



Proposed Algorithms Based on Accelerated Quantized Iterative Hard Thresholding for Compressed Sensing

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Abstract: Compressive sampling (CS) is a signal recovery technique that can effectively recover a sparse signal using fewer measurements than its dimension. Different recovery algorithms such as convex optimization, greedy algorithms, and iterative hard thresholding are used for exact recovery. The iterative hard thresholding algorithms are faster than convex optimization for compressed sensing recovery problems. In this paper, three proposed algorithms are introduced. These three proposed algorithms are based on Accelerated Quantized Iterative Hard Thresholding (AQIHT). They are double-over-relaxation AQIHT (AQIHT_{DR}), conjugate gradient AQIHT (AQIHT_{CG}) and a conjugate gradient double-over-relaxation AQIHT (AQIHT_{CGDR}). The double-over-relaxation algorithm (DR) is based on two over-relaxation steps and the conjugate gradient algorithm (CG) is based on computing the directional update and step size for efficient recovery. Extensive matlab simulation programs are executed to simulate the performance of the three proposed schemes. In addition, they are compared with the related ones. The performance metrics are signal to noise ratio (SNR), error (E) and iteration time. The proposed schemes have superior performance over the traditional ones. Moreover, the proposed mixed scheme has the best performance when compared to all other schemes.

Keywords: Compressed sensing, Sub-Nyquist sampling, AQIHT, Optimization techniques, Recovery algorithms

1. INTRODUCTION

CS is a new theory of sampling in many applications, including data network, sensor network, digital image and video camera, medical systems and analog-to-digital convertors [1]. It is an innovative method of sampling signals at a sub-Nyquist rate. The Shannon/ Nyquist sampling theorem states that an analogue signal can be reconstructed perfectly from its samples, if it was sampled at a rate at least twice the highest frequency present in the signal. For many signals, such as audio or images, the Nyquist rate can be very high leading to a very large number of samples. These samples must be compressed in order to store or transmit them which then places a high requirement on the equipment needed to sample the signal. Compressive Sensing overcomes these issues by using linear sampling operators that combine sampling and compression in a single step which reduces the number of measurements required.

Compressive sensing is an under-determined linear system which M-dimensional measurement vector y represented as:

$$y = \Phi x = \Phi \Psi s = \Theta s \quad (1)$$

where $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix, $x \in \mathbb{R}^{N \times 1}$ is signal to be sensed, $\Psi \in \mathbb{R}^{N \times N}$ is representation basis (basis matrix) it is an original or transform domain in which signal is sparse or compressible, $s \in \mathbb{R}^{N \times 1}$ is k-sparse vector $\Theta = \Phi \Psi$ [2].

However since $\Phi \in \mathbb{R}^{M \times N}$ and $M < N$, this makes the reconstruction of the signal x ill-posed. So, for good reconstruction we need to exploit the sparsity of x . Compressive sensing is concerned with low coherence $\mu(\Phi, \Psi)$ between the measurement matrix (Φ) and the basis matrix (Ψ) which leads to fewer measurements (M) required for signal reconstruction as shown in the following equation [3-5] :



$$M \geq c \cdot \mu^2(\Phi, \Psi) \cdot k \cdot \log(N) \quad (2)$$

where k is the sparsity of the signal, N is the length of the original signal and c is some positive constant.

The Restricted Isometry Property (RIP) is a sufficient condition on CS matrices which can ensure the performance of signal recovery. The verification of a matrix Φ having RIP of high order is computationally intensive. Therefore, an easier method of obtaining such a matrix is to use random matrices such as Bernoulli or Gaussian measurement matrices [6]. CS reconstruction can be solved using a convex relaxation of the recovery problem such as Basis Pursuit. *Basis Pursuit* (BP) depends on Linear Programming (LP) and can recover the signal with strong guarantees but it takes a long time to obtain the desired solution [7]. Furthermore, greedy algorithms such as Matching Pursuit MP, Orthogonal Matching Pursuit OMP [8] are based on selecting one column at each iteration until reaching the desired solution. In addition, iterative algorithms were proposed which based on gradient descent method such as the iterative hard thresholding (IHT) algorithm [9]. Contrary to IHT which uses fixed step size μ , the normalized IHT (NIHT) algorithm uses μ adaptively in each iteration to enhance convergence speed. QIHT is based on reconstructing sparse signal from quantized measurements [10].

This paper proposes three algorithms which are called double-over-relaxation AQIHT (AQIHT_{DR}), conjugate gradient AQIHT (AQIHT_{CG}) and a conjugate gradient double-over-relaxation AQIHT (AQIHT_{CGDR}). The proposed AQIHT_{CGDR} algorithm enhances the performance of the QIHT algorithm. Then, the performance comparisons are done between the three algorithms and the QIHT.

This paper is organized as follows: Section 2 presents some related previous recovery optimization algorithms. The proposed AQIHT algorithms are investigated in Section 3. The simulation analysis and the performance comparisons are discussed in Section 4. Finally, conclusions are made in Section 5.

1. PREVIOUS RELATED RECOVERY ALGORITHMS

In this section, some previous recovery algorithms will be introduced as follows:

A. QIHT algorithm [11]

It is an iterative algorithm that can reconstruct a sparse signal from quantized measurements. The quantized measurement vector $y \in \mathbb{R}^M$ was generated using an optimal Lloyd-Max 1-bit Quantizer Q_b as shown in equation:

$$y = Q_b(\Phi x_0)$$

$$\bar{x}^n = H_s(x^{n-1} + \mu \Phi^T(y - Q_b(\Phi x^{n-1}))) \quad (3)$$

where b represents the number of bits used to quantize each measurement and $b=1$, Q_b is the quantization operator defined at a resolution of 1-bit per measurement and y requires a bit budget $\beta = bM$ bits. QIHT uses hard thresholding operator H_s that keeps only s largest coefficients in magnitude and sets the other coefficients to zero. QIHT uses fixed step size that is set with $\mu = \frac{1}{M} \left(1 - \sqrt{\frac{2K}{M}}\right)$. The QIHT provides better performance than IHT while reconstruction was tested for $1 \leq b \leq 5$.

B. AIHT algorithm [12]

Accelerated methods aim to find an estimate x^n that satisfies two conditions:

1- x^n is k -sparse.

2- x^n satisfies $\|y - \Phi x^n\|_2 \leq \|y - \Phi \bar{x}^n\|_2$

Accelerated methods utilize two different approaches to satisfy these conditions. The first of which is double-over-relaxation approach that updates all elements of \bar{x}^n which uses two relaxation steps \bar{X}_1^n and \bar{X}_2^n . The second of which is conjugate gradient approach that only updates non zero elements in \bar{x}^n . The accelerated methods improve convergence speed for IHT and NIHT algorithms. These methods have the same guarantees such as IHT while a single DORE step converges in much fewer iterations than the IHT iteration [13].

2. PROPOSED ALGORITHMS

In this section, the three proposed algorithms will be discussed:

C. AQIHT_{DR}:

AQIHT_{DR} is an iterative hard thresholding algorithm that aims to reconstruct sparse signal from 1-bit quantized measurement. Then computing two relaxation steps \bar{X}_1^n and \bar{X}_2^n using a linear combination of the prior two estimates x^{n-1} , x^{n-2} with the line search parameters to minimize the cost function. The pseudo code of AQIHT_{DR} is shown below:



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Given :  $y, q, \Phi, s, Q, \mu$ , stopiter, maxiter
Initialize :  $x^0 = 0$ 
Steps
For  $n = 1$  to maxiter
     $x^n = H_s(x^{n-1} + \mu \Phi^T(q - Q(\Phi x^{n-1})))$ 
     $r^n = y - \Phi x^n$ 
If  $n > 2$ 
    1st over-relaxation step:
     $d^n = (\Phi x^n - \Phi x^{n-1})$ 
     $a_1^n = d^{T^n} * r^n / (d^{T^n} * d^n)$ 
     $\bar{X}_1^n = x^n + a_1^n(x^n - x^{n-1})$ 
     $p^n = (1 + a_1^n) * \Phi x^n - a_1^n * \Phi x^{n-1}$ 
     $r_1^n = y - p^n$ 
    2nd over-relaxation step:
     $\bar{d}^n = (p^n - \Phi x^{n-2})$ 
     $a_2^n = \bar{d}^{T^n} * r_1^n / (\bar{d}^{T^n} * \bar{d}^n)$ 
     $\bar{X}_2^n = \bar{X}_1^n + a_2^n(\bar{X}_1^n - x^{n-2})$ 
    Thresholding step:
     $r_2^n = y - \Phi * H_s(\bar{X}_2^n)$ 
    If  $(r_2^{nT} * r_2^n) / (r^{nT} * r^n) < 1$ 
         $x^n = H_s(\bar{X}_2^n)$ 
    End if
End if
break if  $x^{n-1} - x^n / x^n < \text{stopiter}$ 
End for
output  $x = x^n$ 

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The used notation can be defined as follows: x^n is the signal vector estimated after the nth iteration. x^{n-1} and x^{n-2} are the prior two estimates before the nth iteration. d^n is the difference estimated and d^{T^n} is the conjugate of the difference after the nth iteration. a_1^n is the first line search parameter estimated after the nth iteration. \bar{X}_1^n is the n^{th} iterative reconstructed signal after the first relaxation step. r_1^n denotes the first residual after the nth iteration. a_2^n is the second line search parameter estimated after the nth iteration. \bar{X}_2^n is the n^{th}

iterative reconstructed signal after the second relaxation step. r_2^n denotes the second residual after the nth iteration. Output x denotes the reconstructed signal.

The AQIHT_{DR} can be described as follows:

- Firstly, the sparse signal $x^0 = 0$ represents the initialization of algorithm.
- Calculating an initial update \bar{x}^n as in equation (1), then \bar{x}^n is combined with two previous estimates x^{n-1} , x^{n-2} and calculating the line search parameters a_1 and a_2 to minimize the quadratic cost functions $\|y - \Phi \bar{X}_1^n\|_2^2, \|y - \Phi \bar{X}_2^n\|_2^2$.
- Then, the new estimate \bar{X}_2^n is thresholded using hard thresholding operator H_s to guarantee it is k-sparse.
- Then if $\|y - \Phi H_s(\bar{X}_2^n)\|_2^2 > \|y - \Phi \bar{x}^n\|_2^2$ we set $x^n = \bar{x}^n$ otherwise we use $x^n = H_s(\bar{X}_2^n)$
- In the end updating the estimated x^n until reaching to the defined criterion $x^{n-1} - x^n / x^n < \text{stopiter}$.

D. AQIHT_{CG}:

The AQIHT_{CG} based on reconstructing sparse signal from 1-bit quantized measurement using an iterative hard thresholding algorithm. Then the QIHT is accelerated by using the conjugate gradient method. The conjugate gradient method is a greedy algorithm that uses directional updates that are conjugate to the previously chosen directions. The selection of new elements is based on projecting the index set of nonzero entries in \bar{x}^n on the inner product between the residual and the dictionary elements. Then updating x_Γ (which is an $|\Gamma|$ - dimensional sub vector of x) in direction p_Γ^n which is equivalent to updating x using a directional vector p^n of dimension N, with all elements zero apart from the element indexed by Γ . The pseudo code of AQIHT_{CG} is shown below:



Given $y, \Phi, \Gamma = \text{supp}(\bar{x}^1)$, $\text{maxiter}=3, \text{stopiter}$
initialize: $\bar{x}^1, r^1 = y - \Phi \bar{x}^1, \nabla^1 = \Phi^T * r^1, p^0, c^0 = 0$

steps

For $n=1$ to maxiter

$$z^n = \Phi * \text{Proj}_\Gamma(\nabla^n)$$

$$b_1^n = -c^{n-1T} * z^n / c^{n-1T} * c^{n-1}$$

$$p_\Gamma^n = \nabla_\Gamma^n + b_1^n * p_\Gamma^{n-1}$$

$$c^n = z^n + b_1^n * c^{n-1}$$

$$a^n = r^{nT} * c^n / (c^{nT} * c^n)$$

$$x^{n+1} = x^n + a^n * p^n$$

$$r^{n+1} = r^n - a^n * c^n$$

$$\nabla^{n+1} = \Phi^T * r^{n+1}$$

break if $(r^{nT} * r^n - r^{n+1T} * r^{n+1}) / y^T * y < \text{stopiter}$

End for

output $x = x^n$

The used notation can be defined as follow: Γ denotes the index set of nonzero entries in estimated \bar{x}^1 . $\text{Proj}_\Gamma(\nabla^n)$ stands for projecting the estimated gradient ∇^n on to the fixed support set Γ followed by setting to zero all the entries not on the support. ∇_Γ^n is a sub-vector of ∇ that containing only those elements of ∇ with indices in Γ after the n th iteration. p_Γ^n is the update direction after the n th iteration. z^n and c^n are vectors. a^n is the step-size after the n th iteration. The conjugate gradient algorithm can be summarized as follows:

- Firstly, the sparse signal \bar{x}^n is calculated from QIHT equation (1). The residual $r^1 = y - \Phi \bar{x}^1$ and the gradient $\nabla^1 = \Phi^T * r^1$ represent the initialization of algorithm.
- The directional vector p^n of dimension N contains zero elements except the element indexed by Γ from the gradient.
- Then c^n is calculated by the projection of ∇_Γ^n on to the atoms of Φ_Γ .

- Then, obtain step-size a^n which only depends on the current residual and the c^n .
- At each step, x^{n+1} can be calculated iteratively by calculating a new direction and step size in each iteration.
- In the end updating the estimated x^{n+1} until reaching to the defined criterion $(r^{nT} * r^n - r^{n+1T} * r^{n+1}) / y^T * y < \text{stopiter}$ or reach the maximum number of steps which equals 3.

E.AQIHT_{CGDR}:

AQIHT_{CGDR} combines two algorithms. The first of which, conjugate gradient method aims to update only the nonzero element in \bar{x}^n from eq (1). Three conjugate gradient steps are made in each QIHT iteration. At each step, x^n is calculated iteratively by computing a new direction and step size in each iteration. In the first step, the new direction is computed by applying the support that contains non zero indices of \bar{x}^n on the inner product between the residual and the dictionary elements followed by setting to zero all the entries not on the support. In the other steps, the new direction is a combination of the current gradient projected onto the same support set and the previous direction. The second algorithm, double-overrelaxation approach, is used to reduce the specific cost function $\|y - \Phi \bar{X}_2^n\|_2^2$ and is formed of two steps. The first relaxation step \bar{X}_1^n is computed by using a linear combination of the current estimate x^n and prior estimate x^{n-1} with the line search parameter a_1^n to minimize $\|y - \Phi \bar{X}_1^n\|_2^2$. The second relaxation step \bar{X}_2^n is computed by using a linear combination of the first relaxation step \bar{X}_1^n and prior estimate x^{n-2} with the line search parameter a_2^n to minimize the $\|y - \Phi \bar{X}_2^n\|_2^2$. Then the estimated \bar{X}_2^n is thresholded by using the thresholding operator H_s to guarantee that it is k -sparse. The pseudo code of AQIHT_{CGDR} can be found by combining the two codes mentioned above.



2. SIMULATION ANALYSIS

A comparison between QIHT and the proposed three accelerated QIHT approaches will be provided. They are compared with variation of bit rate and its effect on the algorithms according to SNR, error and iteration time.

A. Initialization:

In the matlab program, the normalized input sparse signal (x) with length N=1024 and sparsity level k=16 is sensed using 1000 Gaussian random distribution matrix (A) to obtain the measurement vector (y).

In the recovery process comparison between (QIHT, AQIHTDORE, AQIHTCG and AQIHTCGDORE) algorithms and all algorithms were stopped once

$\|x^{n-1} - x^n\| \|x^n\|^{-1} \leq 10^{-4}$ or if max number of iteration n=1000.

B. Metric Terms:

Run time which is defined as the time used to reach to the desired output is computed. It can be calculated by using tic and toc in the simulation, signal to noise ratio SNR=

$\left[-20 \log_{10} \frac{\|x\|}{\|\hat{x}\|} \right]$ and Error E = $\|x - \hat{x}\|_2$ where \hat{x} represents the estimated signal after the nth iterations and where x represents the initial signal.

C. Results and Discussions:

This part studies the effect of variation of the measurement which is noted by $\beta = bM$ where $\beta \in \{64, 128, \dots, 1280\}$ and b=1 on the performance of the four algorithms as shown in Figures 1-3.

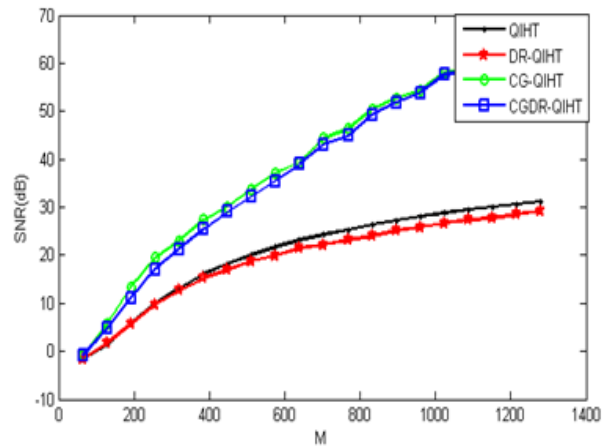


Figure 1: Comparison of average SNR, for QIHT algorithm and proposed approaches AQIHT_{DR}, AQIHT_{CG} and AQIHT_{CGDR} using 1-bit quantization.

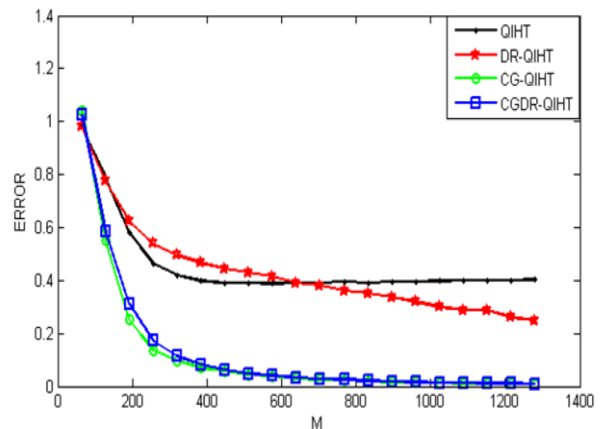


Figure 2: Comparison of average error, for QIHT algorithm and proposed approaches AQIHT_{DR}, AQIHT_{CG} and AQIHT_{CGDR} using 1-bit quantization.

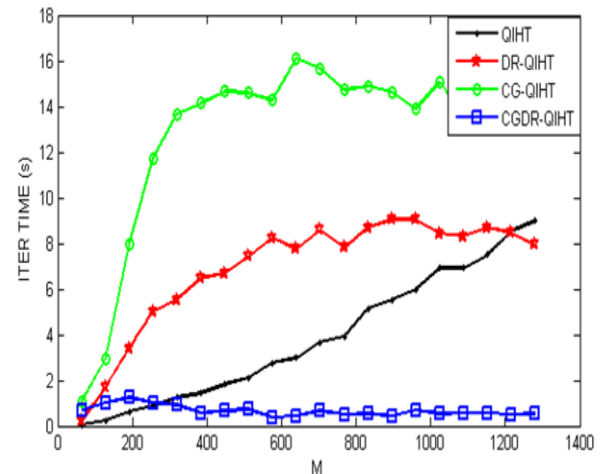


Figure 3: Comparison of average iteration time, for QIHT algorithm and proposed approaches AQIHT_{DR}, AQIHT_{CG} and AQIHT_{CGDR} using 1-bit quantization.



Figure 1 shows that SNR increases with increasing M, $AQIHT_{CGDR}$ and $AQIHT_{CG}$ have superior performance than QIHT and $AQIHT_{DR}$. The reason for this is that $AQIHT_{CGDR}$ and $AQIHT_{CG}$ are based on conjugate gradient method that aims to optimize fixed indices at each iteration which determined by nonzero elements in initial update \bar{x}^n . At (M) =576, the $AQIHT_{CG}$ has higher SNR than $AQIHT_{CGDR}$ by 1.72 dB and QIHT has higher SNR than $AQIHT_{DR}$ by 1.78 dB. $AQIHT_{CG}$ is better in reconstructing optimal solution than $AQIHT_{DR}$ because it is based on greedy strategy.

Figure 2 indicates that with increasing the value of M, the error decreases due to increasing the number of measurements that contain the important information that represents the signal. It is noticeable that for $AQIHT_{CG}$ and $AQIHT_{CGDR}$, the error closes to zero when $\beta = 800$. At (M) =576, the QIHT provides less error than $AQIHT_{DR}$ by 0.03 and $AQIHT_{CG}$ provides slightly less error than $AQIHT_{CGDR}$ by 0.01. Figure 3 shows that iteration time increases with increasing M in all algorithms except $AQIHT_{CGDR}$ remains relatively constant. $AQIHT_{CGDR}$ takes the least iteration time to reach the solution because it combines between $AQIHT_{CG}$ which provides higher SNR and $AQIHT_{DR}$ which gives higher convergence that reaches the solution in minimum iteration time. From this Figure, we can notice that $AQIHT_{DR}$ takes less time than $AQIHT_{CG}$ because it is based on iterative strategy. At (M) =576, $AQIHT_{CGDR}$ reaches optimal solution at less time than $AQIHT_{CG}$ by 13.93 and QIHT takes less time than $AQIHT_{DR}$ by 5.46.

3. CONCLUSIONS

Three Accelerated QIHT approaches are based on iterative and greedy strategies for reconstructing a sparse signal from quantized measurement using 1-bit quantization in a compressed sensing system had been proposed in this paper. Examination of the performance of a conjugate gradient method, a double-over-relaxation approach and a conjugate gradient double-over-relaxation method while varying bit rate was conducted. Results show that conjugate gradient method presents only slightly better signal to noise ratio than the conjugate gradient double-over-relaxation method. The conjugate gradient double-over-relaxation method shows improvement in QIHT performance by 13.72.

However, conjugate gradient double-over-relaxation method presents the best convergence speed and shows a 2.41 improvement over the related one. Therefore, the conjugate gradient double-over-relaxation method appears to be the best method for reconstructing a sparse signal from quantized measurement.

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