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A Study on Block Design with Censored Data

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Abstract: The analysis of data from block designs arises in different fields. Various parametric and nonparametric procedures are available for analysis of such data. But analysis of such design with censored data has not been as widely studied.. In this paper we want to study the performance of few existing tests when some of the observations are right censored. Two different kinds of scores are used for this purpose. Results are obtained through simulation Discussion are made based on the obtained results. For few situations results are shown graphically for easy visual comparison.

Keywords: Block Design, Gehan Scores, Logrank Scores, Censored data, Simulation

1. Introduction

A problem frequently faced by applied statistician is the analysis of time to event data or censored data. Such type of data arises in a number of applied fields, such as medicine, life science, public health, epidemiology, engineering, economics, demography, agriculture, etc. The time to event data present themselves in different ways which create special problems in analyzing such data. The analysis of data from block designs, numerous nonparametric procedures are available. But the analysis when the data is subjected to censoring has not been as widely studied for block designs. Sampford and Taylor (1959) have investigated normal theory approaches to the problem when censoring exists. Patel (1975) developed a statistic for testing equality of treatment effects when there is one observation per cell that uses Gehan (1965) type scores. Woolson and Lachenbruch(1981) derived a class of linear rank tests under local power alternatives for the test of equal treatment effects in the one observation per cell case. Patel's test, as well as a test using logrank scores, are two special cases of this class. Test procedures for designs with more than one observation per cell have received little attention.

In this paper we have studied the performance of some tests that are used in block design analysis when some of the observation are right censored.

2. Description of the test:

Let us consider the procedures for testing no treatment effects in a randomized block design with t treatments, b blocks, and n observations in each cell. The observations are subject to right censoring. Using the notation given by Groggel,et.al(1987) ,we denote the true response variable of interest by Y_{ijk} and C_{ijk} denote the censoring time for the k^{th} observation in the cell corresponding to treatment i and block j. Here, we assume C_{ijk} are independent and their distribution does not depend on i when H_0 is true, and they are independent of the Y_{ijk}. Let X_{ijk} , which is the minimum of Y_{ijk} and C_{ijk} , be the observed value. Also define δ_{ijk} to be one if the ijk-th observation is not censored and zero if it is censored.

Assume that the random variables Y_{ijk} are independent with absolutely continuous distribution function F_{ij}. Then the null hypothesis of interest is

$$H_0 : F_{1j} = F_{2j} = \dots = F_{tj} \quad \text{for all j.}$$

Now before use the observation in test statistic ,replace each observation with a score which depends on the magnitude of X_{ijk} as well as whether the observation is censored or not. Two different kinds of scores that may be used

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for censored data are Gehan and logrank scores. Groggel,et.al(1987) defined Gehan scores and Logrank scores as like following eq.(1) and eq(2). The Gehan scores, U_{ijk}, for the ijk-th observation is simply the number of observations in block j, known to be less than Y_{ijk} minus the number of observations known to exceed Y_{ijk}. That is,

$$U_{ijk} = \sum_{a=1}^{t}\sum_{b=1}^{n}[I(X_{ijk} > X_{ajb})\delta_{ajb} - I(X_{ijk} < X_{ajb})\delta_{ijk}], \qquad (1)$$

where I(A) is equal to one if event A occurs and zero otherwise. Denoting R_{ijk} the rank of X_{ijk} within block j, the log-rank score for the ijk-th observation, L_{ijk}, can be written as

$$L_{ijk} = \sum_{a=1}^{t}\sum_{b=1}^{n}\frac{I(X_{ijk} \geq X_{ajb})\delta_{ajb}}{(tn+1-R_{ajb})} - \delta_{ijk} \qquad (2)$$

Here, the summation term in L_{ijk} is simply the number of uncensored observations less than or equal to X_{ijk} divided by the inverse rank of X_{ijk} within block j.

3.1 Woolson-Lachenbruch Type Statistic

Groggel et al(1987) obtain a test statistic for block design using the approach of Woolson and Lachenbruch(1981) when there is more than one observation in a cell. Derivation of test statistic may be as follows: Let S_{ijk} denote the score (Gehan or Logrank e.g. eq(1) or eq.(2)) for the ijk-th observation. Again, let $W_i = \sum_{j=1}^{b}\sum_{k=1}^{n} S_{ijk}$ and $\underline{W}' = (W_1, W_2, \ldots, W_t)$.

When Ho is true, by using properties of finite sampling distributions as outlined in Miller(1981), , we have

$$E_o(W_i) = 0, \quad Var_o(W_i) = \frac{n(t-1)}{t(nt-1)}A,$$

And $Cov_o(W_i, W_a) = \dfrac{-n}{t(nt-1)}A$,

Where $A = \sum_{i=1}^{t}\sum_{j=1}^{b}\sum_{k=1}^{n} S_{ijk}^2$. W

when Ho is true,

$$\underline{H} = [(nt-1)/(nA)]^{1/2}\underline{W}$$

is a vector with zero mean and idempotent covariance matrix of rank t-1. When Ho is true, $\underline{H}$ as n $\to \infty$ (b fixed) follows asymptotically normal (Breslow(1970). Again when Ho is true, and as b $\to \infty$ (n fixed) $\underline{H}$ follows asymptotically normal [Liaponov's central limit theorem or by following similar steps as in Woolson and Lechenbruch (1981)]. Here we have to mind that we must eliminate blocks in which all of the scores are identical (e,g. all observations in a block are censored) to obtain our result as b $\to \infty$.

From the earlier results when Ho is true, the conditional distribution of $\underline{H}'\underline{H}$ is limiting (n $\to \infty$ or b $\to \infty$) chi-square with t-1 degrees of freedom. Hence an approximate test for H_0 is obtained by rejecting H_0 if

$$\mathbf{T} = \frac{(tn-1)}{nA}\sum_{i=1}^{t} W_i^2 \qquad (3)$$

exceeds the upper 100% cutoff point for a chi-square distribution.

We denote T by TG when used Gehan scores and by TL when used logrank scores.

3.2 Friedman-Type Statistic based on Mack-Skillings Test

Mack and Skillings(1980) developed a statistic based on rank for randomized block design when number of observations per cell. Replacing rank r_{ijk} by scores S_{ijk} as mentioned above, Groggel et. al develop a Friedman type statistic and the resulting statistic is given as

$$\mathbf{FT} = \frac{12b}{N(n+b)}\sum_{i=1}^{t}[\sum_{j=1}^{b}\sum_{k=1}^{n} r_{ijk} - \frac{N+b}{2}]^2 \qquad (4)$$

Where N = tbn. When we use Gehan score in FT it is denoted by FTG and when using logrank score it is denoted by FTL. This statistic has asymptotic chi-square distribution with t-1 degrees of freedom as b and /or n go to infinity. One minor drawback to this statistic is the fact that a number of ties in the S_{ijk} can occur when there is a high percentage of censored observations within a block. Mid-ranks may be used to handle ties.

4. Description of Simulation Procedure

The goals of simulation are to

(1) Investigate the power of the tests

(2) Investigate how well each procedure controls the level of significance using the large sample approximations

(3) Investigate how those block effects affect the performance of the tests

Here we have used the uniform censoring distribution as like the Latta (1981) and Lee, Desu and Gehan (1975).Designs with different combinations of t = treatment, b = number of blocks, and n = number of observations per cell were shown in respective tables.

For the k-th observation in the cell corresponding to treatment i and block j, we generated a true response value, Y_{ijk}, from a distribution and compared it to a censoring time, C_{ijk}, which was generated from a uniform distribution. The response values and censoring times were generated using subroutine RND in a COMPAQ Micro Computer and using the inverse integral transform for the exponential and Weibull deviates as given in Hahn and Shapiro(1967). The observed value, X_{ijk} , was set to be the minimum of Y_{ijk} and C_{ijk} and the censoring indicator, d_{ijk}, is one if $X_{ijk} = Y_{ijk}$ and zero otherwise.

Response values were generated from the exponential distribution with parameter w_{ijk} and Weibull distribution with shape parameter equal to 4 and scale parameter w_{ij}. The treatment effects, τ_i , i =1,. . ., t, were used to modify the distribution parameter w_{ij}. The value of τ_i were indicated in the tables.

The block effect was a shift in the response distribution parameter so that for this situation $w_{ij} = \tau_i + \beta_j$. The values of β_j used are given in table 5(b) and 6(b). In this case the distribution of C_{ijk} was uniform (0,M) where M was chosen to give an expected percent censored of 25% when $w_{ij} = 1$.

The appropriate values of M are 3.9207 for exponential and 3.9207 for Weibull.

Every configuration in the study was replicated 10000 times and the proportion of rejections of null hypothesis was recorded using α = .10, .05 and .01 and the appropriate chi-square or F distribution. Each experiment was also performed without censoring so the effects of the censoring could be observed.

Table 1 Empirical Levels of Test Statistics under Exponential Distribution With Censored Data at level α = .05

t b n	Treament effects τ_i	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Logrank Scores	FTG-Gehan Scorse	FTL-Logrank Scores
3 3 2	1 1 1	.0439	.0345	.0402	.0364	.0296	.0354
5		.0450	.0489	.0446	.0450	.0396	.0434
8		.0477	.0475	.0460	.0510	.0472	.0484
10		.0496	.0488	.0476	.0442	.0436	.0470
3 5 2		.0472	.0449	.0392	.0494	.0378	.0368
5		.0477	.0463	.0464	.0508	.0448	.0428
8		.0468	.0450	.0474	.0506	.0468	.0434
10		.0495	.0504	.0506	.0508	.0478	.0490
3 10 2		.0453	.0469	.0410	.0464	.0402	.0462
5		.0495	.0509	.0508	.0460	.0410	.0444
8		.0481	.0498	.0498	.0604	.0528	.0520
10		.0438	.0458	.0422	.0494	.0438	.0472
5 3 2	1 1 1 1 1	.0442	.0442	.0436	.0342	.0294	.0262
5		.0392	.0392	.0384	.0472	.0392	.0396
8		.0410	.0410	.0460	.0460	.0454	.0450
10		.0466	.0466	.0480	.0460	.0470	.0498
5 5 2		.0488	.0490	.0408	.0414	.0300	.0352
5		.0436	.0436	.0500	.0454	.0404	.0404
8		.0444	.0444	.0494	.0466	.0392	.0478
10		.0424	.0424	.0478	.0446	.0460	.0444
5 10 2		.0484	.0484	.0458	.0438	.0374	.0398
5		.0474	.0474	.0474	.0466	.0446	.0406
8		.0486	.0486	.0536	.0466	.0450	.0422
10		.0398	.0398	.0444	.0524	.0464	.0492

Table 2. Empirical Powers of Test Statistics under Exponential Distribution with Censred Data at level α = .05

t b n	Treament effects τ_i	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Logrank Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 5 2	1 3 8	.1123	.0619	.1134	.5406	.4950	.5050
5		.3442	.2408	.3262	.9864	.9708	.9626
8		.5235	.3594	.5114	.9994	.9986	.9974
10		.6536	.4686	.6596	1.000	1.000	1.000
3 5 2		.2190	.1710	.2168	.8430	.7896	.7804
5		.5521	.4482	.5510	1.000	.9990	.9978
8		.7750	.6718	.7816	1.000	1.000	1.000
10		.8796	.7992	.8790	1.000	1.000	1.000
3 10 2		.4404	.3938	.4358	.9934	.9880	.9834
5		.8800	.8402	.8794	1.000	1.000	1.000
8		.9854	.9724	.9854	1.000	1.000	1.000
10		.9954	.9926	.9954	1.000	1.000	1.000
5 3 2	1 2 3 4 5	.0788	.0788	.0698	.3972	.3086	.3066
5		.1706	.1706	.2566	.9176	.8472	.8154
8		.2614	.2614	.4212	.9968	.9806	.9724
10		.3638	.3638	.5272	.9984	.9952	.9936
5 5 2		.1304	.1304	.1534	.6830	.5832	.5662
5		.3468	.3468	.4528	.9956	.9832	.9778
8		.5466	.5466	6698	1.000	1.000	.9992
10		.6736	.6736	.7872	1.000	1.000	1.000
5 10 2		.3112	.3106	.3514	.9662	.9172	.9050
5		.7314	.7314	.7876	1.000	1.000	.9998
8		.9226	.9226	.9522	1.000	1.000	1.000
10		.9678	.9678	.9822	1.000	1.000	1.000

Table 3. Empirical Levels of Test Statistics under Weibull Distribution with Censored Data at level α = .05

t b n	Treament effects τ_i	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Logrank Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 3 2	1 3 8	.0524	.0524	.0394	.0398	.0796	.0510
5		.0472	.0472	.0518	.0498	.0560	.0466
8		.0494	.0494	.0462	.0490	.0498	.0418
10		.0486	.0486	.0474	.0458	.0436	.0392
3 5 2		.0522	.0522	.0496	.0454	.0532	.0652
5		.0452	.0452	.0500	.0496	.0572	.0530
8		.0440	.0442	.0486	.0486	.0620	.0492
10		.0526	.0526	.0442	.0472	.0548	.0450
3 10 2		.0512	.0510	.0494	.0486	.0594	.0604
5		.0510	.0510	.0448	.0480	.0584	.0594
8		.0470	.0470	.0452	.0504	.0578	.0534
10		.0470	.0470	.0454	.0470	.0504	.0520
5 3 2	1 1 1 1 1	.0548	.0546	.0466	.0428	.0308	.0264
5		.0504	.0504	.0474	.0418	.0400	.0298
8		.0438	.0438	.0480	.0452	.0420	.0376
10		.0482	.0482	.0466	.0496	.0420	.0418
5 5 2		.0488	.0488	.0436	.0450	.0588	.0390
5		.0472	.0472	.0524	.0496	.0424	.0366
8		.0458	.0458	.0474	.0532	.0500	.0464
10		.0458	.0458	.0470	.0494	.0466	.0412
5 10 2		.0470	.0470	.0452	.0400	.0532	.0666
5		.0444	.0444	.0466	.0504	.0620	.0524
8		.0422	.0422	.0480	.0508	.0620	.0484
10		.0468	.0468	.0460	.0486	.0564	.0480

Table 4. Empirical Powers of Test Statistics under Weibull Distribution with Censored Data at level α = .05

t b n	Treament effects τ_i	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Logrank Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 3 2	1 3 8	.1626	.1624	.2298	.1920	.5460	.5132
5		.4564	.4564	.6146	.6474	.8544	.8516
8		.6876	.6876	.8470	.8776	.9496	.9524
10		.7834	.7834	.9114	.9400	.9746	.9778
3 5 2		.3558	.3560	.4130	.3746	.8604	.8286
5		.7778	.7778	.8574	.9824	.9908	.9910
8		.9408	.9408	.9722	8784	.9992	.9992
10		.9802	.9802	.9942	.9968	.9998	1.000
3 10 2	1 2 3 4 5	.7090	.7090	.7494	.7300	.9972	.9950
5		.9890	.9890	.9942	.9954	1.000	1.000
8		1.000	1.000	1.000	1.000	1.000	1.000
10		1.000	1.000	1.000	1.000	1.000	1.000
5 3 2		.1410	.1412	.1704	.7300	.4334	.4000
5		.3100	.3100	.4602	.9954	.6538	.6482
8		.4908	.4906	.7048	1.000	.8154	.8210
10		.6138	.6138	.8092	1.000	.8826	.8870
5 5 2		.2452	.2452	.2908	.1602	.7814	.7398
5		.5940	.5940	.7218	.5028	.9386	.9402
8		.8386	.8386	.9168	.7542	.9840	.9848
10		.9108	.9108	.9640	.8648	.9952	.9952
5 10 2		.5528	.5526	.6044	.2998	.9938	.9898
5		.9432	.9432	.9642	.7650	1.000	1.000
8		.9962	.9962	.9980	.9450	1.000	1.000
10		.9992	.9992	1.000	.9806	1.000	1.000

Table 5(a): Empirical Level of the Test Statistics under Exponential Distribution at level α = .05

t b n	Treat-ment Effects τ_i	Block Effects β_j	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Lgrak Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 5 2	1 1 1	0 .1 .2 .3 .4	.0492	.0436	.0440	.0472	.0390	.0368
			.0450	.0428	.0470	.0408	.0448	.0472
			.0435	.0420	.0580	.0508	.0472	.0434
			.0424	.0415	.0440	.0490	.0496	.0526

Table 5(b): Empirical Powers of the Test Statistics under Exponential Distribution at level α = .05

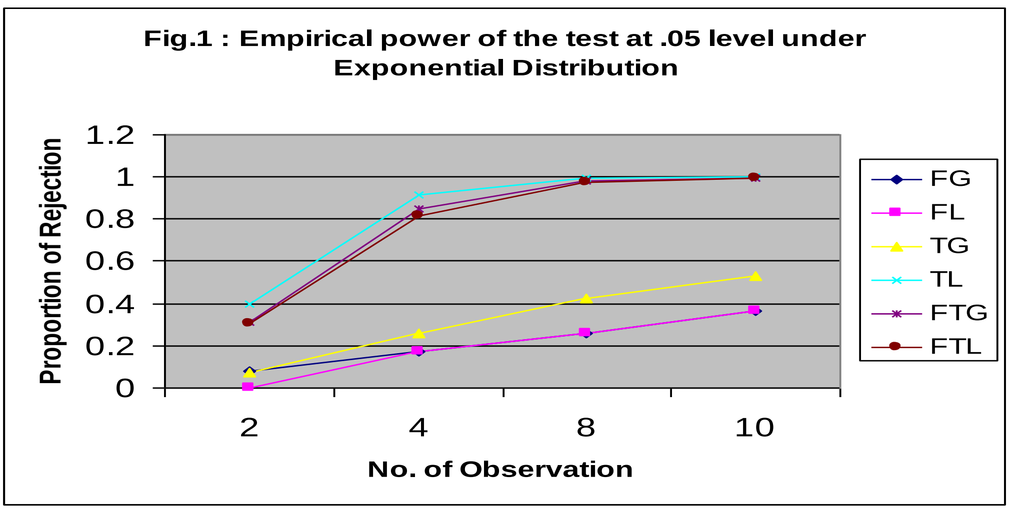
t br n	Treat-ment Effects τ_i	Block Effects β_j	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Lgrak Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 5 2	1 3 8	0 .1 .2 .3 .4	.2802	.1524	.1610	.7874	7354	.7332
5			.4840	.3402	.4460	.9996	.9972	.9948
8			.6524	.5924	.6430	1.000	1.000	1.000
10			.7904	.7205	.7710	1.000	1.000	1.000

Table 6(a): Empirical Level of the Test Statistics under Weibull Distributionat level α = .05

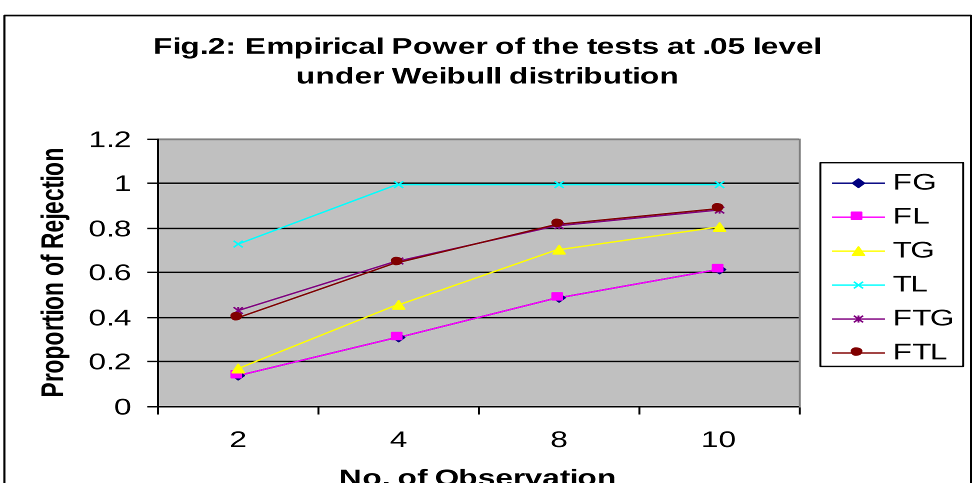
t br n	Treat-ment Effects τ_i	Block Effects β_j	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Lgrak Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 5 2	1 1 1	0 -.5 .5 -1 1	.0532	.0518	.0454	.0450	.0762	.0620
5			.0570	.0564	.0558	.0518	.0650	.0612
8			.0430	.0440	.0440	.0482	.0601	.0602
10			.0470	.0474	.0468	.0538	.0520	.0560

Table 6(b): Empirical Powers of the Test Statistics under Weibull Distribution at level α = .05

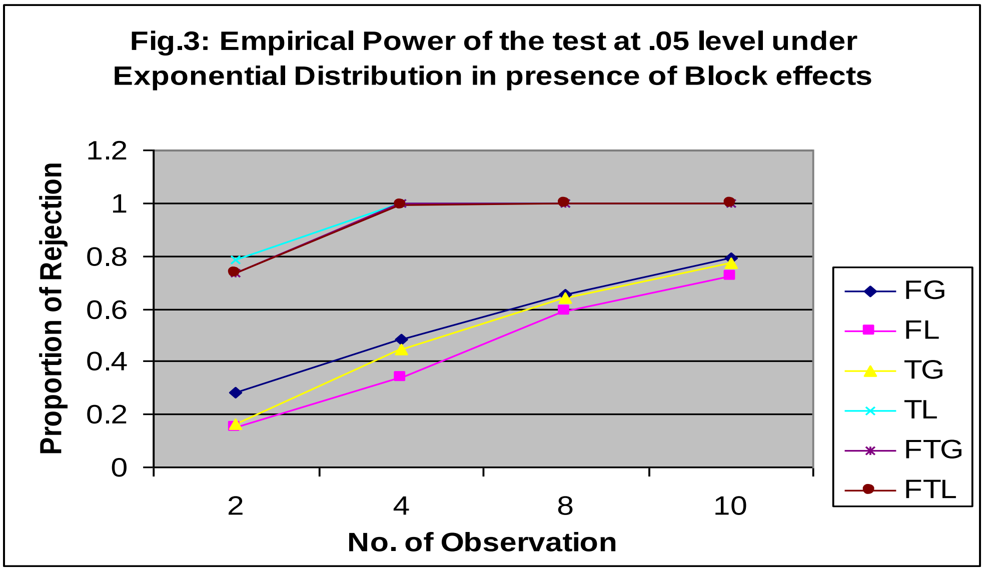
t b n	Treat-ment Effects τ_i	Block Effects β_j	FG-Gehan Scores	FL-Logrank Scores	TG-Gehan Scores	TL-Lgrak Scores	FTG-Gehan Scores	FTL-Logrank Scores
3 5 2	1 3 8	0 -.5 .5 -1 1	.6454	.6408	.6878	.6178	.7820	.7140
5			.9780	.9760	.9940	.9856	.7712	.7042
8			1.000	.9994	1.000	1.000	.7625	.6942
10			1.000	1.000	1.000	1.000	.6724	.6452



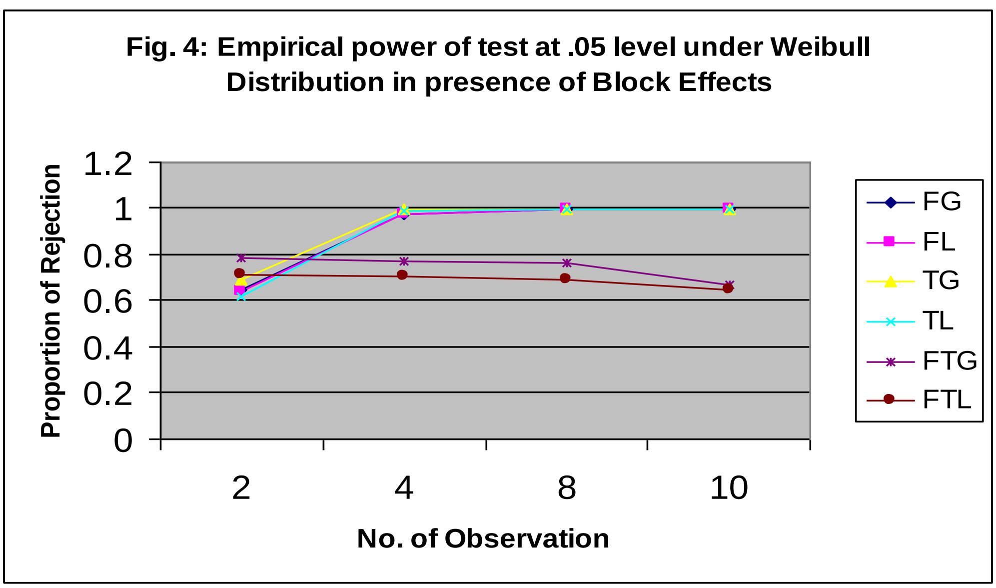
Here t = 5,b = 3,n = 2; τ_i = 1, 2, 3, 4, 5



Here t = 5,b = 3,n = 2; τ_i = 1, 2, 3, 4, 5



t = 3,b = 5,n = 2; τ_i = 1, 3, 4; β_j = 0, 0.1,0.2,0.3,0.4



t = 3,b = 5,n = 2; τ_i = 1, 3, 4; β_j = 0, 0.1,0.2,0.3,0.4

5. Results and Discussion

Table 1 contains the results of simulation under exponential distribution. From the table we see that all the tests control the theoretical level of significance. Only Mack-Skillings statistics seems to be conservative in some situations for both the scores.

Table 2 contains the powers of test statistics under exponential distribution. We have seen that Woolson-Lachenbruch type statistic performs better when logrank scores are used rather than Gehan scores. Friedman Type Mack and Skillings test do well for both type of scores although its power is slightly less than Woolson-Lachenbruch test under logrank scores but better tan Woolson test under Gehan scores. Power of F test is less under both the scores than the other two tests.

Table 3 contains the simulation results for test statistics under the Weibull distribution. Table displays the empirical level of the test statistics. We observe that all the tests control the levels in both the scores. We have also notice that when the cell sizes increases all the statistics satisfies the level very well.

Comparison of the powers are given in table 4 . We see that power of Mack-Skillings test under both the scores is more than the other two tests. Here again, we have seen that power of F test is less than the both other test statistics. We have also observed that power of Friedman Type Mack-Skillings test under both scores are almost equal.

Table 5(a,b) shows the level and power of tests in presence of block effects β_j. The block effects on Y_{ijk} is the distribution parameter w_{ij} i.e , $w_{ij} = \tau_i + \beta_j$. The value of β_j are shown in table 5.

From the table 5(a,b)and 6(a,b) we have seen that in presence of block effects , power of all the tests statistics decreases for the both type of scores .But overall performance regarding two tests remain almost same .That is ,the conclusion obtain in the above (the Table 2 and Table 4) also valid in this situation.

6. SUMMARY

From the simulation study we have the found the following:

1. All the test statistics control the level when cell sizes increases.
2. Power of Woolson-Lachenbruch is more powerful under logrank scores under exponential distribution
3. Friedman Type Mack-Skillings test is powerful for both scores under Weibull distribution.

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