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دراسة المرونة الحرارية ذات الرتب الكسرية لجسم لانهائي يحتوى على فجوة كروية باستخدام القيم الذاتية

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الملخص:

في هذا البحث تم اعتبار مسألة مرونة حرارية لجسم لانهائي يحتوى على فجوة كروية وذلك في سياق نظرية المرونة الحرارية ذات الرتب الكسرية. وبالأخذ في الاعتبار الشروط الحدية بتعرض السطح الداخلي للفجوة لومضة حرارية في حالة انعدام الاجهاد، وبتحويل النموذج الرياضي في الصورة اللابعدية وتطبيق تحويلات لابلاس تم وضع المعادلات في صورة المعادلات التفاضلية المصفوفية والمتجهة. تم تطبيق طريقة القيم الذاتية للحصول على الحلول التحليلية بصورة تامة وبدون تقريبات. وبأخذ تطبيق عددي واجراء تحويل لابلاس العكسي تم الحصول على النتائج العددية وبرسمها تم توضيح تأثير بارامتر الرتب الكسرية على الكميات الفيزيائية المعتبرة.

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ORIGINAL ARTICLE

Eigenvalue approach to fractional order thermoelasticity for an infinite body with a spherical cavity

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Abstract In this article, we consider the problem of a thermoelastic infinite body with a spherical cavity in the context of the theory of fractional order thermoelasticity. The inner surface of the cavity is taken traction free and subjected to a thermal shock. The form of a vector–matrix differential equation has been considered for the governing equations in the Laplace transform domain. The analytical solutions are given by the eigenvalue approach. The graphical results indicate that the fractional parameter effect plays a significant role on all the physical quantities.

1. Introduction

Biot (1956) modified the classical uncoupled theory of thermoelasticity by eliminating the paradox that elastic changes have no effect on the temperature. The heat equations for both theories predict infinite speeds of propagation for heat waves. So, various generalized theories of thermoelasticity were developed. Lord and Shulman, 1967 established the first model generalized thermoelasticity theory (LS). Green and Lindsay (1972) proposed the temperature rate dependent thermoelasticity (GL) theory. During the second half of twentieth century, a large amount of work has been devoted to solving thermoelastic problems. This is due to their many applications in widely diverse fields. In the contexts of the thermoelasticity theories, the counterparts of our problem have been considered by using analytical and numerical methods (Abbas, 2008, 2012, 2014; Abbas and Abo-Dahab, 2014; Abbas and Kumar, 2014; Abbas and Othman, 2012; Abbas and Zenkour, 2013; Abd-alla and Abbas, 2002; Dhaliwal and Sherief, 1980; Sherief and Anwar, 1988, 1989; Sherief et al., 2004; Zenkour and Abbas, 2014a,b).

Fractional calculus has been used successfully to modify many existing models of physical processes e.g., viscoelasticity, chemistry, electronics, wave propagation and biology. One can state that the whole theory of fractional derivatives and integrals was established in the second half of the nineteenth century. Various definitions and approaches of fractional derivatives have become the main purpose of many studies. Youssef (2010) and Youssef and Al-Lehaibi (2010) established the fractional order generalized thermoelasticity of both weak and strong heat conductivity in the context of generalized

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thermoelasticity were considered, and the corresponding variational theorem for fractional order generalized thermoelasticity was developed. A new model of fractional heat equation established by Ezzat (2011b) and Ezzat and El-Karamany (2011a,b). In addition, Sherief et al. (2010) established a new model by using the form of the heat conduction law. Kumar et al. (2013) studied the plane deformation due to thermal source in fractional order thermoelastic media.

In this work, we consider fractional order generalized thermoelasticity of an infinite body with spherical cavity under thermal shock. The inversion of Laplace transform has been carried out numerically by applying a method of numerical inversion of Laplace transform based on Stehfest technique (Stehfest, 1970). Numerical results for physical quantities are represented graphically.

2. The Governing equation

The heat conduction equation takes the form, (El-Karamany and Ezzat, 2011; Ezzat, 2011a),

$$\left(K_{ij}T_{,j}\right)_{,i} = \left(\frac{\partial}{\partial t} + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}}\right)(\rho c_e T + \gamma T_0 e), \quad 0 < \alpha \leq 1. \tag{1}$$

The equations of motion without body force take the form

$$\sigma_{ij,j} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{2}$$

.

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma(T - T_0)]\delta_{ij}, \tag{3}$$

where T is the temperature; λ, μ are Lame's constants; T_0 is the reference temperature; α is the fractional parameter; c_e is the specific heat at constant strain; K_{ij} is the thermal conductivity; ρ is the density of the medium; τ_o is the thermal relaxation time; σ_{ij} are the components of stress tensor; t is the time; δ_{ij} is the Kronecker delta symbol; α_t is the coefficient of linear thermal expansion; u_i are the displacement vector components and e_{ij} are the components of strain tensor.

Now, we suppose elastic and homogenous infinite body with spherical cavity occupying the region $a \leq r < \infty$. Because of the symmetry, all the state functions can be expressed in terms of the space variable r and the time variable t. In a spherical co-ordinate system (r, ϕ, ψ), the displacement components have the form

$$u_r = u(r,t), \quad u_\phi = u_\psi = 0. \tag{4}$$

The strain–displacement relations are

$$e_{rr} = \frac{\partial u}{\partial r}, \ e_{\phi\phi} = \frac{u}{r}, \ e_{\psi\psi} = \frac{u}{r}, \ e_{r\phi} = e_{r\psi} = e_{\phi\psi} = 0, \tag{5}$$

$$e = \frac{\partial u}{\partial r} + 2\frac{u}{r}. \tag{6}$$

Thus, the stress–strain relations are

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) - \gamma(T - T_0), \tag{7}$$

$$\sigma_{\phi\phi} = 2\mu\frac{u}{r} + \lambda\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) - \gamma(T - T_0), \tag{8}$$

$$\sigma_{\psi\psi} = 2\mu\frac{u}{r} + \lambda\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) - \gamma(T - T_0). \tag{9}$$

The equation of motion and energy equation have the form:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\phi\phi} - \sigma_{\psi\psi}\right) = \rho\frac{\partial^2 u}{\partial t^2}, \tag{10}$$

$$K\frac{1}{r_2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \left(\frac{\partial}{\partial t} + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}}\right)\left(\rho c_e T + \gamma T_0\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right)\right) \tag{11}$$

For simplicity, we will use the following non-dimensional variables (Othman and Abbas, 2012).

$$(r', u') = \frac{(r,u)}{c\chi}, \ (t', \tau_o') = \frac{(t, \tau_o)}{\chi}, \ (\sigma'_{rr}, \sigma'_{\phi\phi}, \sigma'_{\psi\psi}) = \frac{1}{\lambda + 2\mu}(\sigma_{rr}, \sigma_{\phi\phi}, \sigma_{\psi\psi}), \ T' = \frac{\gamma(T - T_0)}{\lambda + 2\mu}, \tag{12}$$

where, $c^2 = \frac{\lambda+2\mu}{\rho}, \ \chi = \frac{K}{\rho c_e c^2}$.

From Eq. (12) into Eqs. (7)–(11) one may obtain (after dropping the superscript $'$ for convenience)

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2u}{r^2} - \frac{\partial T}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} = \left(\frac{\partial}{\partial t} + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}}\right)\left(T + \varepsilon\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right)\right), \tag{14}$$

$$\sigma_{rr} = \frac{\partial u}{\partial r} + 2\beta\frac{u}{r} - T, \tag{15}$$

$$\sigma_{\phi\phi} = \sigma_{\psi\psi} = \beta\frac{\partial u}{\partial r} + (1 + \beta)\frac{u}{r} - T, \tag{16}$$

where $\beta = \frac{\lambda}{\lambda+2\mu}, \ \varepsilon = \frac{\gamma T_0}{\rho^2 c^2 c_e}$.

From preceding description, we assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have

$$u(r,0) = \frac{\partial u(r,0)}{\partial t} = 0, \ T(r,0) = \frac{\partial T(r,0)}{\partial t} = 0. \tag{17}$$

The boundary conditions may be expressed as

$$\sigma_{rr}(a,t) = 0, \ T(a,t) = H(t), \quad \sigma_{rr}(r,t)|_{r\to\infty} = 0, \ T(r,t)|_{r\to\infty} = 0, \tag{18}$$

where $H(t)$ is the Heaviside unit step function.

3. Laplace Transform domain

Applying the Laplace transform define by the formula

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}dt. \tag{19}$$

Hence, the Eqs. (13)–(18) take the form

$$\frac{d^2\bar{u}}{dr^2} + \frac{2}{r}\frac{d\bar{u}}{dr} - \frac{2\bar{u}}{r^2} - \frac{d\bar{T}}{dr} = s^2\bar{u}, \tag{20}$$

$$\frac{d^2\bar{T}}{dr^2} + \frac{2}{r}\frac{d\bar{T}}{dr} = \left(s + s^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\left(\bar{T} + \varepsilon\left(\frac{d\bar{u}}{dr} + \frac{2\bar{u}}{r}\right)\right), \tag{21}$$

$$\bar{\sigma}_{rr} = \frac{d\bar{u}}{\partial r} + 2\beta\frac{\bar{u}}{r} - \bar{T}, \tag{22}$$

$$\bar{\sigma}_{\phi\phi} = \bar{\sigma}_{\psi\psi} = \beta\frac{d\bar{u}}{dr} + (1+\beta)\frac{\bar{u}}{r} - \bar{T}, \tag{23}$$

$$\bar{\sigma}_{rr}(a,s) = 0, \bar{T}(a,s) = \frac{1}{s}, \vec{\sigma}_{rr}(r,s)|_{r\to\infty} = 0, \bar{T}(r,s)|_{r\to\infty} = 0. \tag{24}$$

Using Eq. (20) with the differentiating Eq. (21) with respect to r we get

$$\frac{d^2}{dr^2}\left(\frac{d\bar{T}}{dr}\right) + \frac{2}{r}\frac{d}{dr}\left(\frac{d\bar{T}}{dr}\right) - \frac{2}{r^2}\left(\frac{d\bar{T}}{dr}\right) = \left(s + s^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\left(s^2\bar{u} + (1+\varepsilon)\frac{d\bar{T}}{dr}\right). \tag{25}$$

Eqs. (20) and (25) can be written in a vector–matrix differential equation as follows

$$L\vec{V} = A\vec{V}, \tag{26}$$

where $L \equiv \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2}{r^2}$, $\vec{V} = [\bar{u}\,\frac{d\bar{T}}{dr}]^T$ and $A = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$,

and $M_{11} = s^2$, $M_{12} = 1$, $M_{21} = s^2\left(s + s^{1+\alpha}\frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\right)$, $M_{22} = (1+\beta\varepsilon)\left(s + s^{1+\alpha}\frac{\tau_o^\alpha}{\alpha!}\right)$.

3.1. Solution of the vector–matrix differential equation

The using of eigenvalue approach (Das et al., 1997) to solve the Eq. (26), the form of characteristic equation of the matrix A as follows

$$M_{11}M_{22} - M_{12}M_{21} - (M_{22} + M_{11})\lambda + \lambda^2 = 0 \tag{27}$$

where $\lambda = \lambda_1$, $\lambda = \lambda_2$, are the roots of the characteristic Eq. (27). The eigenvector $\vec{X} = [x_1, x_2]^T$, corresponding to eigenvalue λ can be calculated as:

$$x_1 = M_{12}, \ , x_2 = \lambda - M_{11}. \tag{28}$$

For further reference, we shall use the following notations:

$$\vec{X}_1 = [\vec{X}]_{\lambda=\lambda_1}, \ \vec{X}_2 = [\vec{X}]_{\lambda=\lambda_2}. \tag{29}$$

Thus, the solution of Eq. (26) takes the form:

$$\vec{V} = r^{-1/2}\vec{X}_1\left(A_1 I_{3/2}(m_1 r) + A_2 K_{3/2}(m_1 r)\right) + r^{-1/2}\vec{X}_2\left(A_3 I_{3/2}(m_2 r) + A_4 K_{3/2}(m_2 r)\right), \tag{30}$$

where A_1, A_2, A_3, A_4 are constants to be determined from the boundary condition of the problem, $m_1 = \sqrt{\lambda_1}, m_2 = \sqrt{\lambda_2}$, and $I_{3/2}, K_{3/2}$ are the modified of Bessel's functions with order 3/2.

Thus, the field variables can be written for r and s as:

$$\bar{u}(r,s) = r^{-1/2}x_1^1\left(A_1 I_{3/2}(m_1 r) + A_2 K_{3/2}(m_1 r)\right) + r^{-1/2}x_1^2\left(A_3 I_{3/2}(m_2 r) + A_4 K_{3/2}(m_2 r)\right), \tag{31}$$

$$\bar{T}(r,s) = \frac{r^{-1/2}x_2^1}{m_1}\left(A_1 I_{1/2}(m_1 r) - A_2 K_{1/2}(m_1 r)\right) + \frac{r^{-1/2}x_2^2}{m_2}\left(A_3 I_{1/2}(m_2 r) - A_4 K_{1/2}(m_2 r)\right), \tag{32}$$

where x_i^j is the component number i of the eigenvector number j. To complete the solution we have to know the constants A_1, A_2, A_3, and A_4, by using the boundary conditions Eq. (24).

3.2. Numerical inversion of the Laplace transforms

The Stehfest method (Stehfest, 1970) is used in time domain for the final solution of displacement, temperature and stress distributions. In this method, the inverse $f(t)$ of the Laplace transform $f(s)$ is approximated by the relation

$$f(t) = \frac{\ln 2}{t}\sum_{j=1}^{N} V_j F\left(\frac{\ln 2}{t} j\right), \tag{33}$$

where V_j is given by the following equation:

$$V_i = (-1)^{\left(\frac{N}{2}+1\right)} \sum_{k=\frac{i+1}{2}}^{\min\left(i,\frac{N}{2}\right)} \frac{k^{\left(\frac{N}{2}+1\right)}(2k)!}{\left(\frac{N}{2}-k\right)!k!(i-k)!(2k-1)!}. \tag{34}$$

The parameter N is the number of terms used in the summation in Eq. (33). Thus, the solutions of all variables in physical space–time domain are given by

$$u(r,t) = \frac{\ln 2}{t}\sum_{i=1}^{N} V_i \bar{u}\left(r, \frac{\ln 2}{t} i\right), \tag{35}$$

$$T(r,t) = \frac{\ln 2}{t}\sum_{i=1}^{N} V_i \bar{T}\left(r, \frac{\ln 2}{t} i\right) \tag{36}$$

4. Numerical results and discussion

The copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows (Othman and Abbas, 2012)

$\lambda = 7.76 \times 10^{10}(\text{kg})(\text{m})^{-1}(\text{s})^{-2}$, $\mu = 3.86 \times 10^{10}(\text{kg})(\text{m})^{-1}(\text{s})^{-2}$, $T_0 = 293(\text{K})$,

$\text{K} = 3.68 \times 10^2(\text{kg})(\text{m})(\text{K})^{-1}(s)^{-3}$,
$c_e = 3.831 \times 10^2(\text{m})^2(\text{K})^{-1}(\text{s})^{-2}$, $\tau_o = 0.02$,

$\rho_o = 8.954 \times 10^3(\text{kg})(\text{m})^{-3}$, $\alpha_t = 17.8 \times 10^{-6}(\text{K})^{-1}$, $a = 1$.

Numerical calculations are carried out for the temperature, the displacement, the radial and hoop stress distributions along the r-direction in the context of fractional order thermoelasticity theory ($\alpha = 0.1, \alpha = 0.3$) and Lord-Shulman theory (LS at$\alpha = 1$). The computation was performed for one value of time, namely for $t = 0.2$. From Figs. 1–4, it is seen that the temperature starts with its maximum value at the origin and decreases until attaining zero beyond a wave front for the generalized theory, which agree with the boundary conditions. It is noticed that the temperature T decreases with increasing the distance and decreasing the value of fractional parameter α. Fig. 2 shows the displacement distribution u with redial distance r for different values of α, and it is seen that the magnitude of displacement component u increases with the decrease in the value of fractional parameter α. In all cases, (*i.e.*, $\alpha = 0.1, \alpha = 0.3$ *and* LS) the displacement component attains maximum negative values and gradually increases until

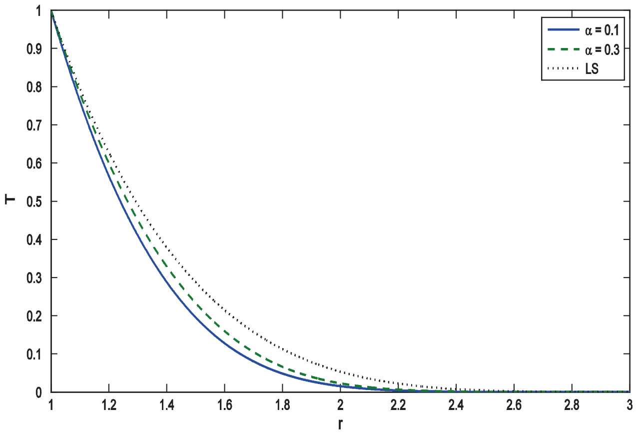

Figure 1 The temperature distribution T with distance r at different values of α.

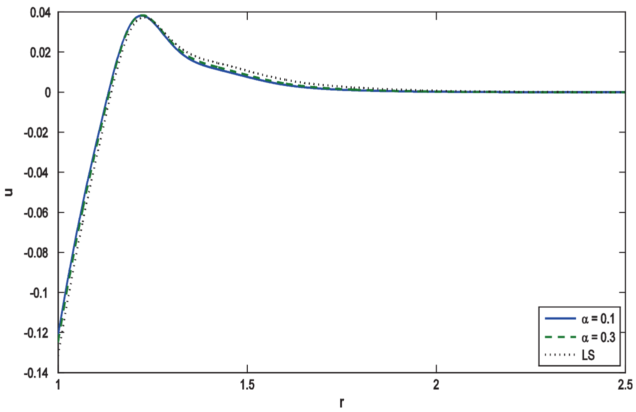

Figure 2 The displacement distribution u with distance r at different values of α.

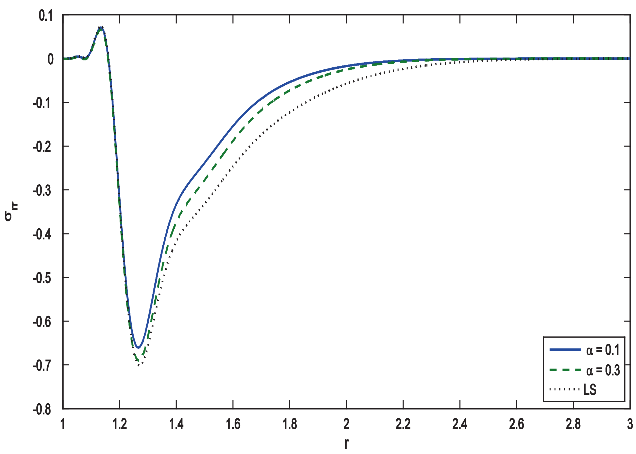

Figure 3 The radial stress distribution σ_{rr} with distance r at different values of α.

it attains a peak value at a particular location in close proximity to the inner surface of cavity and then continuously decreases to zero. Fig. 3 displays the variation of redial stress with redial distance r for different theories and it is noticed

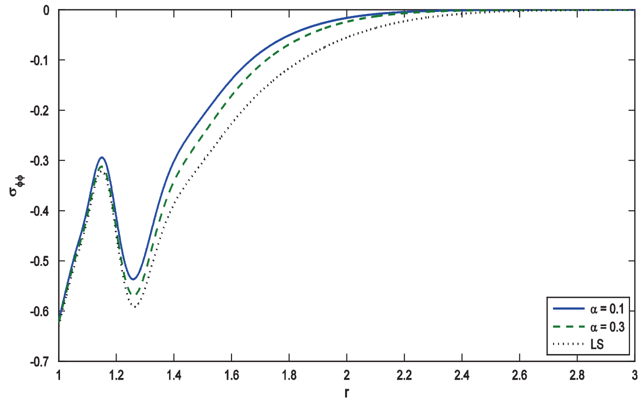

Figure 4 The hoop stress distribution $\sigma_{\phi\phi}$ with distance r at different values of α.

that the stress, always starts from the zero value and terminates at the zero value to obey the boundary conditions. Fig. 4 shows the variation of hoop stress with respect to radial distance r and it is noticed that the hoop stress has a maximum magnitude at the boundary. It is noticed that the absolute value of stresses decreases with the decrease in the value of fractional parameter α.

Finally, we have noticed that the fractional order has a small effect in the displacement while it has a great effect on the distribution of the other field quantities.

5. Conclusions

The problem of investigating the temperature, displacement, and stresses in an infinitely body containing spherical cavity under generalized thermoelasticity theory with fractional order derivative. The inner surface of the cavity is subjected to a thermal shock with the traction free. According to this work, the fractional parameter effect plays a significant role on all distributions. Thus, we can consider the generalized thermoelasticity with fractional order derivative as an improvement on studying elastic materials.

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