquad (16)$$

Equations (14), (15) and (16) are non-linear equations and can be solved using any iterative technique. After obtaining MLE of λ, δ and γ, the MLE of $R_{s,k}$ is obtained by using invariance property of MLEs, that is,

$$\hat{R}_{s,k}^{Mrss} = \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^j \frac{\hat{\delta}}{[\hat{\lambda}(k + j - i) + \hat{\delta}]} \qquad (17)$$

5. BAYES ESTIMATION OF $R_{S,K}$

In this section, Bayes estimates are obtained by assuming parameters λ, δ and γ are unknown and independent random variables with gamma priors (a_i, b_i), i=1, 2, 3. The pdf of a gamma random variable X with parameters (a_i, b_i) is

$$f(x) = \frac{b_i^{a_i}}{\Gamma(a_i)} x^{a_i - 1} e^{-x b_i} \, ; x > 0, a_i > 0, b_i > 0$$

where $a_i, b_i > 0$, i = 1, 2, 3. Thus the joint prior λ, δ and γ is

$$g(\lambda,\delta,\gamma)=\frac{b_1^{a_1}}{\Gamma(a_1)}\frac{b_2^{a_2}}{\Gamma(a_2)}\frac{b_3^{a_3}}{\Gamma(a_3)}\lambda^{a_1-1}\delta^{a_2-1}\gamma^{a_3-1}e^{-b_1\lambda}e^{-b_2\delta}e^{-b_3\gamma};\ \lambda,\delta,\gamma>0,\ a_i,b_i>0, i=1,2,3$$

substituting L(λ, δ, γ) and g(λ, δ, γ) respectively, the corresponding joint posterior distribution is given by

$$\pi(\lambda,\delta,\gamma\mid z,t)=A\lambda^{n+a_1-1}\delta^{m+a_2-1}\gamma^{n+m+a_3-1}e^{-\frac{\gamma}{2}\left(\sum_{i=1}^{n}z_i^2+\sum_{j=1}^{m}t_j^2\right)}e^{-(\lambda-1)\sum_{i=1}^{n}\ln\left(1-e^{-\frac{\gamma}{2}z_i^2}\right)}$$
$$.\,e^{-(\delta-1)\sum_{j=1}^{m}\ln\left(1-e^{-\frac{\gamma}{2}t_j^2}\right)}$$

where

$$A^{-1}=\Gamma(n+a_1)\Gamma(m+a_2)\int_0^\infty\left[\gamma^{n+m+a_3-1}e^{-a_1\left(\sum_{i=1}^{n}z_i^2+\sum_{j=1}^{m}t_j^2\right)}e^{\sum_{i=1}^{n}\ln\left(1-e^{-\frac{\gamma}{2}z_i^2}\right)+\sum_{j=1}^{m}\ln\left(1-e^{-\frac{\gamma}{2}t_j^2}\right)}\right]$$
$$\left[\left(b_1+\sum_{i=1}^{n}\ln\left(1-e^{-\frac{\gamma}{2}z_i^2}\right)\right)^{n+a_1}\left(b_2+\sum_{j=1}^{m}\ln\left(1-e^{-\frac{\gamma}{2}t_j^2}\right)\right)^{m+a_2}\right]^{-1}d\gamma\,.$$

Here, Bayes estimator of $R_{s,k}$ is obtained as the posterior expectation of reliability of s-out-of-k system under squared error (SE) loss function :

$$\hat{R}_{s,k,B}=\int_0^\infty\int_0^\infty\int_0^\infty R_{s,k}\,\pi(\lambda,\delta,\gamma\mid z,t)\,d\lambda\,d\delta\,d\gamma \tag{18}$$

It is difficult to solve equation (18) analytically due to non-existence of closed form expression. Hence, one can use Lindley's approximation.

Lindley (Lindley (1980)) proposed the following approximation method which takes the ratio of the integrals as a whole and produces a single numerical result. If n is large, according to Lindley's approximation, any ratio of the integrals of the form

$$I(x)=E[u(\theta_1,\theta_2,\theta_3)]=\frac{\int_{(\theta_1,\theta_2,\theta_3)}u(\theta_1,\theta_2,\theta_3)e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)}d(\theta_1,\theta_2,\theta_3)}{\int_{(\theta_1,\theta_2,\theta_3)}e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)}d(\theta_1,\theta_2,\theta_3)}$$

where, $u(\theta)=u(\theta_1,\theta_2,\theta_3)$ is a function of θ_1,θ_2 and θ_3, $L(\theta_1,\theta_2,\theta_3)$ is log of likelihood, $G(\theta_1,\theta_2,\theta_3)$ is log of joint prior of θ_1,θ_2 and θ_3 can be written as

$$I(x)=u(\theta_1,\theta_2,\theta_3)+(u_1a_1+u_2a_2+u_3a_3+a_4+a_5)+\frac{1}{2}[A(u_1\sigma_{11}+u_2\sigma_{12}+u_3\sigma_{13})$$
$$+B(u_1\sigma_{21}+u_2\sigma_{22}+u_3\sigma_{23})+C(u_1\sigma_{31}+u_2\sigma_{32}+u_3\sigma_{33})]$$

where

$\hat{\theta}_1,\hat{\theta}_2$ and $\hat{\theta}_3$ are the MLEs of θ_1,θ_2 and θ_3 respectively.

$$a_i = \rho_1\sigma_{i1} + \rho_2\sigma_{i2} + \rho_3\sigma_{i3}\ ,\ i = 1,2,3,$$
$$a_4 = u_{12}\sigma_{12} + u_{13}\sigma_{13} + u_{23}\sigma_{23},$$
$$a_5 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22} + u_{33}\sigma_{33})$$
$$A = \sigma_{11}L_{111} + 2\sigma_{12}L_{121} + 2\sigma_{13}L_{131} + 2\sigma_{23}L_{231} + \sigma_{22}L_{221} + \sigma_{33}L_{331},$$
$$B = \sigma_{11}L_{112} + 2\sigma_{12}L_{122} + 2\sigma_{13}L_{132} + 2\sigma_{23}L_{232} + \sigma_{22}L_{222} + \sigma_{33}L_{332},$$
$$C = \sigma_{11}L_{113} + 2\sigma_{12}L_{123} + 2\sigma_{13}L_{133} + 2\sigma_{23}L_{233} + \sigma_{22}L_{223} + \sigma_{33}L_{333}$$

and subscripts 1,2,3 on right-hand sides refer to θ_1, θ_2 and θ_3 respectively, and

$$\rho_i = \frac{\partial\rho}{\partial\theta_i}, i = 1,2,3; \quad u_i = \frac{\partial u(\theta_1,\theta_2,\theta_3)}{\partial\theta_i}, i = 1,2,3,$$
$$u_{ij} = \frac{\partial^2 u(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j}, i, j = 1,2,3; \quad L_{ij} = \frac{\partial^2 L(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j}, i, j = 1,2,3,$$
$$L_{ijk} = \frac{\partial^2 L(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j\partial\theta_k}, i, j, k = 1,2,3$$

and σ_{ij} is the $(i, j)^{th}$ element of the inverse of the matrix having elements $\{-L_{ij}\}$.

In our case $(\theta_1,\theta_2,\theta_3) \equiv (\lambda,\delta,\gamma)$ and $u = u(\lambda,\delta,\gamma) = R_{s,k}$ for the prior distribution, we have,

$$\rho_1 = \frac{a_1 - 1}{\lambda} - b_1, \rho_2 = \frac{a_2 - 1}{\delta} - b_2, \rho_3 = \frac{c_3 - 1}{\gamma} - b_3$$

Also the values of L_{ij} can be obtained as follows for i, j = 1,2,3

$$L_{11} = -\frac{n}{\lambda^2}, L_{22} = -\frac{m}{\delta^2}$$

$$L_{13} = L_{31} = \sum_{i=1}^{n} \frac{z_i^2 e^{-\frac{\gamma}{2}z_i^2}}{2(1 - e^{-\frac{\gamma}{2}z_i^2})}, \quad L_{23} = L_{32} = \sum_{j=1}^{m} \frac{t_j^2 e^{-\frac{\gamma}{2}t_j^2}}{2(1 - e^{-\frac{\gamma}{2}t_j^2})}$$

$$L_{33} = -\frac{n+m}{\gamma^2} - (\lambda - 1)\sum_{i=1}^{n} \frac{z_i^4 e^{-\frac{\gamma}{2}z_i^2}}{4(1 - e^{-\frac{\gamma}{2}z_i^2})} - (\delta - 1)\sum_{j=1}^{m} \frac{t_j^4 e^{-\frac{\gamma}{2}t_j^2}}{4(1 - e^{-\frac{\gamma}{2}t_j^2})}$$

and the values of L_{ijk} for i, j, k = 1, 2, 3

$$L_{111}=\frac{2n}{\lambda^3}, L_{222}=\frac{2m}{\delta^3},$$

$$L_{133}=L_{331}=-\sum_{i=1}^{n}\frac{z_i^4 e^{-\frac{\gamma}{2}z_i^2}}{4(1-e^{-\frac{\gamma}{2}z_i^2})},\quad L_{233}=L_{332}=-\sum_{j=1}^{m}\frac{t_j^4 e^{-\frac{\gamma}{2}t_j^2}}{4(1-e^{-\frac{\gamma}{2}t_j^2})},$$

$$L_{333}=\frac{2(n+m)}{\gamma^3}+(\lambda-1)\sum_{i=1}^{n}\frac{z_i^6 e^{-\frac{\gamma}{2}z_i^2}(1-e^{-\frac{\gamma}{2}z_i^2})}{8(1-e^{-\frac{\gamma}{2}z_i^2})^3}+(\delta-1)\sum_{j=1}^{m}\sum_{j=1}^{m}\frac{t_j^6 e^{-\frac{\gamma}{2}t_j^2}(1-e^{-\frac{\gamma}{2}t_j^2})}{8(1-e^{-\frac{\gamma}{2}t_j^2})^3}$$

Here, $u=u(\lambda,\delta,\gamma)=R_{s,k}$,

$$u_1=\frac{\partial u}{\partial\lambda}=\sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^{j+1}\frac{\delta(k+j-i)}{[\lambda(k+j-i)+\delta]^2},$$

$$u_2=\frac{\partial u}{\partial\delta}=\sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^{j+1}\frac{\lambda(k+j-i)}{[\lambda(k+j-i)+\delta]^2},$$

$$u_{11}=\frac{\partial^2 u}{\partial\lambda^2}=\sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^{j}\frac{2\delta(k+j-i)^2}{[\lambda(k+j-i)+\delta]^3},$$

$$u_{22}=\frac{\partial^2 u}{\partial\delta^2}=\sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^{j+1}\frac{2\lambda(k+j-i)}{[\lambda(k+j-i)+\delta]^3}$$

$$u_{12}=\frac{\partial^2 u}{\partial\lambda\partial\delta}=\sum_{i=s}^{k}\sum_{j=0}^{i}\binom{k}{i}\binom{i}{j}(-1)^{j+1}\frac{(k+j-i)[\lambda(k+j-i)-\delta]}{[\lambda(k+j-i)+\delta]^3}$$

$u_3=\frac{\partial u}{\partial\gamma}=0$ and $u_{13}=u_{23}=u_{31}=u_{33}=0$. Hence

$$a_4=u_{12}\sigma_{12},\quad a_5=\frac{1}{2}(u_{11}\sigma_{11}+u_{22}\sigma_{22}),$$
$$A=\sigma_{11}L_{111}+\sigma_{33}L_{331},\quad B=\sigma_{22}L_{222}+\sigma_{33}L_{332},$$
$$C=2\sigma_{13}L_{133}+2\sigma_{23}L_{233}+\sigma_{33}L_{333}$$

Then, the Bayes estimator of $R_{s,k}$ is

$$\hat{R}^B_{s,k}=R_{s,k}+(u_1a_1+u_2a_2+a_4+a_5)+\frac{1}{2}[A(u_1\sigma_{11}+u_2\sigma_{12}) + B(u_1\sigma_{21}+u_2\sigma_{22})+C(u_1\sigma_{31}+u_2\sigma_{32})] \quad (19)$$

All the parameters are evaluated at $(\hat{\lambda},\hat{\delta},\hat{\gamma})$.

6. SIMULATION STUDY

A simulation study is conducted to compare the performance of the proposed estimators of system reliability for different sample sizes. 100000 random samples of size n, m, m_x, m_y, r_x and r_y were generated for the stress and strength populations, when (λ, δ, γ) = (4.5, 0.8, 1), (8, 0.3, 1.5) and (1.07, 1, 1) with the true values of stress-strength reliability for (s, k) = (1,3) are 0.9440, 0.8888, 0.7624 and when (s, k) = (2, 4) are 0.9038, 0.8126, 0.6180 respectively. The Bayesian estimates under squared error loss function using different informative priors are considered with gamma prior having c_1 = 7, c_2 = 3, c_3 = 1, d_1 = 3, d_2 = 2, d_3 = 1 (prior1) and c_1 = 1, c_2 = 1, c_3 = 1, d_1 = 1, d_2 = 1, d_3 = 1 (prior2).

From the tables 1,2 and 3, it is observed that MSEs of $\hat{R}_{s,k}^{Msrs}, \hat{R}_{s,k}^{Mrss}$ and $\hat{R}_{s,k}^{B}$ decreases with the increase in the sample size.

Table 1. MLEs under SRS and RSS, Bayes estimators and MSEs for the estimators of $R_{s,k}$

$\lambda=4,\ \delta=0.8, \gamma=1, r_x=5, r_y=5$, prior 1										
(s,k)	$R_{s,k}$	n	m	(m_x, m_y)	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{Mrss}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{Mrss}$)	MSE($\hat{R}_{s,k}^{B}$)
(1,3)	0.9440	10	10	(2,2)	0.9431	0.9367	0.9381	0.0126	0.0119	0.0032
		10	15	(2,3)	0.9598	0.9382	0.9333	0.0081	0.0046	0.0012
		10	25	(2,5)	0.9482	0.9479	0.9456	0.0083	0.0053	0.0049
		15	25	(3,5)	0.9460	0.9456	0.9459	0.0074	0.0052	0.0062
		25	25	(5,5)	0.9459	0.9438	0.9447	0.0028	0.0026	0.0018
(2,4)	0.9038	10	10	(2,2)	0.9485	0.9301	0.9291	0.0287	0.0248	0.0094
		10	15	(2,3)	0.9323	0.9286	0.9254	0.0216	0.0087	0.0086
		10	25	(2,5)	0.9205	0.9132	0.9101	0.0157	0.0029	0.0016
		15	25	(3,5)	0.9166	0.9028	0.9086	0.0101	0.0016	0.0012
		25	25	(5,5)	0.9046	0.9027	0.9016	0.0082	0.0012	0.0009

Table 2. MLEs under SRS and RSS, Bayes estimators and MSEs for the estimators of $R_{s,k}$

$\lambda=8,\ \delta=0.3, \gamma=1.5, r_x=5, r_y=5$, prior 2										
(s,k)	$R_{s,k}$	n	m	(m_x, m_y)	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{Mrss}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{Mrss}$)	MSE($\hat{R}_{s,k}^{B}$)
(1,3)	0.8888	10	10	(2,2)	0.8974	0.8765	0.8892	0.0402	0.0116	0.0162
		10	15	(2,3)	0.8912	0.8886	0.8864	0.0151	0.0102	0.0109
		10	25	(2,5)	0.8910	0.8824	0.8842	0.0126	0.0089	0.0088
		15	25	(3,5)	0.8982	0.8721	0.8801	0.0086	0.0042	0.0072
		25	25	(5,5)	0.8886	0.8844	0.8832	0.0077	0.0051	0.0081
(2,4)	0.8126	10	10	(2,2)	0.8283	0.8119	0.8240	0.0490	0.0325	0.0429
		10	15	(2,3)	0.8191	0.8142	0.8176	0.0330	0.0274	0.0332
		10	25	(2,5)	0.8146	0.8115	0.8141	0.0303	0.0017	0.0301
		15	25	(3,5)	0.8139	0.8102	0.8116	0.0135	0.0039	0.0132
		25	25	(5,5)	0.8127	0.8123	0.8120	0.0042	0.0029	0.0012

Table 3. MLEs under SRS and RSS, Bayes estimators and MSEs for the estimators of $R_{s,k}$

$\lambda=1.07,\ \delta=1, \gamma=1, r_x=5, r_y=5$, prior 1										
(s,k)	$R_{s,k}$	n	m	(m_x, m_y)	$\hat{R}_{s,k}^{Msrs}$	$\hat{R}_{s,k}^{Mrss}$	$\hat{R}_{s,k}^{B}$	MSE($\hat{R}_{s,k}^{Msrs}$)	MSE($\hat{R}_{s,k}^{Mrss}$)	MSE($\hat{R}_{s,k}^{B}$)
		10	10	(2,2)	0.7595	0.7584	0.7587	0.0224	0.0217	0.0216

(1,3)	0.7624	10	15	(2,3)	0.7688	0.7662	0.7631	0.0158	0.0126	0.0116
		10	25	(2,5)	0.7651	0.7620	0.7646	0.0013	0.0010	0.0012
		15	25	(3,5)	0.7648	0.7602	0.7632	0.0012	0.0009	0.0011
		25	25	(5,5)	0.7618	0.7522	0.7512	0.0006	0.0004	0.0003
(2,4)	0.6180	10	10	(2,2)	0.6085	0.6041	0.6082	0.0286	0.0237	0.0258
		10	15	(2,3)	0.6194	0.6166	0.6132	0.0243	0.0143	0.0229
		10	25	(2,5)	0.6192	0.6181	0.6176	0.0130	0.0122	0.0181
		15	25	(3,5)	0.6188	0.6183	0.6181	0.0126	0.0122	0.0116
		25	25	(5,5)	0.6185	0.6181	0.6183	0.0089	0.0030	0.0023

7. CONCLUSIONS

The reliability of a k component system in which s identical strength components sustain a common stress is studied. Here, each component of a system compresses on two dependent elements which are connected in a series. The k strengths $(X_1, Y_1), (X_2, Y_2), \ldots, (X_k, Y_k)$ follow bivariate generalized Rayleigh distribution and the stress T to follow generalized Rayleigh distribution. The reliability of the s-out-of k system is estimated by using maximum likelihood and Bayesian approaches. The maximum likelihood estimates are obtained under simple random sampling and as well as ranked set sampling schemes. Simulation results indicate that the MSEs for the estimators of $R_{s,k}$ decreases as the sample size increases. The MSEs of the ML estimator are greater than that of the Bayes estimator for priors considered. Also in this study, the MSEs of ML estimates based on RSS is lesser than that of ML estimators based on SRS indicating superiority of estimation under RSS.

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