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On the Moment Estimation of Parameters of Triangular Credibility Distribution

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Abstract: In this paper, the problem of estimating underlying parameters of a membership function namely, triangular membership function is considered. Estimation of parameters has been carried out by using two different approaches which are the extensions of an estimation technique available in statistical theory namely, method of moments. A numerical comparative study about the efficiency of those two approaches in estimating the parameters of triangular membership function has also been carried out.

Keywords: Fuzzy variable, Triangular membership function, Triangular credibility distribution, Method of moments, Empirical membership function, Empirical moments.

1. Introduction

Researchers working in various domains of investigation often encounter systems having uncertainties of different forms. The causes of uncertainties can be broadly classified into two. One is objective uncertainty due to randomness and other is subjective uncertainty due to fuzziness. In order to handle situations involving fuzziness, Zadeh [11] initiated a theory called fuzzy set theory in which membership functions play vital role. Generally, the membership functions are the one which helps to characterize and understand the behavior of a system possessing fuzziness. In some cases, for an entity involved in the system being studied while it may not be possible to associate a single numerical quantity, a set of possible numeric quantities could be assigned in a graded manner. Membership functions introduced by Zadeh [11] are mathematical tools meant for performing this task. For example, the membership function

$$\mu_A(x) = \begin{cases} 0.2 & if \quad x = 7 \\ 0.8 & if \quad x = 8 \\ 0.5 & if \quad x = 9 \\ 0 & otherwise \end{cases}$$

describes values which an entity A (say, intelligence level measured in a 10 point scale) can take when $\{7,8,9\}$ are the possible numerical values A can assume. Given the values in the set $\{7,8,9\}$, the above membership function gives maximum support to the value 8 for being associated with A when compared to other values. The values different from those three cannot be associated with the entity A. Among the three possible values listed above which receive the support, one with least support is 7 next to 9.

In fuzzy set theory, several membership functions possessing nice mathematical properties are available. Equipossible, Triangular and Trapezoidal membership functions are some popular membership functions available in the literature. These membership functions depend on certain constants. To understand the characteristics of a fuzzy system it is necessary to know their values. For example, consider the triangular membership function

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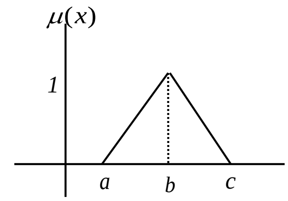
$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a} & \text{if } a\le x\le b\\ \dfrac{x-c}{b-c} & \text{if } b\le x\le c\\ 0 & \text{otherwise}\end{cases}$$



consisting of three constants namely, a,b and c. The above membership function assigns positive membership values when the argument lies in the interval (a,c) and zero for other values. The above membership function has its maximum value (membership value 1) at only one point namely, b and decreases towards zero in either direction when the argument tends away from b and move either towards a or c. It should be noted that the values of membership function lie in the interval $[0,1]$. Consider a situation where the time taken to reach office from home by a person is treated as an imprecise quantity which can lie in between 40 and 55. The above membership function can be used to express this situation by taking $a=40$, $b=45$ and $c=55$ if there is enough room to justify 45 is the time taken with the maximum belief (the membership value attaining maximum at that point) and the belief level decreases for values deviating from 45 in either direction. It may be noted that the above function completely eliminates the values of time taken outside the interval $(40,55)$ by assigning the membership value zero for them.

It is to be noted that once the form of membership function has been identified, the underlying parameters of the membership functions need to be estimated to understand the imprecise situation. Methods like method of moments, method of least squares, method of maximum likelihood are some popular methods available in statistical theory for estimating the parameters involved in probability distributions. Developing methods similar to these estimation procedures for estimating the parameters involved in membership functions require tools comparable with distribution function, moments, etc., available in statistical theory. However such tools are not available in fuzzy set theory developed by Zadeh [11]. Li and Liu [2] developed a theory called credibility theory meant for understanding fuzzy situations in which majority of tools comparable with those available in statistical theory are present. Refinements in credibility theory were carried out by Liu [3], Li and Liu [4] and Liu [5]. In this work, a method of estimation similar to the method of moments available in statistical estimation theory is considered by making use of the tools available in credibility theory to estimate the parameters involved in triangular membership function.

So far no investigations have been carried out particularly in estimating the credibility distribution or credibility membership functions. This paper considers the problem of estimating fuzzy membership function available in credibility theory. This study is motivated by the works of Chen and Ralescu [1], Wang, Gao and Guo [9] and Wang and Peng [10] which are related to uncertainty distributions available in Uncertainty Theory. It is to be noted that uncertainty theory developed by Liu [6] is a new branch of mathematics meant for handling imprecise situations.

In statistical theory of estimation, the parameters involved in a distribution are estimated using a sample which contains values of crisp nature. In this paper the parameters involved in a membership function are estimated with the help of an experimental data obtained from one or more experts'. The expert (experts') is expected to give a sequence of values covering the range of values which the variable under study can assume along with the membership value representing their belief degree level corresponding to those values. Therefore, the data provided by the experts' will be of the form (x_i,θ_i), $i=1,2,...,n$ where x_i is the i^{th} possible value that the variable under study can take and θ_i is the corresponding belief degree level (ranging from 0 to 1) given by the expert. It is also assumed that the experts' assign the membership value 1 to only one possible value and 0 to the possible values x_1 and x_n. This requirement creates scope for two different approaches in estimating the three parameters involved in triangular membership function. It is to be noted that the triangular membership function assigns the membership 1 for exactly one value. Hence, one may be inclined to use for the parameter b, the value reported by the expert with membership 1 and estimate the remaining two parameters else all the three parameters can be estimated using the experimental data. This paper compares the performances of above mentioned two approaches in estimating triangular membership function.

This paper is organized as follows. Section two briefly introduces the basic concepts of credibility theory and discusses certain properties with reference to triangular membership function. Estimation of fuzzy triangular membership using two different estimation approaches (extension of method of moments) is illustrated with an experts' experimental data in section three. In section four, a comparative study about the two different approaches of estimation are presented. Section five summarizes the findings of this work.

2. CREDIBILITY THEORY

A. Credibility measure space

The set function Cr is called a credibility measure if it satisfies the axioms of normality, monotonicity, self-duality and Maximality defined in Liu and Liu [2]. The triplet (Θ, P, Cr) where Θ is a non-empty set, P being $\sigma-$algebra over Θ is known as credibility space.

B. Fuzzy variable

Fuzzy variable ξ is a measurable function from the credibility space (Θ, P, Cr) to the set of real numbers.

C. Membership function

The membership function μ of a fuzzy variable ξ defined on the credibility measure space (Θ, P, Cr) is defined as

$$\mu(x) = \left(2\left(Cr\left(\xi = x\right)\right)\right) \wedge 1,\ x \in R\,. \tag{1}$$

Triangular membership function is the one which is employed in this paper and is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a} & if\ \ a \le x \le b \\ \dfrac{x-c}{b-c} & if\ \ b \le x \le c \\ 0 & otherwise \end{cases} \tag{2}$$

where (a,b,c) are known real values.

D. Credibility inversion theorem

Let ξ be a fuzzy variable with membership function μ. Then for any set B of real numbers

$$Cr\{\xi \in B\} = \frac{1}{2}\left(\operatorname*{Sup}_{x\in B} \mu(x) + 1 - \operatorname*{Sup}_{x\in B^c} \mu(x)\right) \tag{3}$$

E. Credibility distribution

Let ξ be a fuzzy variable defined on the credibility measure space (Θ, P, Cr) with membership function μ. Then the credibility distribution is defined as

$$\Phi(x) = Cr\{\theta \in \Theta / \xi(\theta) \le x\}\quad \forall\ x \in R\,. \tag{4}$$

F. Triangular credibility distribution

Let ξ be a fuzzy variable with triangular membership function μ given in (2). Then the distribution function of the fuzzy variable defined by Liu [7] is

$$\Phi(x) = \begin{cases} 0 & if\ \ x \le a \\ \dfrac{x-a}{2(b-a)} & if\ \ a \le x \le b \\ \dfrac{x+c-2b}{2(c-b)} & if\ \ b \le x \le c \\ 1 & if\ \ x \ge c \end{cases} \tag{5}$$

G. Expected value

Let ξ be a fuzzy variable with the credibility distribution function $Cr(\xi \le r)$. Then by the definition of Liu [7], the expected value of the fuzzy variable is given by

$$E(\xi) = \int_0^{\infty} Cr(\xi \ge r)dr - \int_{-\infty}^{0} Cr(\xi \le r)dr \tag{6}$$

H. Moments

Let ξ be a fuzzy variable . Then the k^{th} moment of fuzzy variable defined by Liu [7] is

$$E(\xi^K) = K\int_0^\infty r^{K-1} Cr(\xi \geq r)\, dr \tag{7}$$

where the expected value $E(\xi^k)$ is called the k^{th} moment of fuzzy variable and k is a positive integer.

Using the above definition, first three moments of triangular credibility distribution have been derived and are given below

$$E(\xi) = \frac{a+2b+c}{4} \tag{8}$$

$$E(\xi^2) = \frac{a^2+ab+2b^2+bc+c^2}{6} \tag{9}$$

$$E(\xi^2) = \frac{a^3+a^2b+ab^2+2b^3+b^2c+bc^2+c^3}{8} \tag{10}$$

It is interesting to note that the moments of triangular credibility distributions derived are same as the moments of uncertainty zigzag distribution given by Liu [8].

3. Estimation of Parameters in Fuzzy Triangular Membership Function

In this section, estimation of parameters in triangular membership function using a method of estimation similar to the method of moments available in statistical theory is considered. To maintain the readability the method of moments used in statistical theory of estimation is explained below.

A. *Method of Moments*

Let X be a random variable. Assume that $\theta_1, \theta_2, \cdots, \theta_k$ be the k unknown parameters of the distribution $F_X(\theta)$ to be estimated . Denote by $\mu_1', \mu_2', \cdots, \mu_k'$ the first k raw population moments which are in general functions of the unknown parameters $\theta_1, \theta_2, \cdots, \theta_k$. Equating the raw population moments with the corresponding raw sample moments $\hat{\mu}_1', \hat{\mu}_2', \cdots, \hat{\mu}_k'$,the system of equations

$$\begin{aligned}
&\mu_1'(\theta_1, \theta_2, \ldots, \theta_k) = \hat{\mu}_1' \\
&\mu_2'(\theta_1, \theta_2, \ldots, \theta_k) = \hat{\mu}_2' \\
&\vdots \\
&\mu_k'(\theta_1, \theta_2, \ldots, \theta_k) = \hat{\mu}_k'
\end{aligned}$$

are formed and solutions of the parameters $\theta_1, \theta_2, \cdots, \theta_k$ in terms of the sample moments $\hat{\mu}_1', \hat{\mu}_2', \cdots, \hat{\mu}_k'$ are obtained. These solutions are taken as the moment estimates of the parameters.

This method of moments can be modified suitably for the purpose of estimating parameters in a fuzzy membership function. One possible modification is to adopt the method of moments explained above on using empirical moments (computed using the empirical membership function) based on experimental data in the place of sample moments towards the estimation of a membership function.

The empirical membership function based on the experimental data can be constructed by using the definition of Liu [8] which is stated below

B. *Empirical Membership function*

Let $(x_1, \alpha_1), (x_2, \alpha_2), \ldots, (x_n, \alpha_n)$ be a set of experimental data such that $x_1 < x_2 < \cdots < x_n$. The empirical membership function is defined as

$$\mu_n(x) = \begin{cases} \alpha_i + \dfrac{(\alpha_{i+1}-\alpha_i)(x-x_i)}{(x_{i+1}-x_i)} & \text{if } x_i \leq x \leq x_{i+1}, 1 \leq i < n \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Here, x_i represents the i^{th} value in experimental dataset for which the expert has belief level α_i. It is assumed that x_1 is the largest value below which all other values have belief level 0 and x_n is the smallest value above which all other values have the belief level 0. It is to be noted that these values serve as infimum and supremum of the possible values of the fuzzy variable.

The above empirical membership function can be written as

$$\mu_n(x) = \begin{cases} w_{x,i} & if \quad x_i \le x \le x_{i+1}, 1 \le i < n \\ 0 & otherwise. \end{cases} \tag{12}$$

where $w_{x,i} = \alpha_i + \frac{(\alpha_{i+1} - \alpha_i)(x - x_i)}{(x_{i+1} - x_i)}$ is the empirical membership value associated with the possible values x lying between x_i and x_{i+1}.

Now for obtaining the empirical moments, the empirical distribution function needs to be determined. On using the empirical membership function defined in (12) the empirical distribution function is obtained as

$$\Phi_n(x) = \begin{cases} 0 & if \quad x < x_1 \\ \frac{w_{x,i}}{2} & if \quad x_1 \le x < x_r \\ 1 - \frac{w_{x,i}}{2} & if \quad x_r \le x < x_n \\ 1 & if \quad x \ge x_n \end{cases} \tag{13}$$

where x_r is the point for which the domain experts assigns the membership value 1.

The empirical distribution function given above can be used to compute the empirical moments by using the formula given in (7). The first three empirical moments are given by

$$E(\hat{\xi}) = \int_0^\infty Cr(\xi \ge r)\, dr \tag{14}$$

$$E(\hat{\xi}^2) = 2\int_0^\infty r \;\; Cr(\xi \ge r)\, dr \tag{15}$$

and

$$E(\hat{\xi}^3) = 3\int_0^\infty r^2 \;\; Cr(\xi \ge r)\, dr \tag{16}$$

The number of empirical moments to be computed depends on the choice of method of moments one intends to use. If it is decided to take the value corresponding to membership 1 as an estimate of b, one needs to consider the first two empirical and theoretical moments only. That is, if x_r is the value corresponding to membership 1 as provided by the expert then x_r is taken as the estimated value of b and the other two parameters namely, a and c are estimated by solving the following two equations

$$\frac{a + 2x_r + c}{4} = E(\hat{\xi})$$

and

$$\frac{a^2 + ax_r + 2x_r^{\,2} + x_r c + c^2}{6} = E(\hat{\xi}^2)$$

Alternatively, the method of moments can be applied to estimate all the three parameters with the help of first three moments and its corresponding empirical moments. That is, the parameters a, b and c are obtained by solving the following three equations

$$\frac{a + 2b + c}{4} = E(\hat{\xi})$$

$$\frac{a^2 + ab + 2b^2 + bc + c^2}{6} = E(\hat{\xi}^2)$$

$$\frac{a^3 + a^2 b + ab^2 + 2b^3 + b^2 c + bc^2 + c^3}{8} = E(\hat{\xi}^3)$$

To illustrate the above two procedures of estimating the parameters, let us consider a hypothetical data which is of the form (x_i, θ_i) where x_i's are the possible values that the variable under study can take and θ_i's are the corresponding belief degree level(membership value) given by the domain experts. Let the data be

$$(3,0), (5,0.2), (7,0.65), (9,1), (11,0.8), (13,0.5), (15,0)$$

Then the empirical membership function of the experimental data is given by

$$\mu_n(\theta)=\begin{cases} 0.1\theta-0.3 & if\ \ 3\le\theta<5 \\ \dfrac{0.45\theta-1.85}{2} & if\ \ 5\le\theta<7 \\ \dfrac{0.35\theta-1.15}{2} & if\ \ 7\le\theta<9 \\ 1.9-0.1\theta & if\ \ 9\le\theta<11 \\ \dfrac{4.9-0.3\theta}{2} & if\ \ 11\le\theta<13 \\ \dfrac{7.5-0.5\theta}{2} & if\ \ 13\le\theta<15 \\ 0 & otherwise \end{cases} \tag{17}$$

From the empirical membership function given above the empirical triangular credibility distribution function is derived using the credibility inversion theorem given by Liu [7]. It is given by

$$\Phi_n(\theta)=\begin{cases} 0 & if\ \ \theta<3 \\ \dfrac{0.1\theta-0.3}{2} & if\ \ 3\le\theta<5 \\ \dfrac{0.45\theta-1.85}{4} & if\ \ 5\le\theta<7 \\ \dfrac{0.35\theta-1.15}{4} & if\ \ 7\le\theta<9 \\ 1-\dfrac{1.9-0.1\theta}{2} & if\ \ 9\le\theta<11 \\ 1-\dfrac{4.9-0.3\theta}{4} & if\ \ 11\le\theta<13 \\ 1-\dfrac{7.5-0.5\theta}{4} & if\ \ 13\le\theta<15 \\ 1 & if\ \ \theta\ge 15 \end{cases} \tag{18}$$

It may be observed that a triangular distribution is suitable for the given experimental data. Note that, in experimental data the belief degree level of domain experts for the possible value 9 is 1. Hence, 9 is taken as the estimated value of parameter b. Therefore, for estimating the other two parameters namely, a and c involved in triangular distribution, it is sufficient if first two moments of true distribution and first two empirical moments are available.

The first two moments of triangular credibility distribution on substituting the value of b are

$$E(\xi^{\bullet})=\frac{a+18+c}{4} \text{ and}$$
$$E(\xi^{\bullet 2})=\frac{a^2+9a+162+9c+c^2}{6} \tag{19}$$

Now the empirical moments can be obtained by using the empirical triangular credibility distribution given in (18). The first empirical moment is given by

$$\begin{aligned} E(\hat{\xi}) &= \int_0^3 dx+\int_3^5\left(1-\frac{0.1x-0.3}{2}\right)dx+\int_5^7\left(1-\frac{0.45x-1.85}{4}\right)dx+\int_7^9\left(1-\frac{0.35x-1.15}{4}\right)dx \\ &+\int_9^{11}\left(\frac{1.9-0.1x}{2}\right)dx+\int_{11}^{13}\left(\frac{4.9-0.3x}{4}\right)dx+\int_{13}^{15}\left(\frac{7.5-0.5x}{4}\right)dx \\ &=9.45 \end{aligned} \tag{20}$$

Similarly the second empirical moment is calculated as

$$E(\hat{\xi}) = 2\left\{\begin{array}{l} \int_0^3 x dx + \int_3^5 \left(x - \frac{0.1x^2 - 0.3x}{2}\right)dx + \int_5^7 \left(x - \frac{0.45x^2 - 1.85x}{4}\right)dx + \int_7^9 \left(x - \frac{0.35x^2 - 1.15x}{4}\right)dx \\ \quad + \int_9^{11} \left(\frac{1.9x - 0.1x^2}{2}\right)dx + \int_{11}^{13} \left(\frac{4.9x - 0.3x^2}{4}\right)dx + \int_{13}^{15} \left(\frac{7.5x - 0.5x^2}{4}\right)dx \end{array}\right\}$$
$$= 101.8333 \tag{21}$$

By equating the two true distribution moments given in (19) with the empirical moments given in (20) and (21), the value of two unknown parameters namely, a and c has been estimated as $(3.785198, 16.014802)$. Hence, the estimated value of the parameters a, b and c involved in fuzzy triangular membership function using the first approach (estimating two parameters only) is $(3.785198, 9, 16.014802)$.

Now for estimating the parameters using second approach of method of moments in which all the three parameters are treated as unknown, the third empirical moment is needed. It is computed as

$$E(\hat{\xi}) = 3\left\{\begin{array}{l} \int_0^3 x^2 dx + \int_3^5 \left(x^2 - \frac{0.1x^3 - 0.3x^2}{2}\right)dx + \int_5^7 \left(x^2 - \frac{0.45x^3 - 1.85x^2}{4}\right)dx + \int_7^9 \left(x^2 - \frac{0.35x^3 - 1.15x^2}{4}\right)dx \\ \quad + \int_9^{11} \left(\frac{1.9x^2 - 0.1x^3}{2}\right)dx + \int_{11}^{13} \left(\frac{4.9x^2 - 0.3x^3}{4}\right)dx + \int_{13}^{15} \left(\frac{7.5x^2 - 0.5x^3}{4}\right)dx \end{array}\right\}$$
$$= 1199.25 \tag{22}$$

By equating the first three theoretical moments given in (8),(9) and (10) and empirical moments given in (20),(21) and (22) of triangular membership function, the values of underlying parameters have been estimated as $(3.323186, 9.445478, 15.585857)$.

It is to be mentioned that in most of the studies involving imprecise quantities, obtaining the higher order theoretical moments and empirical moments are difficult. Therefore, the idea of assigning values to some parameters depending on the experts' belief level could reduce the difficulty to some extent.

The following section presents a comparative study on the efficiency of two approaches considered in this work.

4. Comparative Study

In order to determine the efficiency of two approaches of method of moments available in statistical theory of estimation, the absolute errors involved in estimation procedure have been calculated using

$$Err = \sum_i \left|\mu_n(\theta_i) - \alpha_i\right| \tag{23}$$

where $\mu_n(\theta_i)$ is the empirical membership value of the i^{th} possible value based on the estimated values of the unknown parameters a, b and c and α_i is the belief degree given to the i^{th} possible value by the experts.

The absolute error due to the first approach is 0.3664715 and second approach of estimation is 0.4235127. Hence, for the data considered, the first approach of estimation is more efficient than the second approach.

Further in order to make a detailed study on the two methods, five data sets of different lengths have been considered including the one considered under section three. The data sets considered in this study are

$(3,0),(5,0.2),(7,0.65),(9,1),(11,0.8),(13,0.5),(15,0)$

$(3,0),(5,0.2),(6,0.65),(8,1),(10,0.8),(11,0.5),(13,0)$

$(3,0),(6,0.2),(10,0.65),(14,1),(19,0.8),(22,0.5),(25,0)$

$(3,0),(7,0.2),(14,0.65),(18,1),(23,0.8),(29,0.5),(33,0)$ and

$(3,0),(9,0.2),(18,0.65),(23,1),(31,0.8),(37,0.5),(43,0)$

From each of the data set given above thirty experimental sets have been generated by inducing small disturbances namely, $+0.05$ and -0.05 randomly to the experts' belief degree levels. The absolute errors due to first and second approaches in estimating the underlying parameters of triangular membership function are tabulated below. In the following table, the captions A1 and A2 indicate the first and second approaches considered in this paper. As mentioned earlier, the first approach treats the value corresponding to membership 1 as the estimate of b and estimates the remaining two parameters whereas the second approach estimates all the three parameters using the method of moments.

TABLE I. ABSOLUTE ERROR DUE TO TWO ESTIMATION METHODS

S.NO	Data 1		Data 2		Data 3		Data 4		Data 5	
	A1	A2	A1	A2	A1	A2	A1	A2	A1	A2
1	0.368	0.395	0.618	0.639	0.683	0.802	0.273	0.347	0.540	0.564
2	0.428	0.522	0.598	0.649	0.500	0.592	0.483	0.493	0.386	0.507
3	0.389	0.434	0.504	0.542	0.598	0.674	0.483	0.493	0.430	0.435
4	0.368	0.395	0.561	0.578	0.598	0.674	0.483	0.493	0.518	0.624
5	0.428	0.522	0.598	0.649	0.598	0.674	0.366	0.479	0.413	0.518
6	0.368	0.395	0.324	0.348	0.602	0.649	0.430	0.362	0.637	0.546
7	0.459	0.516	0.324	0.348	0.602	0.649	0.366	0.479	0.575	0.417
8	0.327	0.348	0.435	0.447	0.393	0.419	0.398	0.428	0.413	0.518
9	0.459	0.516	0.630	0.659	0.627	0.769	0.542	0.408	0.430	0.435
10	0.395	0.429	0.460	0.482	0.603	0.693	0.430	0.362	0.575	0.417
11	0.334	0.330	0.534	0.552	0.627	0.769	0.398	0.428	0.520	0.472
12	0.297	0.320	0.534	0.552	0.500	0.592	0.542	0.408	0.386	0.507
13	0.487	0.548	0.405	0.436	0.525	0.639	0.531	0.495	0.535	0.449
14	0.327	0.348	0.422	0.453	0.506	0.572	0.366	0.479	0.430	0.435
15	0.466	0.507	0.561	0.578	0.683	0.802	0.398	0.428	0.540	0.564
16	0.340	0.409	0.598	0.649	0.377	0.478	0.644	0.505	0.386	0.507
17	0.334	0.330	0.536	0.572	0.472	0.580	0.600	0.508	0.540	0.564
18	0.428	0.522	0.460	0.482	0.602	0.649	0.398	0.428	0.413	0.518
19	0.428	0.522	0.641	0.666	0.500	0.592	0.483	0.493	0.575	0.417
20	0.340	0.409	0.405	0.436	0.500	0.543	0.366	0.479	0.637	0.546
21	0.332	0.414	0.460	0.482	0.506	0.572	0.531	0.495	0.332	0.327
22	0.466	0.507	0.519	0.533	0.602	0.649	0.430	0.362	0.535	0.449
23	0.368	0.395	0.598	0.649	0.590	0.693	0.408	0.424	0.441	0.453
24	0.297	0.320	0.641	0.666	0.377	0.478	0.408	0.424	0.575	0.417
25	0.340	0.409	0.630	0.659	0.590	0.693	0.483	0.493	0.575	0.417
26	0.327	0.348	0.517	0.555	0.488	0.525	0.382	0.471	0.332	0.327
27	0.556	0.626	0.517	0.555	0.506	0.572	0.408	0.424	0.430	0.435
28	0.297	0.320	0.561	0.578	0.525	0.639	0.531	0.495	0.386	0.507
29	0.459	0.516	0.422	0.453	0.377	0.478	0.382	0.471	0.430	0.435
30	0.428	0.522	0.405	0.436	0.472	0.580	0.430	0.362	0.291	0.400
Average	**0.388**	**0.437**	**0.514**	**0.543**	**0.538**	**0.623**	**0.446**	**0.447**	**0.474**	**0.471**

From the table, it is clear that estimation of parameters involved in triangular membership function by first approach is more efficient in almost all data sets except the last one. However, in that case, the gain in efficiency of the second approach over the first approach is not significant. Hence, the first approach is recommended for all the data sets.

5. CONCLUSION

In this paper, the problem of estimating parameters involved in triangular credibility distribution function has been considered. Two approaches which are the extended versions of an estimation technique available in statistical theory namely, method of moments have been proposed for estimating those parameters. The estimation procedures were illustrated with the help of a hypothetical experts' experimental data. A comparative study on the efficiency of two approaches has also been carried out using some simulated data sets and it was found that the first approach of estimation, which assigns value (corresponding to membership value unity) for one parameter and estimates the remaining two using empirical moments performs better than the second approach of estimation which estimates all the three parameters.

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